Problem 1

Exit Slip Solutions 1) $4\cdot\sin\left(2\cdot x+\frac{\pi}{3}\right)+3=6.75$

Step 1) isolate $\sin\left(2\cdot x+\frac{\pi}{3}\right)$

$4\cdot\sin\left(2\cdot x+\frac{\pi}{3}\right)+3-3=6.75-3$ leads to $4\cdot\sin\left(2\cdot x+\frac{\pi}{3}\right)=3.75$

$4\cdot\sin\left(2\cdot x+\frac{\pi}{3}\right)=\frac{15}{4}$ leads to $\frac{4\cdot\sin\left(2\cdot x+\frac{\pi}{3}\right)}{4}=\frac{1}{4}\left(\frac{15}{4}\right)$

$\sin\left(2\cdot x+\frac{\pi}{3}\right)=\frac{15}{4}$

Step 2) Replace interior function with ANY LETTER OTHER THAN X OR Y

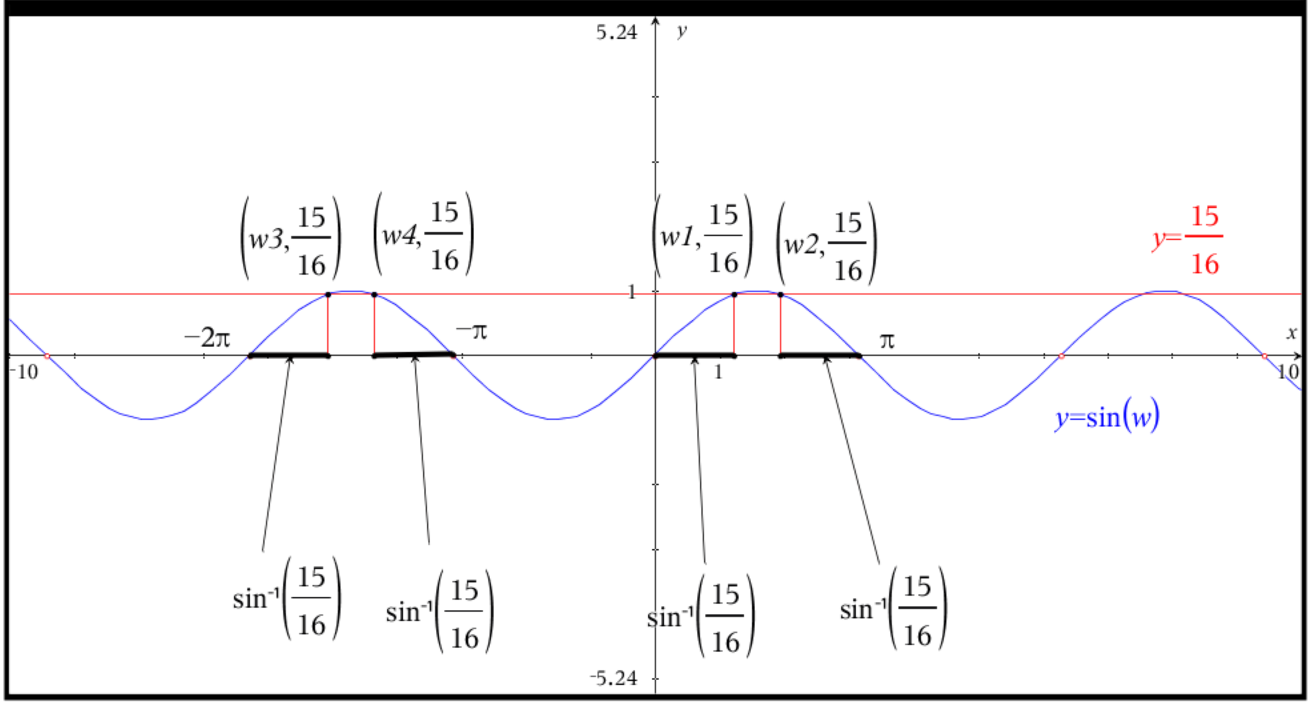
$\sin\left(2\cdot x+\frac{\pi}{3}\right)=\frac{15}{4}$ Let w = $2\cdot x+\frac{\pi}{3}$

$\sin(w)=\frac{15}{4}$

Step 3) Use inverse trigonometry to find unknown angle $\sin^{-1}(\sin(w))=\sin^{-1}\left(\frac{15}{16}\right)$ leads to $w=\sin^{-1}\left(\frac{15}{16}\right)$

Recall how referencing with sine function works

We reference from multiples of π



$\left(w1, \frac{15}{16}\right)$ implies $\left(\sin^{-1}\left(\frac{15}{16}\right), \frac{15}{16}\right)$ but we need the solution related to x not w!

Recall w = $2 \cdot x + \frac{\pi}{3}$ so $\left(2 \cdot x + \frac{\pi}{3}, \frac{15}{16}\right) = \left(\sin^{-1}\left(\frac{15}{16}\right), \frac{15}{16}\right)$, we need to solve $2 \cdot x + \frac{\pi}{3} = \sin^{-1}\left(\frac{15}{16}\right)$ for x

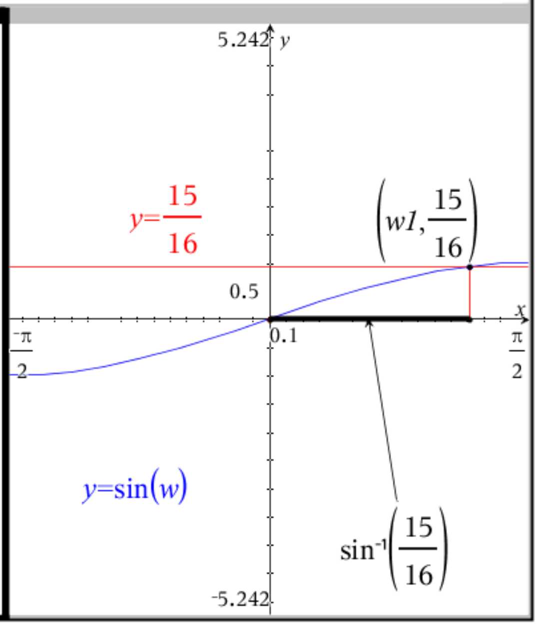
$2 \cdot x + \frac{\pi}{3} = \sin^{-1}\left(\frac{15}{16}\right)$

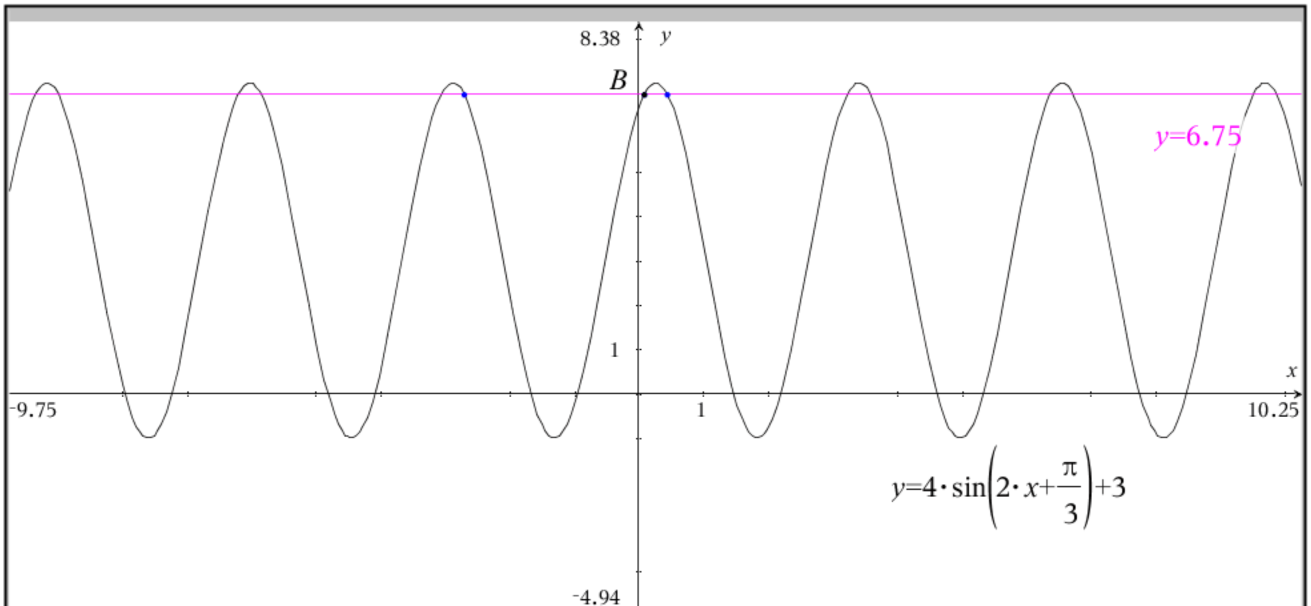
$2 \cdot x + \frac{\pi}{3} - \frac{\pi}{3} = \frac{-\pi}{3} + \sin^{-1}\left(\frac{15}{16}\right)$ leads to $2 \cdot x = \frac{-\pi}{3} + \sin^{-1}\left(\frac{15}{16}\right)$

$\frac{2 \cdot x}{2} = \frac{1}{2}\left(\frac{-\pi}{3} + \sin^{-1}\left(\frac{15}{16}\right)\right)$ leads to $x = \frac{-\pi}{6} + \frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)$

$x = \frac{-\pi}{6} + \frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right) \approx 0.084089$ radians

This is ONE of the infinitely many solutions to the equation $4 \cdot \sin\left(2 \cdot x + \frac{\pi}{3}\right) + 3 = 6.75$





This was ACTUALLY B from our EXIT SLIP B = $\left(\frac{-\pi}{6} + \frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right), \frac{27}{4}\right) \approx (0.084089, 6.75)$

$\left(w2, \frac{15}{16}\right)$ implies $\left(\pi - \sin^{-1}\left(\frac{15}{16}\right), \frac{15}{16}\right)$ but we need the solution related to x not w!

Recall w = $2 \cdot x + \frac{\pi}{3}$ so $\left(2 \cdot x + \frac{\pi}{3}, \frac{15}{16}\right) = \left(\pi - \sin^{-1}\left(\frac{15}{16}\right), \frac{15}{16}\right)$,

we need to solve $2 \cdot x + \frac{\pi}{3} = \pi - \sin^{-1}\left(\frac{15}{16}\right)$ for x

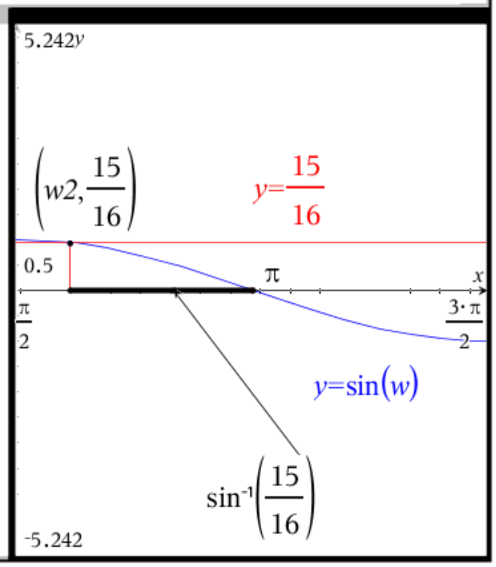
$2 \cdot x + \frac{\pi}{3} = \pi - \sin^{-1}\left(\frac{15}{16}\right)$

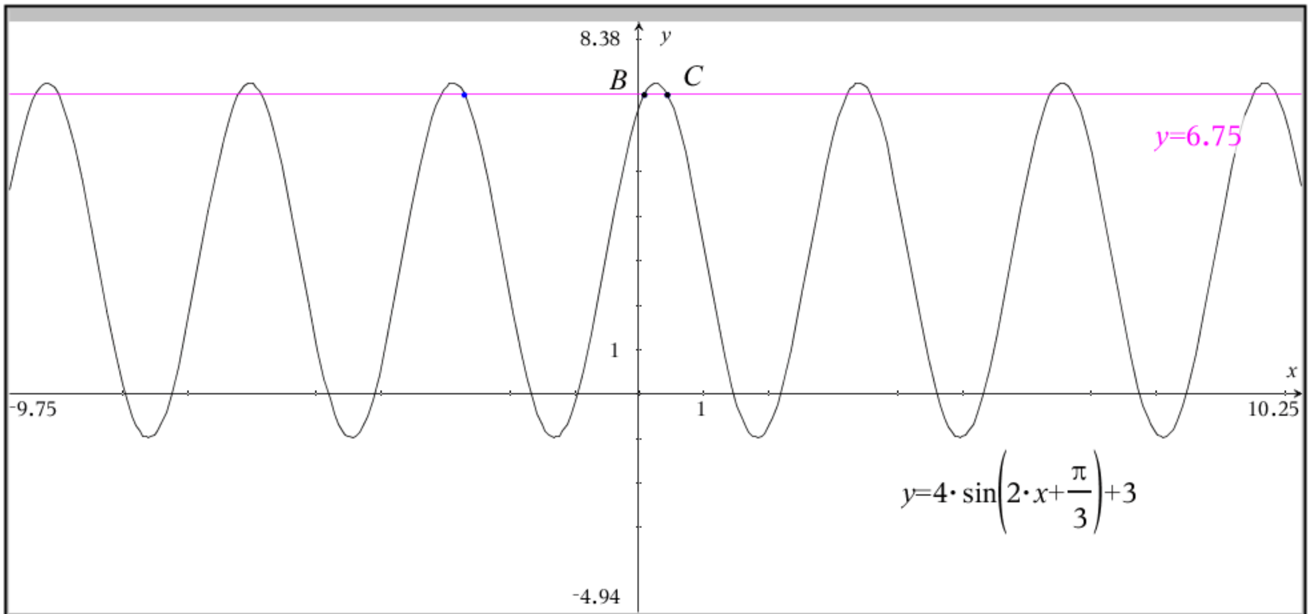
$2 \cdot x + \frac{\pi}{3} - \frac{\pi}{3} = \frac{-\pi}{3} + \pi - \sin^{-1}\left(\frac{15}{16}\right)$ leads to $2 \cdot x = \frac{2\pi}{3} - \sin^{-1}\left(\frac{15}{16}\right)$

$\frac{2 \cdot x}{2} = \frac{1}{2}\left(\frac{2\pi}{3} - \sin^{-1}\left(\frac{15}{16}\right)\right)$ leads to $x = \frac{2\pi}{6} - \frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)$

$x = \frac{\pi}{3} - \frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right) \approx 0.43951$ radians

This is ONE of the infinitely many solutions to the equation $4 \cdot \sin\left(2 \cdot x + \frac{\pi}{3}\right) + 3 = 6.75$





This was ACTUALLY C from our EXIT SLIP C = $\left(\frac{\pi}{3} - \frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right), \frac{27}{4}\right) \approx (0.43951, 6.75)$

$\left(w2, \frac{15}{16}\right)$ implies $\left(\pi - \sin^{-1}\left(\frac{15}{16}\right), \frac{15}{16}\right)$ but we need the solution related to x not w!

Recall w = $2 \cdot x + \frac{\pi}{3}$ so $\left(2 \cdot x + \frac{\pi}{3}, \frac{15}{16}\right) = \left(\pi - \sin^{-1}\left(\frac{15}{16}\right), \frac{15}{16}\right)$,

we need to solve $2 \cdot x + \frac{\pi}{3} = \pi - \sin^{-1}\left(\frac{15}{16}\right)$ for x

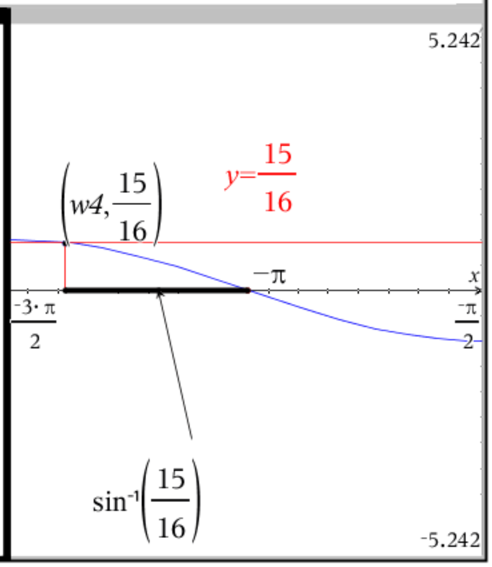
$2 \cdot x + \frac{\pi}{3} = -\pi - \sin^{-1}\left(\frac{15}{16}\right)$

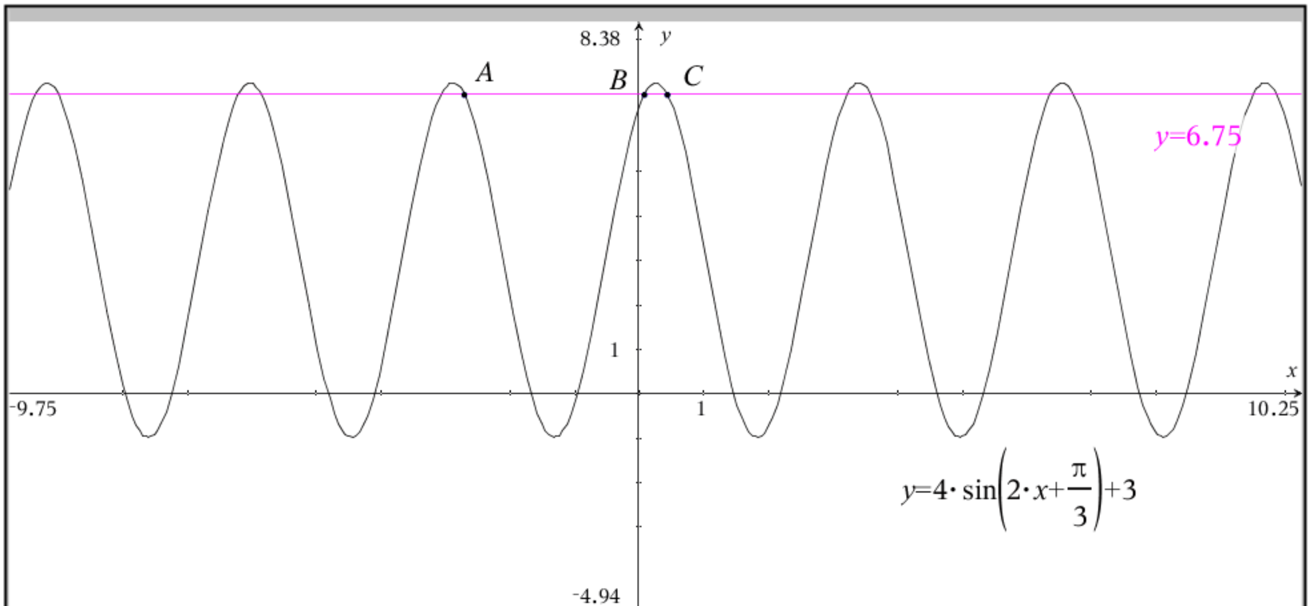
$2 \cdot x + \frac{\pi}{3} - \frac{\pi}{3} = \frac{-\pi}{3} - \pi - \sin^{-1}\left(\frac{15}{16}\right)$ leads to $2 \cdot x = \frac{-4\pi}{3} - \sin^{-1}\left(\frac{15}{16}\right)$

$\frac{2 \cdot x}{2} = \frac{1}{2}\left(\frac{-4\pi}{3} - \sin^{-1}\left(\frac{15}{16}\right)\right)$ leads to $x = \frac{-4\pi}{6} - \frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)$

$x = \frac{-2\pi}{3} - \frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right) \approx -2.70208$ radians

This is ONE of the infinitely many solutions to the equation $4 \cdot \sin\left(2 \cdot x + \frac{\pi}{3}\right) + 3 = 6.75$





This was ACTUALLY A from our EXIT SLIP A = $\left(\frac{-2\pi}{3} - \frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right), \frac{27}{4}\right) \approx (-2.70208, 6.75)$

Recall how referencing with sine function works

We can build two general rules for all of the solutions for this equation off of x1 and x2 solutions

Note the period length of $y=4\cdot\sin\left(2\cdot x+\frac{\pi}{3}\right)+3$ is $\frac{2\cdot\pi}{2}=\pi$

Rule 1 x = x1+period n for all n that are integers

$= \frac{-\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+\pi n$ for all $n \in \mathbb{Z}$

$= \frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+\pi\left(\frac{-1}{6}+n\right)$ for all $n \in \mathbb{Z}$

Rule 2 x = x2+period n for all n that are integers

$= \frac{\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+\pi n$ for all $n \in \mathbb{Z}$

$= -\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+\pi\left(\frac{1}{3}+n\right)$ for all $n \in \mathbb{Z}$

This is difficult to DO because of the changes to the period

Test of Rule 1

$\frac{-\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+\pi n$

n = 0 leads to $\frac{-\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)$ (original value)

n = 1 leads to $\frac{-\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+\pi=\frac{5\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)$

test $4\cdot\sin\left(2\cdot\left(\frac{5\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)\right)+\frac{\pi}{3}\right)+3$

$4\cdot\sin\left(\frac{5\pi}{3}+1\sin^{-1}\left(\frac{15}{16}\right)+\frac{\pi}{3}\right)+3$

$4\cdot\sin\left(\frac{6\pi}{3}+1\sin^{-1}\left(\frac{15}{16}\right)\right)+3$

$4\cdot\sin\left(2\pi+1\sin^{-1}\left(\frac{15}{16}\right)\right)+3=\frac{27}{4}$

Test of Rule 1

$$\frac{-\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+\pi n$$

n = −1 leads to $\frac{-\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)-\pi=\frac{-7\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)$

test $4\cdot\sin\left(2\left(\frac{-7\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)\right)+\frac{\pi}{3}\right)+3$

$$4\cdot\sin\left(\frac{-7\pi}{3}+1\sin^{-1}\left(\frac{15}{16}\right)+\frac{\pi}{3}\right)+3$$

$$4\cdot\sin\left(\frac{-6\pi}{3}+1\sin^{-1}\left(\frac{15}{16}\right)\right)+3$$

$$4\cdot\sin\left(-2\pi+1\sin^{-1}\left(\frac{15}{16}\right)\right)+3=\frac{27}{4}$$

Test of Rule 1

$$\frac{-\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+\pi n$$

n = 2 leads to $\frac{-\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+2\pi=\frac{11\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)$

test $4\cdot\sin\left(2\left(\frac{11\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)\right)+\frac{\pi}{3}\right)+3$

$$4\cdot\sin\left(\frac{11\pi}{3}+1\sin^{-1}\left(\frac{15}{16}\right)+\frac{\pi}{3}\right)+3$$

$$4\cdot\sin\left(\frac{12\pi}{3}+1\sin^{-1}\left(\frac{15}{16}\right)\right)+3$$

$$4\cdot\sin\left(4\pi+1\sin^{-1}\left(\frac{15}{16}\right)\right)+3=\frac{27}{4}$$

Test of Rule 1

$\frac{-\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+\pi n$

n = −2 leads to $\frac{-\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)-2\pi=\frac{-13\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)$

test $4\cdot\sin\left(2\left(\frac{-13\pi}{6}+\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)\right)+\frac{\pi}{3}\right)+3$

$4\cdot\sin\left(\frac{-13\pi}{3}+1\sin^{-1}\left(\frac{15}{16}\right)+\frac{\pi}{3}\right)+3$

$4\cdot\sin\left(\frac{-12\pi}{3}+1\sin^{-1}\left(\frac{15}{16}\right)\right)+3$

$4\cdot\sin\left(-4\pi+1\sin^{-1}\left(\frac{15}{16}\right)\right)+3=\frac{27}{4}$

Test of Rule 2

$\frac{\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+\pi n$

n = 0 leads to $\frac{\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)$ (original value)

n = 1 leads to $\frac{\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+\pi=\frac{4\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)$

test $4\cdot\sin\left(2\cdot\left(\frac{4\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)\right)+\frac{\pi}{3}\right)+3$

$4\cdot\sin\left(\frac{8\pi}{3}-1\sin^{-1}\left(\frac{15}{16}\right)+\frac{\pi}{3}\right)+3$

$4\cdot\sin\left(\frac{9\pi}{3}-1\sin^{-1}\left(\frac{15}{16}\right)\right)+3$

$4\cdot\sin\left(3\pi-1\sin^{-1}\left(\frac{15}{16}\right)\right)+3=\frac{27}{4}$

Test of Rule 2

$\frac{\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+\pi n$

n = −1 leads to $\frac{\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)-\pi=\frac{-2\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)$

test $4\cdot\sin\left(2\cdot\left(\frac{-2\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)\right)+\frac{\pi}{3}\right)+3$

$4\cdot\sin\left(\frac{-4\pi}{3}-1\sin^{-1}\left(\frac{15}{16}\right)+\frac{\pi}{3}\right)+3$

$4\cdot\sin\left(\frac{-3\pi}{3}-1\sin^{-1}\left(\frac{15}{16}\right)\right)+3$

$4\cdot\sin\left(-\pi-1\sin^{-1}\left(\frac{15}{16}\right)\right)+3=\frac{27}{4}$

Test of Rule 2

$\frac{\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+\pi n$

n = −2 leads to $\frac{\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)-2\pi=\frac{-5\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)$

test $4\cdot\sin\left(2\cdot\left(\frac{-5\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)\right)+\frac{\pi}{3}\right)+3$

$4\cdot\sin\left(\frac{-10\pi}{3}-1\sin^{-1}\left(\frac{15}{16}\right)+\frac{\pi}{3}\right)+3$

$4\cdot\sin\left(\frac{-9\pi}{3}-1\sin^{-1}\left(\frac{15}{16}\right)\right)+3$

$4\cdot\sin\left(-3\pi-1\sin^{-1}\left(\frac{15}{16}\right)\right)+3=\frac{27}{4}$

Test of Rule 2

$\frac{\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+\pi n$

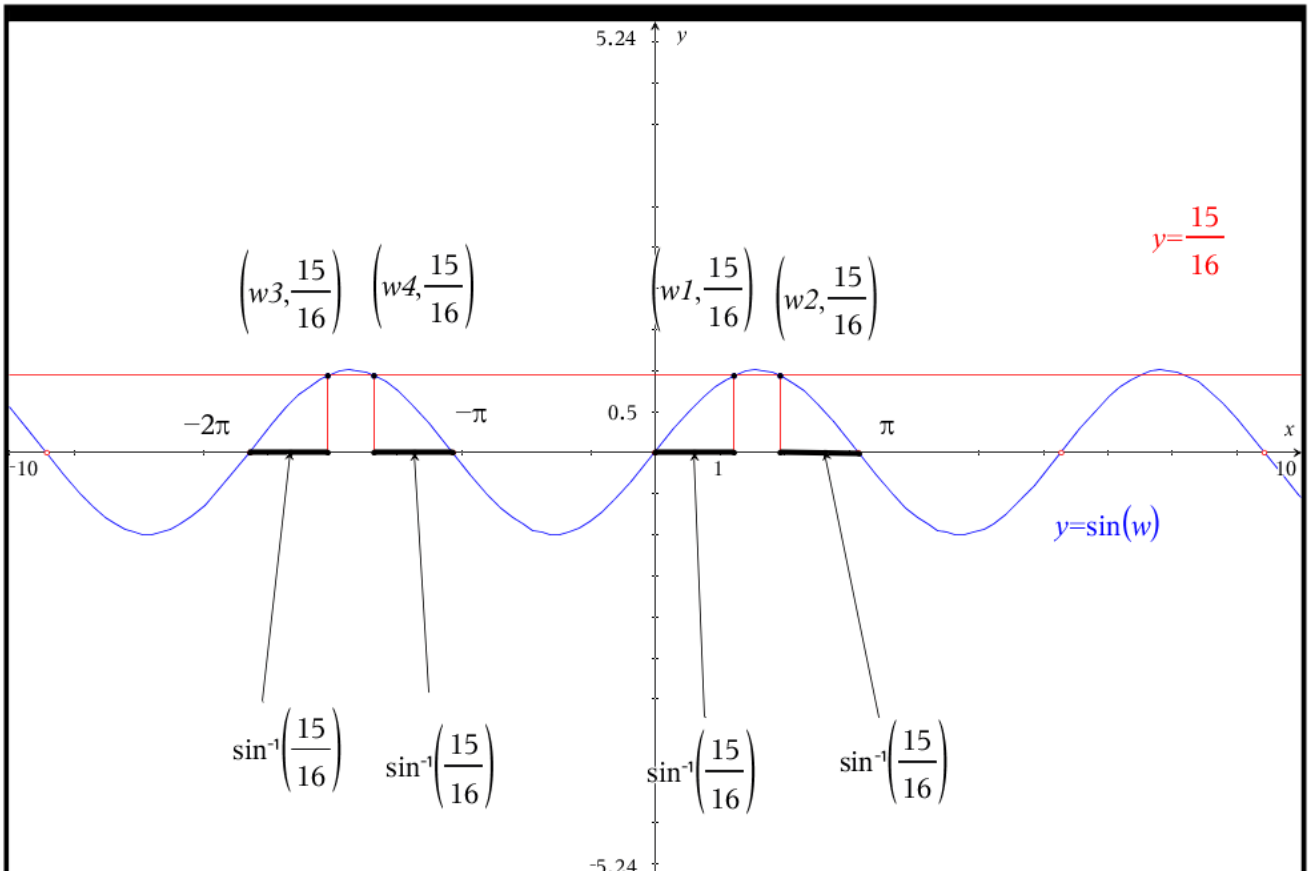
n = 2 leads to $\frac{\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)+2\pi=\frac{7\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)$

test $4\cdot\sin\left(2\cdot\left(\frac{7\pi}{3}-\frac{1}{2}\sin^{-1}\left(\frac{15}{16}\right)\right)+\frac{\pi}{3}\right)+3$

$4\cdot\sin\left(\frac{14\pi}{3}-1\sin^{-1}\left(\frac{15}{16}\right)+\frac{\pi}{3}\right)+3$

$4\cdot\sin\left(\frac{15\pi}{3}-1\sin^{-1}\left(\frac{15}{16}\right)\right)+3$

$4\cdot\sin\left(5\pi-1\sin^{-1}\left(\frac{15}{16}\right)\right)+3=\frac{27}{4}$



Problem 2

Exit Slip Solutions 2) $-5\cdot\cos(x)+3=4.75$

Step 1) isolate $\cos(x)$

$-5\cdot\cos(x)+3-3=4.75-3$ leads to $-5\cdot\cos(x)=1.75$

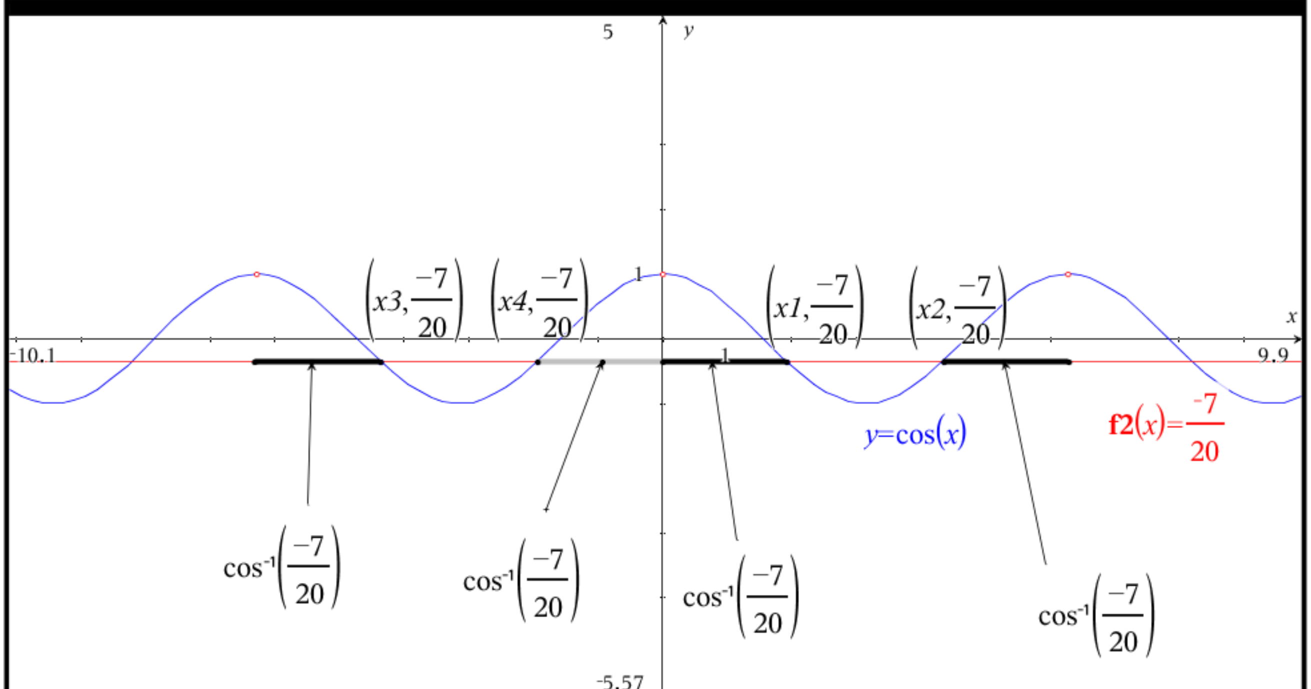
$-5\cos(x)=\frac{7}{4}$ leads to $\frac{-5\cdot\cos(x)}{-5}=\frac{7}{4}\cdot\frac{-1}{5}$

$\cos(x)=\frac{-7}{20}$

Step 2) Use inverse trigonometry to find unknown angle $\cos^{-1}(\cos(x))=\cos^{-1}\left(\frac{-7}{20}\right)$ leads to $x=\cos^{-1}\left(\frac{-7}{20}\right)$

Recall how referencing with cosine function works

We reference from multiples of 2π

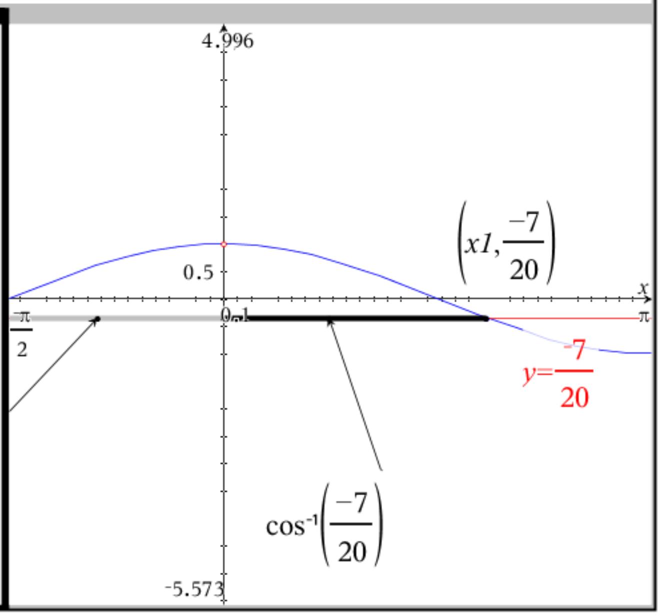


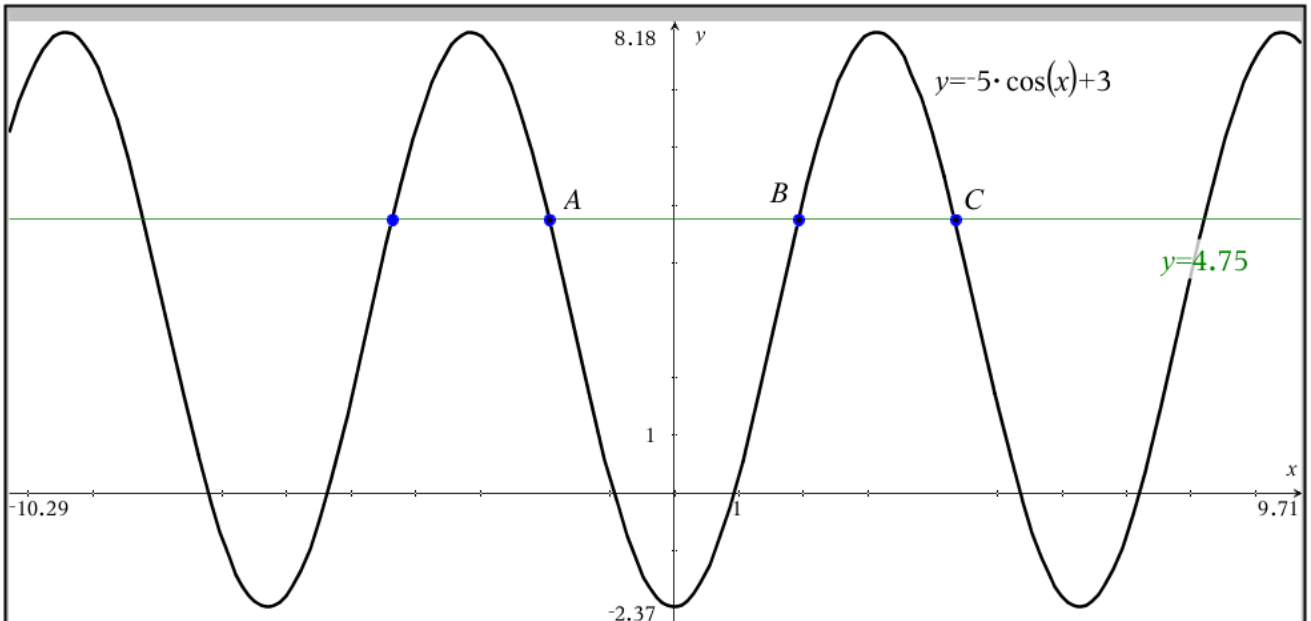
$\left(x1, \frac{-7}{20}\right)$ implies $\left(\cos^{-1}\left(\frac{-7}{20}\right), \frac{-7}{20}\right)$

Since there was no transformation applied to domain, we get one answer directly

$x = \cos^{-1}\left(\frac{-7}{20}\right) \approx 1.92837$ radians

This is ONE of the infinitely many solutions to the equation $-5 \cdot \cos(x) + 3 = 4.75$



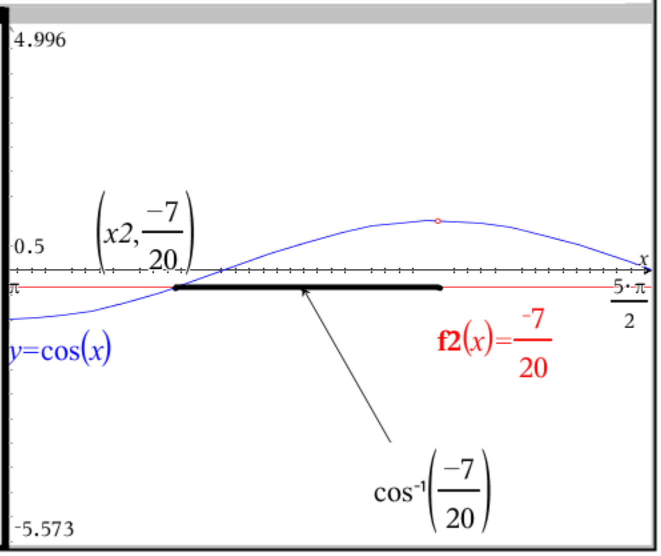


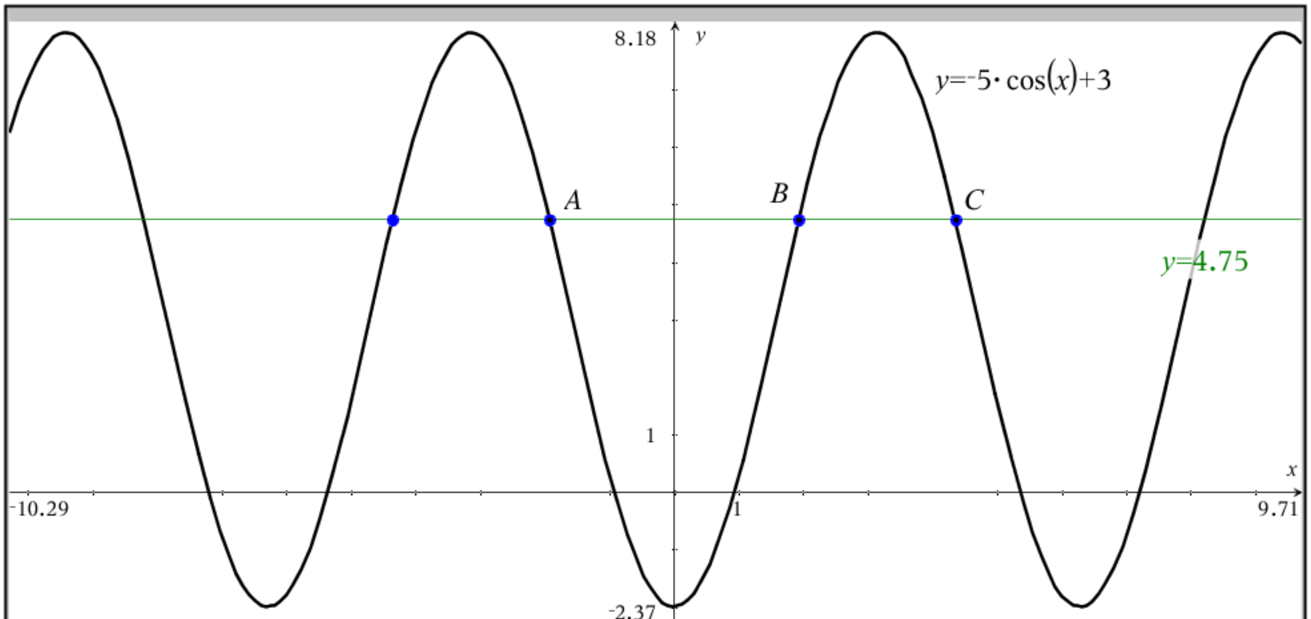
This was ACTUALLY B from our EXIT SLIP B = $\left(\cos^{-1}\left(\frac{-7}{20}\right), \frac{19}{4}\right) \approx (1.92837, 4.75)$

$\left(x2,\frac{-7}{20}\right)$ implies $\left(2\pi-\cos^{-1}\left(\frac{-7}{20}\right),\frac{-7}{20}\right)$

$x=2\pi-\cos^{-1}\left(\frac{-7}{20}\right)\approx 4.35482$ radians

This is ONE of the infinitely many solutions to the equation $-5\cdot\cos(x)+3=4.75$



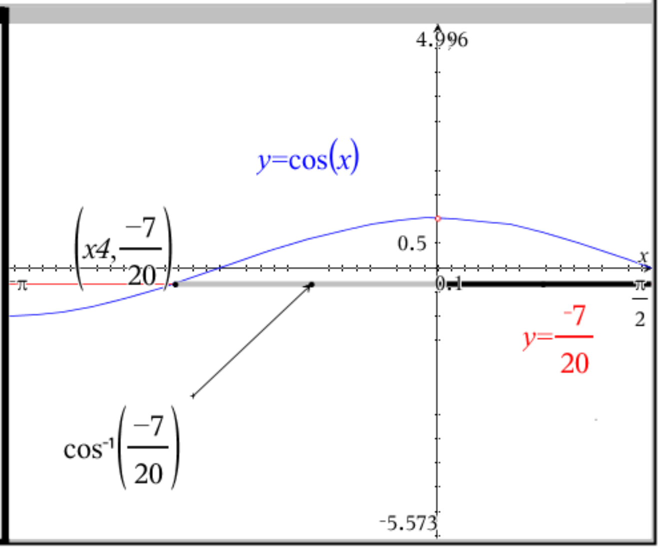


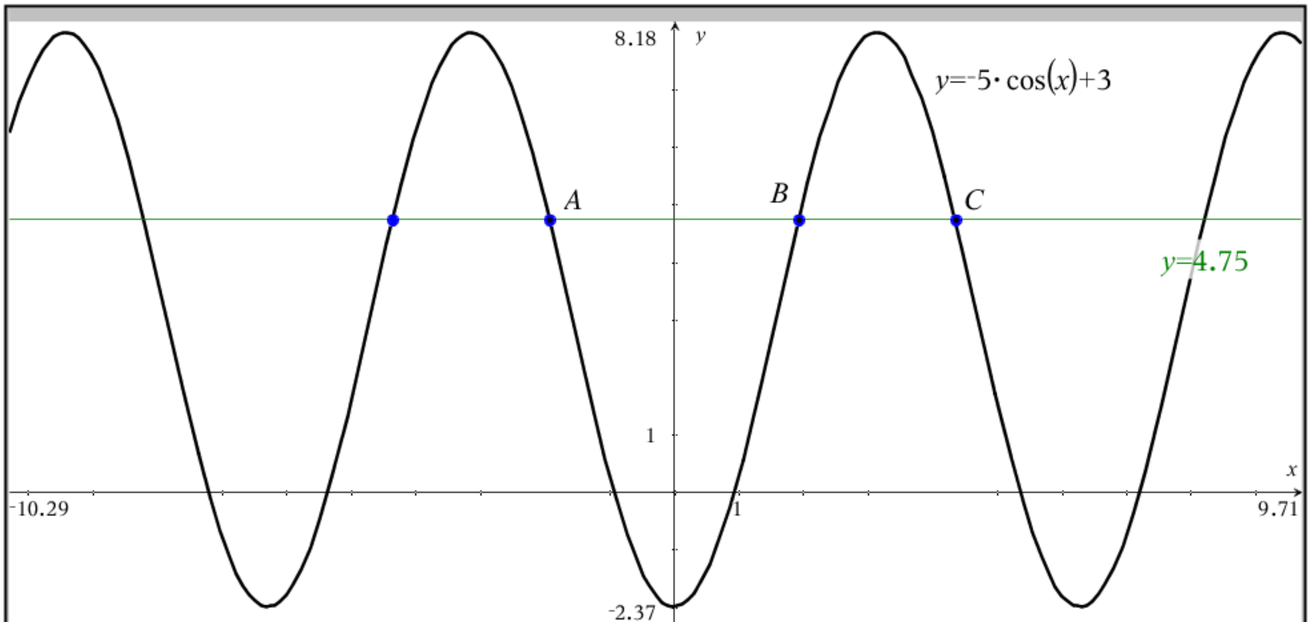
This was ACTUALLY C from our EXIT SLIP C = $\left(2\pi-\cos^{-1}\left(\frac{-7}{20}\right),\frac{19}{4}\right)\approx(4.35482,4.75)$

$\left(x4,\frac{-7}{20}\right)$ implies $\left(-\cos^{-1}\left(\frac{-7}{20}\right),\frac{-7}{20}\right)$

$x=-\cos^{-1}\left(\frac{-7}{20}\right)\approx-\cos^{-1}\left(\frac{-7}{20.}\right)$ ▸ -1.92837 radians

This is ONE of the infinitely many solutions to the equation $-5\cdot\cos(x)+3=4.75$





This was ACTUALLY A from our EXIT SLIP A = $\left(-\cos^{-1}\left(\frac{-7}{20}\right),\frac{19}{4}\right)\approx(-1.92837,4.75)$

Recall how referencing with cosine function works

We reference from multiples of 2π

We can build two general rules for all of the solutions for this equation off of x1 and x2 solutions

Rule 1 x = x1+2πn for all n that are integers

$$= \cos^{-1}\left(\frac{-7}{20}\right)+2\pi n \quad \text{for all } n \in \mathbb{Z}$$

Rule 2 x = x2+2πn for all n that are integers

$$= 2\pi-\cos^{-1}\left(\frac{-7}{20}\right)+2\pi n \quad \text{for all } n \in \mathbb{Z}$$

$$= -\cos^{-1}\left(\frac{-7}{20}\right)+(n+1)2\pi \quad \text{for all } n \in \mathbb{Z}$$

Test of Rule 1

$$\cos^{-1}\left(\frac{-7}{20}\right)+2\pi n$$

n = 0 leads to $\cos^{-1}\left(\frac{-7}{20}\right)$ (original value)

n = 1 leads to $\cos^{-1}\left(\frac{-7}{20}\right)+2\pi$

test $-5\cdot\cos\left(\cos^{-1}\left(\frac{-7}{20}\right)+2\pi\right)+3=\frac{19}{4}$

Test of Rule 1

$\cos^{-1}\left(\frac{-7}{20}\right)+2\pi n$

n = 2 leads to $\cos^{-1}\left(\frac{-7}{20}\right)+4\pi$

test $-5\cdot\cos\left(\cos^{-1}\left(\frac{-7}{20}\right)+4\pi\right)+3=\frac{19}{4}$

Test of Rule 1

$\cos^{-1}\left(\frac{-7}{20}\right)+2\pi n$

n = −1 leads to $\cos^{-1}\left(\frac{-7}{20}\right)-2\pi$

test $-5\cdot\cos\left(\cos^{-1}\left(\frac{-7}{20}\right)-2\pi\right)+3=\frac{19}{4}$

Test of Rule 1

$\cos^{-1}\left(\frac{-7}{20}\right)+2\pi n$

n = −2 leads to $\cos^{-1}\left(\frac{-7}{20}\right)-4\pi$

test $-5\cdot\cos\left(\cos^{-1}\left(\frac{-7}{20}\right)-4\pi\right)+3=\frac{19}{4}$

Test of Rule 2

$2\pi-\cos^{-1}\left(\frac{-7}{20}\right)+2\pi n$

n = 0 leads to $2\pi-\cos^{-1}\left(\frac{-7}{20}\right)$ (original value)

n = 1 leads to $2\pi-\cos^{-1}\left(\frac{-7}{20}\right)+2\pi=4\pi-\cos^{-1}\left(\frac{-7}{20}\right)$

test $-5\cdot\cos\left(4\pi-\cos^{-1}\left(\frac{-7}{20}\right)\right)+3=\frac{19}{4}$

Test of Rule 2

$2\pi - \cos^{-1}\left(\frac{-7}{20}\right) + 2\pi n$

n = 2 leads to $2\pi - \cos^{-1}\left(\frac{-7}{20}\right) + 4\pi = 6\pi - \cos^{-1}\left(\frac{-7}{20}\right)$

test $-5 \cdot \cos\left(6\pi - \cos^{-1}\left(\frac{-7}{20}\right)\right) + 3 = \frac{19}{4}$

Test of Rule 2

$2\pi - \cos^{-1}\left(\frac{-7}{20}\right) + 2\pi n$

n = −1 leads to $2\pi - \cos^{-1}\left(\frac{-7}{20}\right) - 2\pi = -\cos^{-1}\left(\frac{-7}{20}\right)$

test $-5 \cdot \cos\left(-\cos^{-1}\left(\frac{-7}{20}\right)\right) + 3 = \frac{19}{4}$

Test of Rule 2

$2\pi-\cos^{-1}\left(\frac{-7}{20}\right)+2\pi n$

n = −2 leads to $2\pi-\cos^{-1}\left(\frac{-7}{20}\right)-4\pi=-2\pi-\cos^{-1}\left(\frac{-7}{20}\right)$

test $-5\cdot\cos\left(-2\pi-\cos^{-1}\left(\frac{-7}{20}\right)\right)+3=\frac{19}{4}$