#27 p535

Solution to p535

#27 cot x sec x

Related Trigonometric Identities

1) Quotient Identity $\cot x = \frac{\cos x}{\sin x}$

2) Reciprocal Identity $\cot x = \frac{1}{\tan x}$

3) Reciprocal Identity $\sec x = \frac{1}{\cos x}$

4) Reciprocal Identity $\csc x = \frac{1}{\sin x}$

Given cot x sec x

$\cot x \sec x = \left(\frac{\cos x}{\sin x}\right)(\sec x)$ by Quotient Id

$= \left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\cos x}\right)$ by Reciprocal Id

$= \left(\frac{\cos x}{\cos x}\right)\left(\frac{1}{\sin x}\right)$ by comm prop x

$= (1)\left(\frac{1}{\sin x}\right)$ by Multiplicative Id

$= \csc x$ by Reciprocal Id

#28 p535

Solution to p535

#28 sin

Related Trigonometric Identities

1) Quotient Identity $\tan x = \frac{\sin x}{\cos x}$

Given cos x tan x

$\cos x \tan x = (\cos x)\left(\frac{\sin x}{\cos x}\right)$ by Quotient Id

$= \left(\frac{\cos x}{\cos x}\right)(\sin x)$ by comm prop x

$= (1)(\sin x)$ by Multiplicative Id

$= \sin x$

Solution to p535

#29 sin x(csc x− sin x)

Related Trigonometric Identities

1) Reciprocal Identity $\csc x = \frac{1}{\sin x}$

2) cosentities

$\sin^2 x+\cos^2 x=1$

$1-\sin^2 x=\cos^2 x$

$1-\cos^2 x=\sin^2 x$

Given sin x(csc x− sin x)

$\sin x(\csc x - \sin x) = \sin x(\csc x) - \sin^2 x$

by dist prop

$= \sin x\left(\frac{1}{\sin x}\right) - \sin^2 x$ by Reciprocal Id

$= \left(\frac{\sin x}{\sin x}\right) - \sin^2 x$ by algebra

$= 1 - \sin^2 x$ by Multiplicative Id

$= \cos^2 x$ by Pythagorean Id

Solution to p535

#29 sin x(csc x− sin x)

Method 2

Related Trigonometric Identities

1) Reciprocal Identity $\csc x = \frac{1}{\sin x}$

2) Pythagorean Identities

$\sin^2 x+\cos^2 x=1$

$1-\sin^2 x=\cos^2 x$

$1-\cos^2 x=\sin^2 x$

Given sin x(csc x− sin x)

$\sin x(\csc x - \sin x) = \sin x\left(\frac{1}{\sin x} - \sin x\right)$

by Reciprocal Id

$= \sin x\left(\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}\right)$ by algebra

$= \sin x\left(\frac{1-\sin^2 x}{\sin x}\right)$ by algebra

$= \sin x\left(\frac{\cos^2 x}{\sin x}\right)$ by Pythagorean Id

$= \left(\frac{\sin x}{\sin x}\right)\left(\cos^2 x\right)$ by comm prop x

$= 1\ (\cos^2 x)$ by Multiplicative Id

$= \cos^2 x$

Solution to p535

#30 $\sec^2 x(1-\sin^2 x)$

Related Trigonometric Identities

1) Reciprocal Identity $\sec x = \frac{1}{\cos x}$

2) Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $\quad 1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\quad \cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $\quad 1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #30 $\sec^2 x(1-\sin^2 x)$

$\sec^2 x(1-\sin^2 x) = \sec^2 x(\cos^2 x)$ by Pythagorean Id

$= \left(\frac{1}{\cos^2 x}\right)(\cos^2 x)$ by Reciprocal Id

$= \left(\frac{\cos^2 x}{\cos^2 x}\right)$ by algebra

$= 1$ by Multiplicative Id

Solution to p535

#30 $\sec^2 x(1-\sin^2 x)$

Related Trigonometric Identities

1) Reciprocal Identity $\sec x = \frac{1}{\cos x}$

2) Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $\quad 1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\quad \cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $\quad 1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

3) Quotient Identity

$\tan x = \frac{\sin x}{\cos x}$

Given #30 $\sec^2 x(1-\sin^2 x)$

$\sec^2 x(1-\sin^2 x) = \sec^2 x-\sec^2 x(\sin^2 x)$

by distributive prop

$= \sec^2 x-\left(\frac{1}{\cos^2 x}\right)(\sin^2 x)$ by Reciprocal Id

$= \sec^2 x-\left(\frac{\sin^2 x}{\cos^2 x}\right)$ by algebra

$= \sec^2 x-\tan^2 x$ by Quotient Id

$= 1$ by Pythagorean Id

#31 p535

Solution to p535

#31 $\frac{\cot x}{\csc x}$

Related Trigonometric Identities

1) Quotient Identity $\cot x = \frac{\cos x}{\sin x}$

2) Reciprocal Identity $\sin x = \frac{1}{\csc x}$

Given $\frac{\cot x}{\csc x}$

$\frac{\cot x}{\csc x} = \left(\frac{\cot x}{1}\right)\left(\frac{1}{\csc x}\right)$ by algebra

$= \left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\csc x}\right)$ by Quotient Id

$= \left(\frac{\cos x}{\sin x}\right)\left(\frac{\sin x}{1}\right)$ by Reciprocal Id

$= \left(\frac{\sin x}{\sin x}\right)(\cos x)$ by comm prop x

$= (1)(\cos x)$ by Multiplicative Id

$\frac{\cot x}{\csc x} = \cos x$

#32 p535

Solution to p535

#32 $\frac{\csc x}{\sec x}$

Related Trigonometric Identities

1) Quotient Identity $\cot x = \frac{\cos x}{\sin x}$

2) Reciprocal Identity $\csc x = \frac{1}{\sin x}$

2) Reciprocal Identity $\cos x = \frac{1}{\sec x}$

Given $\frac{\csc x}{\sec x}$

$\frac{\csc x}{\sec x} = \left(\frac{\csc x}{1}\right)\left(\frac{1}{\sec x}\right)$ by algebra

$= \left(\frac{1}{\sin x}\right)\left(\frac{1}{\sec x}\right)$ by Reciprocal Id

$= \left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{1}\right)$ by Reciprocal Id

$= \left(\frac{\cos x}{\sin x}\right)$ by algebra

$= \cot x$ by Quotient Id

$\frac{\csc x}{\sec x} = \cot x$

#33 p535

Solution to p535

#33 $\frac{1-\sin^2 x}{\csc^2 x-1}$

Related Trigonometric Identities

1) Reciprocal Identity $\tan x = \frac{1}{\cot x}$

2) Quotient Identity $\tan x = \frac{\sin x}{\cos x}$

3) Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

$\frac{1-\sin^2 x}{\csc^2 x-1} = \frac{\cos^2 x}{\csc^2 x-1}$ by Pythagorean Id

$= \frac{\cos^2 x}{\cot^2 x}$ by Pythagorean Id

$= \left(\frac{\cos^2 x}{1}\right)\left(\frac{1}{\cot^2 x}\right)$ by algebra

$= \left(\frac{\cos^2 x}{1}\right)\left(\frac{\tan^2 x}{1}\right)$ by Reciprocal ID

$= \left(\frac{\cos^2 x}{1}\right)\left(\frac{\sin^2 x}{\cos^2 x}\right)$ by Quotient ID

$= \left(\frac{\cos^2 x}{\cos^2 x}\right)\left(\sin^2 x\right)$ by algebra

$= 1\left(\sin^2 x\right)$ by Multiplicative ID

$\frac{1-\sin^2 x}{\csc^2 x-1} = \sin^2 x$

#34 p535

Solution to p535

#34 $\frac{1}{\tan^2 x+1}$

Related Trigonometric Identities

1) Reciprocal Identity $\cos x = \frac{1}{\sec x}$

2) Quotient Identity $\tan x = \frac{\sin x}{\cos x}$

3) Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given $\frac{1}{\tan^2 x+1}$

$\frac{1}{\tan^2 x+1} = \frac{1}{\sec^2 x}$ by Pythagorean Id

$= \cos^2 x$ by Reciprocal ID

#35 p535

Solution to p535

#35 $(\sec x)\left(\frac{\sin x}{\tan x}\right)$

Related Trigonometric Identities

1) Reciprocal Identity $\sec x = \frac{1}{\cos x}$

2) Quotient Identity $\tan x = \frac{\sin x}{\cos x}$

Given #35 $(\sec x)\left(\frac{\sin x}{\tan x}\right)$

$(\sec x)\left(\frac{\sin x}{\tan x}\right) = \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\tan x}\right)$ by Reciprocal ID

$= \left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\tan x}\right)$ by comm prop x

$= (\tan x)\left(\frac{1}{\tan x}\right)$ by Quotient ID

$= \frac{\tan x}{\tan x}$ by algebra

$= 1$ by Multiplicative ID

$(\sec x)\left(\frac{\sin x}{\tan x}\right) = 1$

#36 p535

Solution to p535

#36 $\left(\frac{\tan^2 x}{\sec^2 x}\right)$

Related Trigonometric Identities

1) Reciprocal Identity $\cos x = \frac{1}{\sec x}$

2) Quotient Identity $\tan x = \frac{\sin x}{\cos x}$

Given #36 $\left(\frac{\tan^2 x}{\sec^2 x}\right)$

$\left(\frac{\tan^2 x}{\sec^2 x}\right) = \left(\frac{\tan^2 x}{1}\right)\left(\frac{1}{\sec^2 x}\right)$ by algebra

$= \left(\frac{\sin^2 x}{\cos^2 x}\right)\left(\frac{1}{\sec^2 x}\right)$ by Quotient ID

$= \left(\frac{\sin^2 x}{\cos^2 x}\right)\left(\frac{\cos^2 x}{1}\right)$ by Reciprocal ID

$= \left(\frac{\cos^2 x}{\cos^2 x}\right)\left(\frac{\sin^2 x}{1}\right)$ by comm prop x

$= 1\left(\sin^2 x\right)$ by Multiplicative ID

$\left(\frac{\tan^2 x}{\sec^2 x}\right) = \sin^2 x$

#37 p535

Solution to p535

#37 $\cos\left(\frac{\pi}{2}-x\right)\cdot\sec x$

Related Trigonometric Identities

1) Reciprocal Identity sec x = $\frac{1}{\cos x}$

2) Cofunction Identity

$\cos\left(\frac{\pi}{2}-x\right)=\sin x$

3) Quotient Identity tan x $=\frac{\sin x}{\cos x}$

Given #37 $\cos\left(\frac{\pi}{2}-x\right)\cdot\sec x$

$\cos\left(\frac{\pi}{2}-x\right)\cdot\sec x = \sin x(\sec x)$ by Cofunction ID

$= \sin x\left(\frac{1}{\cos x}\right)$ by Reciprocal ID

$= \left(\frac{\sin x}{\cos x}\right)$ by algebra

$= \tan x$ by Quotient ID

$\cos\left(\frac{\pi}{2}-x\right)\cdot\sec x = \tan x$

#38 p535

Solution to p535

#38 $\cot\left(\frac{\pi}{2}-x\right)\cdot\cos x$

Related Trigonometric Identities

1) Reciprocal Identity sec x = $\frac{1}{\cos x}$

2) Cofunction Identity

$\cot\left(\frac{\pi}{2}-x\right)=\tan x$

#38 $\cot\left(\frac{\pi}{2}-x\right)\cdot\cos x$

$\cot\left(\frac{\pi}{2}-x\right)\cdot\cos x = \tan x(\cos x)$ by Cofunction ID

$= \frac{\sin x}{\cos x}(\cos x)$ by QuotientID

$= \sin x\cdot\left(\frac{\cos x}{\cos x}\right)$ by comm prop x

$= \sin x\cdot(1)$ by Multiplicative ID

$\cot\left(\frac{\pi}{2}-x\right)\cdot\cos x = \sin x$

#39 p535

Solution to p535

#39 $\frac{\cos^2 x}{1-\sin x}$

Related Trigonometric Identities

1) Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $\quad$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\quad$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $\quad$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #39 $\frac{\cos^2 x}{1-\sin x}$

$\frac{\cos^2 x}{1-\sin x} = \frac{1-\sin^2 x}{1-\sin x}$ by Pythagorean Id

$= \frac{(1-\sin x)(1+\sin x)}{(1-\sin x)}$ by D.O.T.S.

$= \frac{(1-\sin x)}{(1-\sin x)}(1+\sin x)$ by algebra

$= 1\cdot(1+\sin x)$ by Multiplicative ID

$\frac{\cos^2 x}{1-\sin x} = 1+\sin x$

#40 p535

Solution to p535

#40 $\cos x(1+\tan^2 x)$

Related Trigonometric Identities

1) Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $\quad$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\quad$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $\quad$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\sec x = \frac{1}{\cos x}$

Given #40 $\cos x(1+\tan^2 x)$

$\cos x(1+\tan^2 x) = \cos x(\sec^2 x)$ by Pythagorean ID

$= \cos x\left(\frac{1}{\cos^2 x}\right)$ by Reciprocal ID

$= \frac{(\cos x)}{(\cos x)}\left(\frac{1}{\cos x}\right)$ by algebra

$= 1\cdot\left(\frac{1}{\cos x}\right)$ by Multiplicative ID

$= 1\cdot(\sec x)$ by Reciprocal ID

$\cos x(1+\tan^2 x) = \sec x$

#41 p535

Solution to p535

#41 $\sin x \cdot \tan x + \cos x$

Related Trigonometric Identities

1) Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad \cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad 1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

2) Quotient Identity $\tan x = \dfrac{\sin x}{\cos x}$

Given #41 $\sin x \cdot \tan x + \cos x$

$\sin x \cdot \tan x + \cos x = \sin x \cdot \dfrac{\sin x}{\cos x} + \cos x$ by Quotient ID

$= \dfrac{\sin^2 x}{\cos x} + \cos x$ by algebra

$= \dfrac{\sin^2 x}{\cos x} + \dfrac{\cos^2 x}{1} \cdot \dfrac{\cos x}{\cos x}$ by Multiplicative ID

$= \dfrac{\sin^2 x}{\cos x} + \dfrac{\cos^2 x}{\cos x}$ by algebra

$= \dfrac{\sin^2 x + \cos^2 x}{\cos x}$ by algebra

$= \dfrac{1}{\cos x}$ by Pythagorean ID

$= \sec x$ by Reciprocal ID

$\sin x \cdot \tan x + \cos x = \sec x$

#42 p535

Solution to p535

#42 $\csc x \cdot \tan x + \sec x$

Related Trigonometric Identities

1) Reciprocal Identity $\csc x = \dfrac{1}{\sin x}$

2) Reciprocal Identity $\sec x = \dfrac{1}{\cos x}$

3) Quotient Identity $\tan x = \dfrac{\sin x}{\cos x}$

Given #42 $\csc x \cdot \tan x + \sec x$

$\csc x \cdot \tan x + \sec x = \csc x \cdot \dfrac{\sin x}{\cos x} + \sec x$ by Quotient ID

$= \dfrac{1}{\sin x} \cdot \dfrac{\sin x}{\cos x} + \sec x$ by Reciprocal ID

$= \dfrac{\sin x}{\sin x} \cdot \dfrac{1}{\cos x} + \sec x$ by comm prop x

$= 1 \cdot \dfrac{1}{\cos x} + \sec x$ by Multiplicative ID

$= 1 \cdot \sec x + 1 \cdot \sec x$ by Reciprocal ID

$= 2 \sec x$ by algebra

$\csc x \cdot \tan x + \sec x = 2 \sec x$

#43 p535

Solution to p535

#43 $\cot x \cdot \sin x + \tan x \cdot \cos x$

Related Trigonometric Identities

1) Quotient Identity $\tan x = \frac{\sin x}{\cos x}$

2) Quotient Identity $\cot x = \frac{\sin x}{\cos x}$

Given #43 $\cot x \cdot \sin x + \tan x \cdot \cos x$

$\cot x \cdot \sin x + \tan x \cdot \cos x = \frac{\cos x}{\sin x} \cdot \sin x + \tan x \cdot \cos x$ by Quotient ID

$= \frac{\cos x}{\sin x} \cdot \sin x + \frac{\sin x}{\cos x} \cdot \cos x$ by Quotient ID

$= \frac{\sin x}{\sin x} \cdot \cos x + \frac{\cos x}{\cos x} \cdot \sin x$ by comm prop x

$= 1 \cdot \cos x + 1 \cdot \sin x$ by Multiplicative ID

$\csc x \cdot \tan x + \sec x = \cos x + \sin x$

#44 p535

Solution to p535

#44 $\sin x \cdot \sec x + \cos x \cdot \csc x$

Related Trigonometric Identities

1) Quotient Identity $\tan x = \frac{\sin x}{\cos x}$

2) Quotient Identity $\cot x = \frac{\sin x}{\cos x}$

3) Reciprocal Identity $\sec x = \frac{1}{\cos x}$

4) Reciprocal Identity $\csc x = \frac{1}{\sin x}$

Given #44 $\sin x \cdot \sec x + \cos x \cdot \csc x$

$\sin x \cdot \sec x + \cos x \cdot \csc x = \sin x \cdot \frac{1}{\cos x} + \cos x \cdot \csc x$ by Reciprocal ID

$= \frac{\sin x}{\cos x} + \cos x \cdot \csc x$ by algebra

$= \tan x + \cos x \cdot \csc x$ by Quotient ID

$= \tan x + \cos x \cdot \frac{1}{\sin x}$ by Reciprocal ID

$= \tan x + \frac{\cos x}{\sin x}$ by algebra

$= \tan x + \cot x$ by Quotient ID

$\sin x \cdot \sec x + \cos x \cdot \csc x = \tan x + \cot x$

Solution to p535

#44 $\sin x \cdot \sec x + \cos x \cdot \csc x$

Related Trigonometric Identities

1) Reciprocal Identity $\sec x = \frac{1}{\cos x}$

2) Reciprocal Identity $\csc x = \frac{1}{\sin x}$

3) Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad \cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad 1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

$\sin x \cdot \sec x + \cos x \cdot \csc x = \sin x \cdot \frac{1}{\cos x} + \cos x \cdot \csc x$ by Reciprocal ID

$= \frac{\sin x}{\cos x} + \cos x \cdot \csc x$ by algebra

$= \frac{\sin x}{\cos x} + \cos x \cdot \frac{1}{\sin x}$ by Reciprocal ID

$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$ by algebra

$= \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x} + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x}$ by Multiplicative ID

$= \frac{\sin^2 x}{\cos x \cdot \sin x} + \frac{\cos^2 x}{\sin x \cdot \cos x}$ by algebra

$= \frac{\sin^2 x + \cos^2 x}{\cos x \cdot \sin x}$ by algebra

$= \frac{1}{\sin x \cdot \cos x}$ by Pythagorean ID

$= \frac{1}{\sin x} \cdot \frac{1}{\cos x}$ by algebra

$= \csc x \cdot \sec x$ by Reciprocal ID

$\sin x \cdot \sec x + \cos x \cdot \csc x = \csc x \cdot \sec x$

#45 p535

Solution to p535

#45 $\tan^2 x - \tan^2 x \cdot \sin^2 x$

Related Trigonometric Identities

1) Quotient Identity $\tan x = \frac{\sin x}{\cos x}$

2) Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad \cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad 1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

Given #45 $\tan^2 x - \tan^2 x \cdot \sin^2 x$

$\tan^2 x - \tan^2 x \cdot \sin^2 x = \tan^2 x \cdot (1 - \sin^2 x)$ by algebra

$= \tan^2 x \cdot (\cos^2 x)$ by Pythagorean ID

$= \frac{\sin^2 x}{\cos^2 x} \cdot (\cos^2 x)$ by Quotient ID

$= \frac{\cos^2 x}{\cos^2 x} \cdot (\sin^2 x)$ by comm prop x

$= 1 \cdot (\sin^2 x)$ by Multiplicative ID

$\tan^2 x - \tan^2 x \cdot \sin^2 x = \sin^2 x$

#46 p535

Solution to p535

#46 $\sin^2 x \cdot \csc^2 x - \sin^2 x$

Related Trigonometric Identities

1) Quotient Identity $\cot x = \frac{\cos x}{\sin x}$

2) Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad \cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad 1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

Given #46 $\sin^2 x \cdot \csc^2 x - \sin^2 x$

$\sin^2 x \cdot \csc^2 x - \sin^2 x = \sin^2 x \cdot (\csc^2 x - 1)$ by algebra

$= \sin^2 x \cdot (\cot^2 x)$ by Pythagorean ID

$= \sin^2 x \cdot \left(\frac{\cos^2 x}{\sin^2 x}\right)$ by Quotient ID

$= \frac{\sin^2 x}{\sin^2 x} \cdot (\cos^2 x)$ by comm prop x

$= 1 \cdot (\cos^2 x)$ by Multiplicative ID

$\sin^2 x \cdot \csc^2 x - \sin^2 x = \cos^2 x$

#47 p535

Solution to p535

#47 $\sin^2 x \cdot \sec^2 x - \sin^2 x$

Related Trigonometric Identities

1) Quotient Identity $\cot x = \frac{\cos x}{\sin x}$

2) Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad \cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad 1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

Given #47 $\sin^2 x \cdot \sec^2 x - \sin^2 x$

$\sin^2 x \cdot \sec^2 x - \sin^2 x = \sin^2 x \cdot (\sec^2 x - 1)$ by algebra

$= \sin^2 x \cdot (\tan^2 x)$ by Pythagorean ID

$\sin^2 x \cdot \csc^2 x - \sin^2 x = \sin^2 x \cdot (\tan^2 x)$

#48 p535

Solution to p535

#48 $\cos^2 x+\cos^2 x\cdot\tan^2 x$

Related Trigonometric Identities

1) Reciprocal Identity sec x $=\frac{1}{\cos x}$

2) Pythagorean Identities

$\sin^2 x+\cos^2 x=1$	$1+\cot^2 x=\csc^2 x$
$1-\sin^2 x=\cos^2 x$	$\cot^2 x=\csc^2 x-1$
$1-\cos^2 x=\sin^2 x$	$1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #48 $\cos^2 x+\cos^2 x\cdot\tan^2 x$

#48 $\cos^2 x+\cos^2 x\cdot\tan^2 x=\cos^2 x\cdot\left(1+\tan^2 x\right)$ by algebra

$=\cos^2 x\cdot\left(\sec^2 x\right)$ by Pythagorean ID

$=\cos^2 x\cdot\left(\frac{1}{\cos^2 x}\right)$ by Reciprocal ID

$=\frac{\cos^2 x\cdot}{\cos^2 x}$ by algebra

=1 by Multiplicative ID

$\cos^2 x+\cos^2 x\cdot\tan^2 x=1$

#49 p535

Solution to p535

#49 $\frac{\sec^2 x-1}{\sec x-1}$

Given #49 $\frac{\sec^2 x-1}{\sec x-1}$

#49 $\frac{\sec^2(x)-1}{\sec(x)-1}=\frac{(\sec(x)+1)(\sec(x)-1)}{\sec(x)-1}$ by DOTS

$=\frac{\sec(x)-1}{\sec(x)-1}\cdot\frac{(\sec(x)+1)}{1}$ by Comm. Prop x

$=1\cdot\frac{(\sec(x)+1)}{1}$ by Multiplicative ID

=sec (x) +1

$\frac{\sec^2 x-1}{\sec x-1}$ =sec (x) +1

#50 p535

Solution to p535

#50 $\frac{\cos^2 x-4}{\cos x-2}$

Given #50 $\frac{\cos^2 x-4}{\cos x-2}$

#50 $\frac{\cos^2 x-4}{\cos x-2}=\frac{(\cos(x)+2)(\cos(x)-2)}{\cos(x)-2}$ by DOTS

$=\frac{\cos(x)-2}{\cos(x)-2}\cdot\frac{(\cos(x)+2)}{1}$ by Comm. Prop x

$=1\cdot\frac{(\cos(x)+2)}{1}$ by Multiplicative ID

=cos (x) +2

$\frac{\cos^2 x-4}{\cos x-2}$=cos (x) +2

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Solution to p535

#51 $\tan^4 x+2\tan^2 x+1$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$	$1+\cot^2 x=\csc^2 x$
$1-\sin^2 x=\cos^2 x$	$\cot^2 x=\csc^2 x-1$
$1-\cos^2 x=\sin^2 x$	$1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #51 $\tan^4 x+2\tan^2 x+1$

#51 $\tan^4 x+2\tan^2 x+1=(\tan^2 x+1)(\tan^2 x+1)$ by PST

$=(\sec^2 x)(\sec^2 x)$ by Pythagorean ID

$=\sec^4 x$ by Algebra

$\tan^4 x+2\tan^2 x+1=\sec^4 x$

#52 p535

Solution to p535

#52 $1-2\cos^2 x+\cos^4 x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$	$1+\cot^2 x=\csc^2 x$
$1-\sin^2 x=\cos^2 x$	$\cot^2 x=\csc^2 x-1$
$1-\cos^2 x=\sin^2 x$	$1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #52 $1-2\cos^2 x+\cos^4 x$

#52 $1-2\cos^2 x+\cos^4 x$

$=\cos^4 x-2\cos^2 x+1$ by Comm prop x

$=(\cos^2 x-1)(\cos^2 x-1)$ by PST

$=(-1)(1-\cos^2 x)(-1)(1-\cos^2 x)$ by algebra

$=(-1)(\sin^2 x)(-1)(\sin^2 x)$ by Pythagorean ID

$=\sin^4 x$ by Algebra

$1-2\cos^2 x+\cos^4 x=\sin^4 x$

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Solution to p535

#53 $\sin^4 x-\cos^4 x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$	$1+\cot^2 x=\csc^2 x$
$1-\sin^2 x=\cos^2 x$	$\cot^2 x=\csc^2 x-1$
$1-\cos^2 x=\sin^2 x$	$1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #53 $\sin^4 x-\cos^4 x$

#53 $\sin^4 x-\cos^4 x$

$=(\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)$ by DOTS

$=(\sin^2 x-\cos^2 x)(1)$ by Pythagorean ID

$=\sin^2 x-\cos^2 x$ by Multiplicative ID

Now there are several methods of finishing this

$\sin^2 x-\cos^2 x=\sin^2 x-1(\cos^2 x)$

$=\sin^2 x-1(1-\sin^2 x)$

$=\sin^2 x-1+\sin^2 x$

$=2\sin^2 x-1$

There is definitely an advantage in calculus to eliminating a trigonometric function

Solution to p535

#53 $\sin^4 x - \cos^4 x$

Related Trigonometric Identities

1) Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad$ $1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad$ $\cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad$ $1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

Given #53 $\sin^4 x - \cos^4 x$

#53 $\sin^4 x - \cos^4 x$

$= (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$ by DOTS

$= (\sin^2 x - \cos^2 x)(1)$ by Pythagorean ID

$= \sin^2 x - \cos^2 x$ by Multiplicative ID

Now there are several methods of finishing this

$\sin^2 x - \cos^2 x = 1 \cdot (\sin^2 x) - \cos^2 x$

$= 1 \cdot (1 - \cos^2 x) - \cos^2 x$

$= 1 - \cos^2 x - \cos^2 x$

$= 1 - 2\cos^2 x$

There is definitely an advantage in calculus to eliminating a trigonometric function

Solution to p535

#53 $\sin^4 x - \cos^4 x$

Related Trigonometric Identities

1) Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad$ $1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad$ $\cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad$ $1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

Given #53 $\sin^4 x - \cos^4 x$

#53 $\sin^4 x - \cos^4 x$

$= (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$ by DOTS

$= (\sin^2 x - \cos^2 x)(1)$ by Pythagorean ID

$= \sin^2 x - \cos^2 x$ by Multiplicative ID

Others would say that this is enough!

Solution to p535

#53 $\sin^4 x - \cos^4 x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$	$1+\cot^2 x=\csc^2 x$
$1-\sin^2 x=\cos^2 x$	$\cot^2 x=\csc^2 x-1$
$1-\cos^2 x=\sin^2 x$	$1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #53 $\sin^4 x - \cos^4 x$

#53 $\sin^4 x - \cos^4 x$

$=(\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)$ by DOTS

$=(\sin^2 x-\cos^2 x)(1)$ by Pythagorean ID

$=\sin^2 x-\cos^2 x$ by Multiplicative ID

$=(\sin x-\cos x)(\sin x+\cos x)$ by DOTS

(others would say that this is useful)

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Solution to p535

#54 $\sec^4 x - \tan^4 x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$	$1+\cot^2 x=\csc^2 x$
$1-\sin^2 x=\cos^2 x$	$\cot^2 x=\csc^2 x-1$
$1-\cos^2 x=\sin^2 x$	$1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #54 $\sec^4 x - \tan^4 x$

#54 $\sec^4 x - \tan^4 x$

$=(\sec^2 x-\tan^2 x)(\sec^2 x+\tan^2 x)$ by DOTS

$=(1)(\sec^2 x+\tan^2 x)$ by Pythagorean ID

$=\sec^2 x+\tan^2 x$ by Multiplicative ID

Now there are several methods of finishing this

$$\begin{aligned}\sec^2 x+\tan^2 x &= \sec^2 x+1(\tan^2 x)\\ &= \sec^2 x+1(\sec^2 x-1)\\ &= \sec^2 x+\sec^2 x-1\\ &= 2\sec^2 x-1\end{aligned}$$

There is definitely an advantage in calculus to eliminating a trigonometric function

Solution to p535

#54 $\sec^4 x - \tan^4 x$

Related Trigonometric Identities

1) Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad$ $1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad$ $\cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad$ $1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

Given #54 $\sec^4 x - \tan^4 x$

#54 $\sec^4 x - \tan^4 x$

$= (\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$ by DOTS

$= (1)(\sec^2 x + \tan^2 x)$ by Pythagorean ID

$= \sec^2 x + \tan^2 x$ by Multiplicative ID

Now there are several methods of finishing this

$\sec^2 x + \tan^2 x = 1(\sec^2 x) + \tan^2 x$

$= 1(\tan^2 x + 1) + \tan^2 x$

$= \tan^2 x + 1 + \tan^2 x$

$= 2\tan^2 x + 1$

There is definitely an advantage in calculus to eliminating a trigonometric function

Solution to p535

#54 $\sec^4 x - \tan^4 x$

Related Trigonometric Identities

1) Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad$ $1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad$ $\cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad$ $1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

Given #54 $\sec^4 x - \tan^4 x$

#54 $\sec^4 x - \tan^4 x$

$= (\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$ by DOTS

$= (1)(\sec^2 x + \tan^2 x)$ by Pythagorean ID

$= \sec^2 x + \tan^2 x$ by Multiplicative ID

Others would have stopped here!

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Solution to p535

#55 $\csc^3 x - \csc^2 x - \csc x + 1$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad \cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad 1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

Given #55 $\csc^3 x - \csc^2 x - \csc x + 1$

#55 $\csc^3 x - \csc^2 x - \csc x + 1$

$= \csc^2 x(\csc x - 1) - \csc x + 1$ by Distributive Prop

$= \csc^2 x(\csc x - 1) - 1(\csc x - 1)$ by Distributive Prop

$= (\csc x - 1)(\csc^2 x - 1)$ by Grouping

$= (\csc x - 1)(\cot^2 x)$ by Pythagorean ID

Now any additional steps would probably make this more complicated than the original

#56 p535

Solution to p535

#56 $\sec^3 x - \sec^2 x - \sec x + 1$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad \cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad 1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

Given #55 $\sec^3 x - \sec^2 x - \sec x + 1$

#55 $\sec^3 x - \sec^2 x - \sec x + 1$

$= \sec^2 x(\sec x - 1) - \sec x + 1$ by Distributive Prop

$= \sec^2 x(\sec x - 1) - 1(\sec x - 1)$ by Distributive Prop

$= (\sec x - 1)(\sec^2 x - 1)$ by Grouping

$= (\csc x - 1)(\tan^2 x)$ by Pythagorean ID

Now any additional steps would probably make this more complicated than the original