

Directions: Sketch the angle in standard position related to the point (label this angle θ)

Label the reference angle (with REF)

Label the related QUADRANT 1 triangle

Determine the measure of angle θ and the measure of REF

State the related basic six trigonometric ratios BE CAREFUL WITH SIGNS

Point A (12,16)

Related QUADRANT 1 triangle

Work for the measure of angle REF

$$\sin^{-1}\left(\frac{16}{20}\right)$$

$$\cos^{-1}\left(\frac{12}{20}\right) = 53.1^\circ$$

$$\tan^{-1}\left(\frac{16}{12}\right)$$

Work for the measure of angle θ

same as REF

Work for hypotenuse

$$\sqrt{12^2 + 16^2} = \sqrt{400} = 20$$

$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\frac{\tan \theta}{\cot \theta}$
$\frac{16}{20} = \frac{4}{5}$	$\frac{12}{20} = \frac{3}{5}$	$\frac{16}{12} = \frac{4}{3}$	$\frac{20}{16} = \frac{5}{4}$	$\frac{20}{12} = \frac{5}{3}$	$\frac{12}{16} = \frac{3}{4}$

Point B (-20,21)

Related QUADRANT 1 triangle

Work for the measure of angle REF

$$\sin^{-1}\left(\frac{21}{29}\right)$$

$$\cos^{-1}\left(\frac{20}{29}\right) = 46.4^\circ$$

$$\tan^{-1}\left(\frac{21}{20}\right)$$

Work for the measure of angle θ

$180 - \text{ref } \angle$

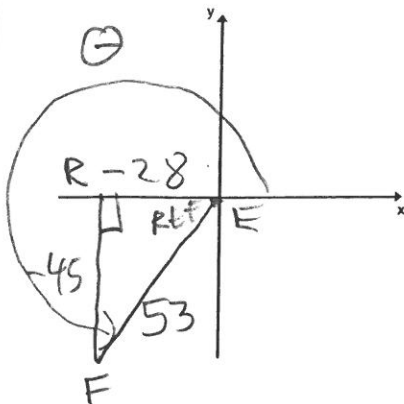
$$180 - 46.4 = 133.6^\circ$$

Work for hypotenuse

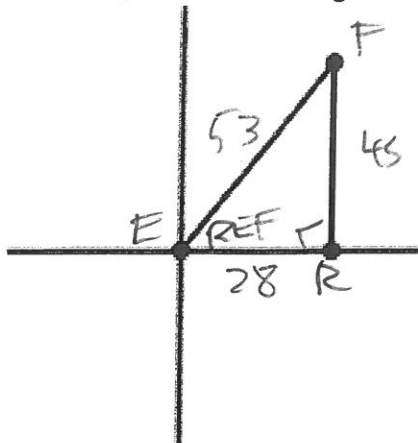
$$\sqrt{21^2 + 20^2} = \sqrt{841} = 29$$

$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\frac{\tan \theta}{\cot \theta}$
$\frac{21}{29}$	$-\frac{20}{29}$	$-\frac{21}{20}$	$\frac{29}{21}$	$-\frac{29}{20}$	$-\frac{20}{21}$

Point C (-28,-45)



Related QUADRANT 1 triangle



Work for the measure of angle REF

$$\sin^{-1}\left(\frac{45}{53}\right)$$

$$\cos^{-1}\left(\frac{28}{53}\right) = 58.1^\circ$$

$$\tan^{-1}\left(\frac{45}{28}\right)$$

Work for the measure of angle θ

$$180 + \text{REF } \angle$$

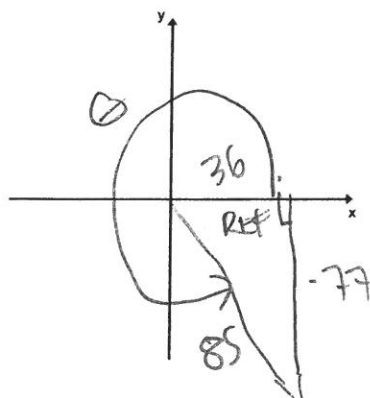
$$180 + 58.1 = 238.1^\circ$$

Work for hypotenuse

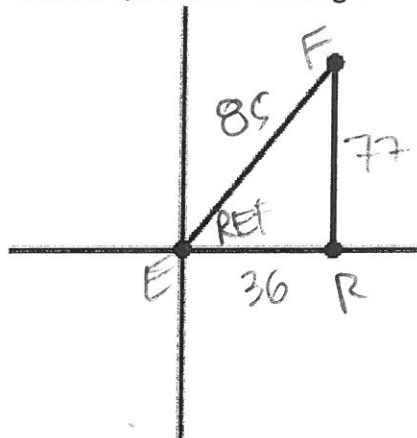
$$\sqrt{28^2 + 45^2} = \sqrt{2809} = 53$$

$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\tan \theta$ $\cot \theta$
$-\frac{45}{53}$	$-\frac{28}{53}$	$+\frac{45}{28}$	$-\frac{53}{45}$	$-\frac{53}{28}$	$\frac{28}{45}$

Point D (36,-77)



Related QUADRANT 1 triangle



Work for the measure of angle REF

$$\sin^{-1}\left(\frac{77}{85}\right)$$

$$\cos^{-1}\left(\frac{36}{85}\right) = 64.9^\circ$$

$$\tan^{-1}\left(\frac{77}{36}\right)$$

Work for the measure of angle θ

$$360 - \text{ref } \angle$$

$$360 - 64.9 = 295.1^\circ$$

Work for hypotenuse

$$\sqrt{77^2 + 36^2} = \sqrt{7225} = 85$$

$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\tan \theta$ $\cot \theta$
$-\frac{77}{85}$	$\frac{36}{85}$	$-\frac{77}{36}$	$-\frac{85}{77}$	$\frac{85}{36}$	$-\frac{36}{77}$

Directions: State the EXACT VALUES of each of the trigonometric expressions WITHOUT A CALCULATOR

SHOW HOW YOU ARRIVED AT THE GIVEN RATIO

$\csc\left(\frac{\pi}{6}\right)$	$\sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$	$\cot\left(\frac{5\pi}{6}\right) = \frac{1}{\tan\left(\frac{5\pi}{6}\right)} = \frac{1}{-\frac{\sqrt{3}}{3}} = -\frac{3}{\sqrt{3}} = -\sqrt{3}$
$\csc\left(\frac{19\pi}{6}\right) = \csc(570^\circ) = \csc(210^\circ) = \frac{1}{\sin 210^\circ} = \frac{1}{-\frac{1}{2}} = -2$	$\sec\left(\frac{15\pi}{4}\right) = \sec(675^\circ) = \sec(315^\circ) = \frac{1}{\cos(315^\circ)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$	$\cot\left(\frac{51\pi}{6}\right) = \cot(1530^\circ) = \cot(90^\circ) = \frac{1}{\tan 90^\circ} = \frac{\cos 90^\circ}{\sin 90^\circ} = \frac{0}{1} = 0$
$\csc(\pi) = \frac{1}{\sin \pi} = \frac{1}{\sin 180^\circ} = \frac{1}{0}$ $\csc \pi = \text{undefined}$	$\sec\left(\frac{3\pi}{2}\right) = \frac{1}{\cos\left(\frac{3\pi}{2}\right)} = \frac{1}{\cos(270^\circ)} = \frac{1}{0}$ $\sec\left(\frac{3\pi}{2}\right) = \text{undefined}$	$\cot\left(\frac{\pi}{2}\right) = \frac{1}{\tan\left(\frac{\pi}{2}\right)} = \frac{1}{\text{undefined}} = \frac{1}{\left(\frac{1}{0}\right)} = 0$ $\cot \frac{\pi}{2} = \frac{\cos \frac{\pi}{2}}{\sin \frac{\pi}{2}} = \frac{0}{1} = 0$

Directions: State when each of the trigonometric functions is 0, 1 and undefined IF POSSIBLE (use degrees and radians)

function	This function is equal to 0 when angle is _____	This function is equal to 1 when angle is _____	This function is equal to undefined when angle is _____
Sine	$x=0 \quad x=180 \quad x=360$	$x=90$	never
Cosine	$x=90 \quad x=270$	$x=0 \quad x=360$	never
Tangent	$x=0 \quad x=180 \quad x=360$	$x=45 \quad x=225$	$x=90 \quad x=270$
Cosecant	never	$x=90$	$x=0 \quad x=180 \quad x=360$
Secant	never	$x=0 \quad x=360$	$x=90 \quad x=270$
Cotangent	$x=90 \quad x=270$	$x=45 \quad x=225$	$x=0 \quad x=180 \quad x=360$

Directions: Determine which quadrant that θ lies in (EXPLAIN WHY you made your selection)

Given conditions	θ lies in this quadrant	REASON FOR SELECTION
$\sin \theta < 0$ and $\cos \theta > 0$	4	$\begin{array}{c} \sin \\ \hline - \\ + \end{array}$ $\begin{array}{c} \cos \\ \hline + \\ + \end{array}$ Both $\begin{array}{c} + \\ \hline - \\ + \end{array}$
$\cot \theta < 0$ and $\sec \theta > 0$	4	$\begin{array}{c} \cot \\ \hline - \\ - \end{array}$ $\begin{array}{c} \sec \theta \\ \hline + \\ + \end{array}$ both $\begin{array}{c} + \\ \hline - \\ + \end{array}$
$\sin \theta < 0$ and $\tan \theta = 1$	3	$\begin{array}{c} \sin \\ \hline - \\ - \end{array}$ $\begin{array}{c} \tan \\ \hline + \\ + \end{array}$ $\begin{array}{c} + \\ \hline - \\ - \end{array}$
$\cot \theta = -1$ and $\cos \theta > 0$	4	$\begin{array}{c} \cot \\ \hline - \\ - \end{array}$ $\begin{array}{c} \cos \\ \hline + \\ + \end{array}$ Both $\begin{array}{c} - \\ \hline + \\ + \end{array}$

Directions: find the values of the specific trigonometric functions of θ based on the given constraints SHOW WORK

Function value	Constraint	Desired value
$\sin \theta = \frac{-7}{8}$	θ lies in quadrant 4	$\cot \theta$ $\sqrt{64-49} = \sqrt{15}$ $\cot \theta = \frac{-7}{\sqrt{15}}$
$\cos \theta = \frac{-1}{8}$	$90^\circ \leq \theta \leq 180^\circ$	$\csc \theta$ $\sqrt{63}$ $\csc \theta = \frac{8}{\sqrt{63}} = \frac{8}{3\sqrt{7}} = \frac{8\sqrt{7}}{21}$
$\tan \theta = \frac{-7}{2}$	$\sin \theta > 0$	$\cos \theta$ $\sqrt{49+4} = \sqrt{53}$ $\cos \theta = \frac{-2}{\sqrt{53}} = \frac{-2\sqrt{53}}{53}$
$\cot \theta = \frac{8}{7}$	$\cos \theta < 0$	$\sec \theta$ $\sqrt{113}$ $\sec \theta = \frac{\sqrt{113}}{-8} = -\frac{\sqrt{113}}{8}$
$\sec \theta = \frac{9}{4}$	$270^\circ \leq \theta \leq 360^\circ$	$\csc \theta$ $\sqrt{65}$ $\csc \theta = -\frac{9}{\sqrt{65}} = -\frac{9\sqrt{65}}{65}$
$\csc \theta = \frac{10}{5}$	$\frac{3\pi}{2} \leq \theta \leq 2\pi$	$\csc \theta$ impossible $\csc \theta < 0$ in Q4

Q4