

	A	B	C	D
=				
1	a		2	
A1	a			

$$y = 2 \csc\left(2x + \frac{3\pi}{4}\right) + 0$$

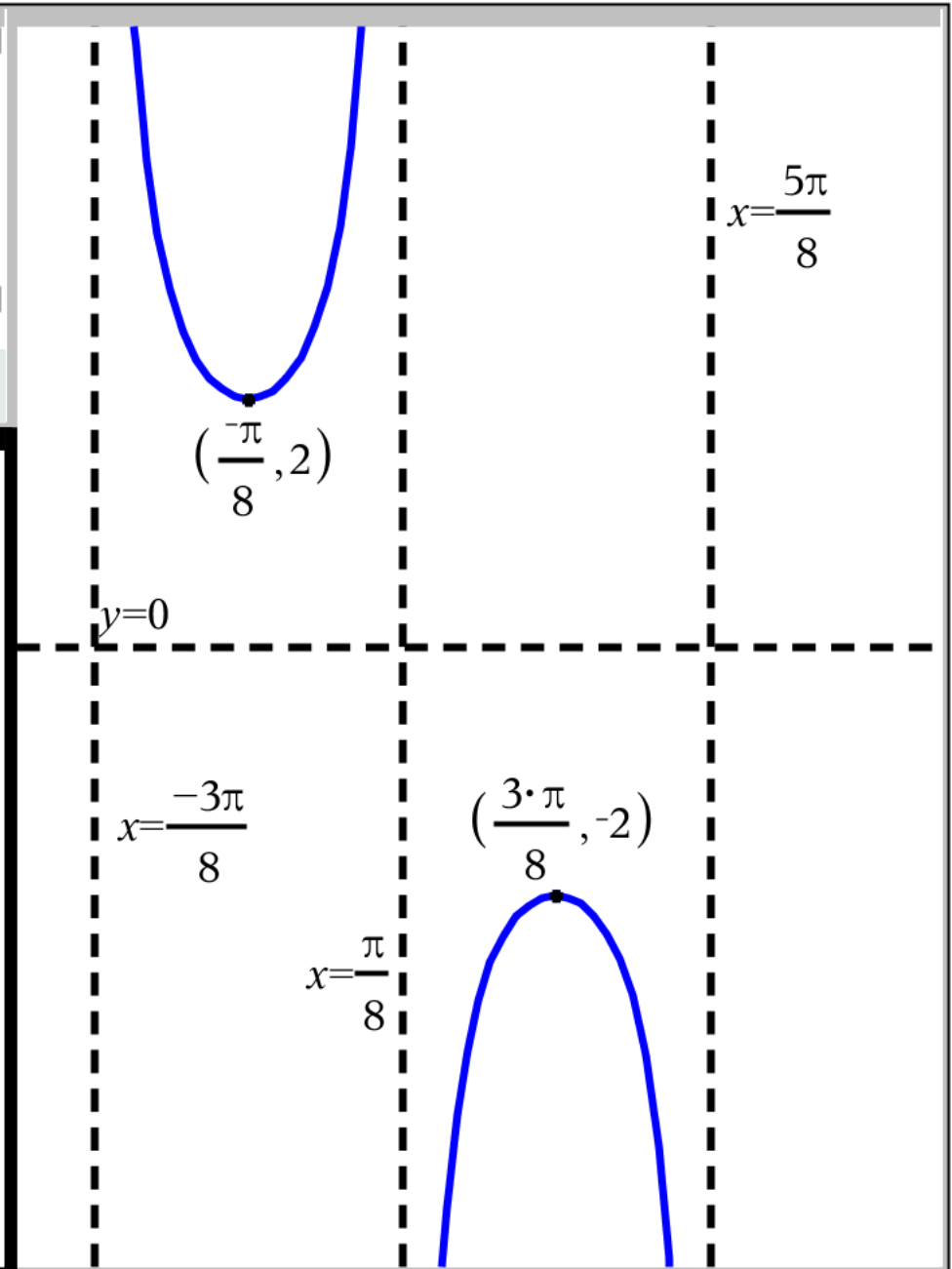
$$y = 2 \csc\left(2\left(x + \frac{3\pi}{8}\right)\right) +$$

$$\text{Period} \left(\frac{-3\pi}{8}, \frac{5\pi}{8}\right)$$

Amplitude 2 Equation of Midline $y = 0$

$$\text{Equation of Asymptotes } x = \frac{-3\pi}{8} \quad x = \frac{\pi}{8} \quad x = \frac{5\pi}{8}$$

$$\text{Equation of All Asymptotes } x = \frac{-3\pi}{8} + \frac{n\pi}{2} \text{ with } n \in \mathbb{Z}$$



Shift to left because start = $-\frac{3 \cdot \pi}{8}$

We know $y = a \csc(b(x + \frac{3 \cdot \pi}{8})) + d$

Period of this function $(\frac{-3 \cdot \pi}{8}, \frac{5 \cdot \pi}{8})$

This gives us a period length of $\frac{5 \cdot \pi}{8} - \frac{-3 \cdot \pi}{8} = \pi$

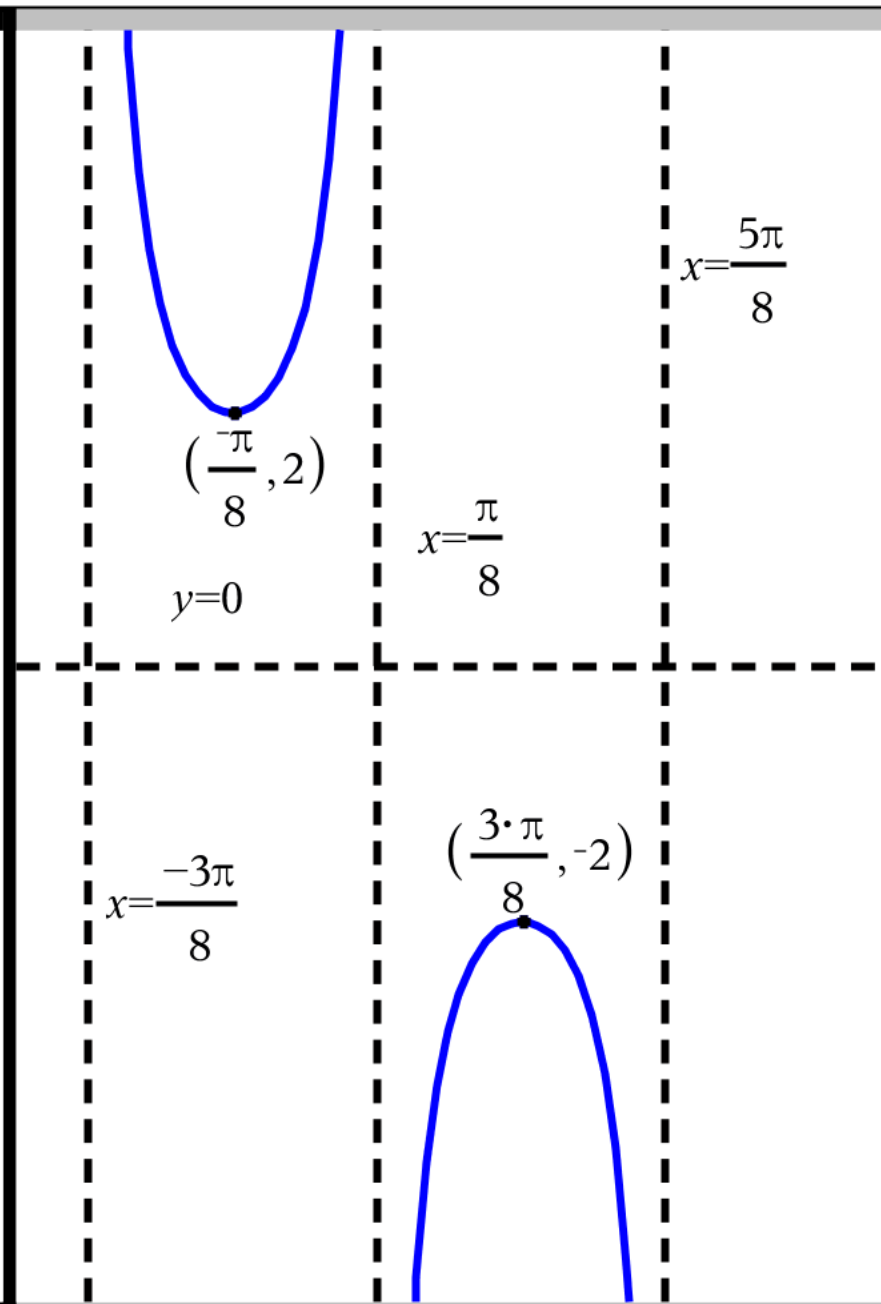
$$|b| = 2 \cdot \pi \div \pi = 2 \cdot \pi \cdot \frac{1}{\pi} = 2$$

Since this IS NOT a reflection,
both a and b are positive

This means $y = a \csc(2(x + \frac{3 \cdot \pi}{8})) + d$

Since the midline is $y=0$, this means

$$y = a \csc(2(x + \frac{3 \cdot \pi}{8})) + 0$$



Finding a this is the distance from the midline to the local extremes

$$|-2 - 0| = 2$$

$$|0 - 2| = 2$$

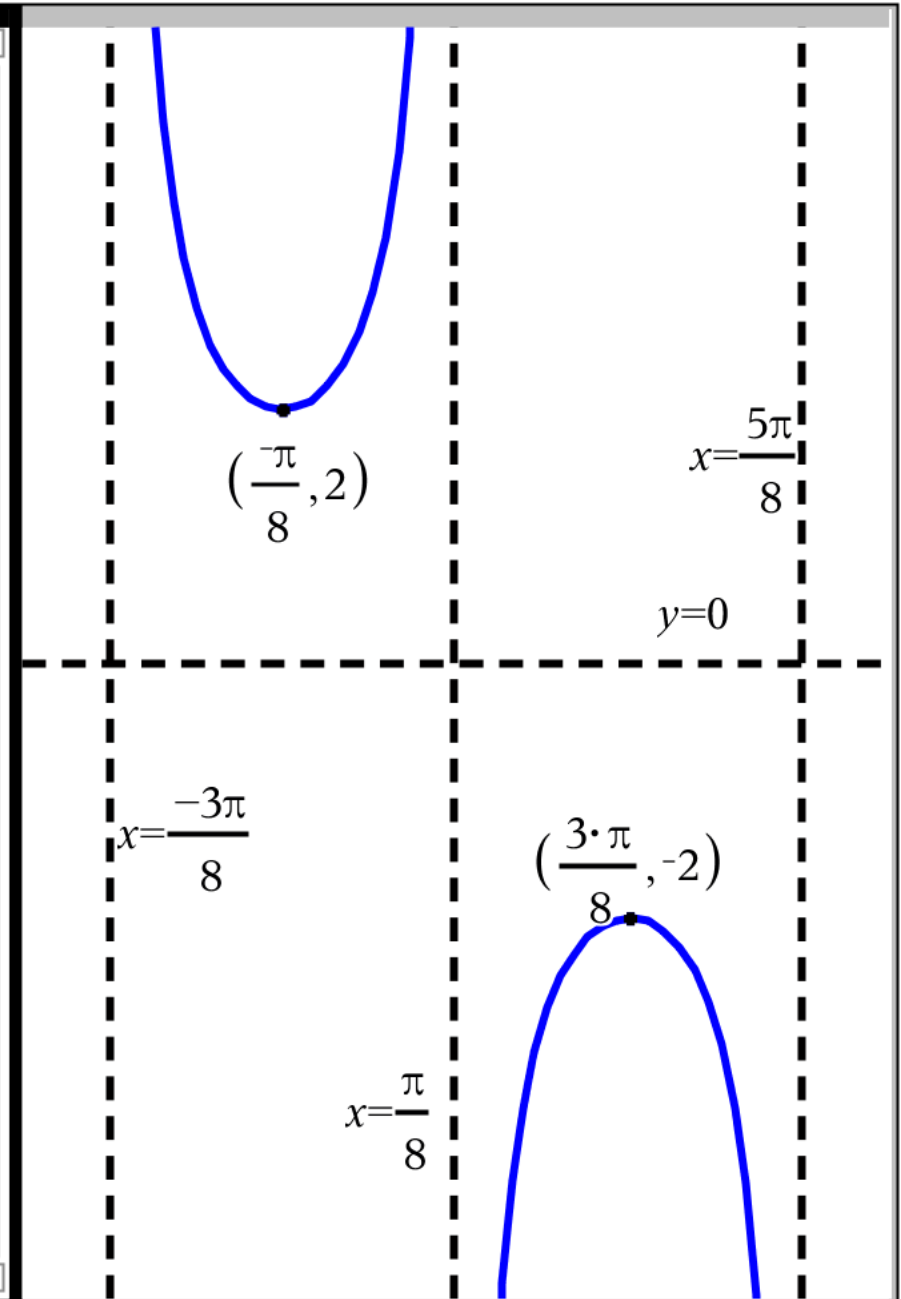
This is NOT a VERTICAL reflection

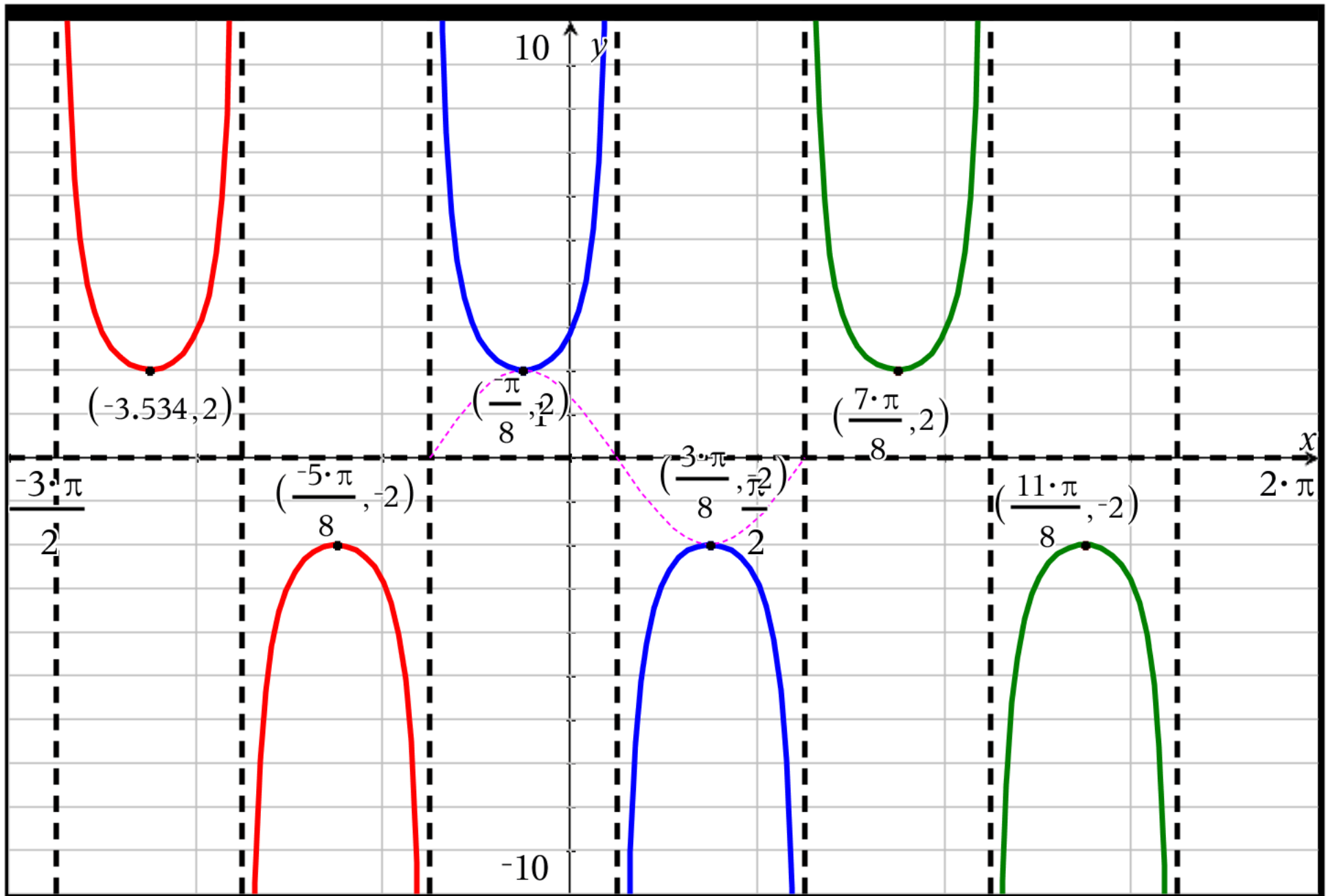
So $a=2$

So we know that

$$y = 2 \csc\left(2\left(x + \frac{3 \cdot \pi}{8}\right)\right) + 0$$

$$y = 2 \csc\left(2x + \frac{3 \cdot \pi}{4}\right) + 0$$





A	B	C	D
=			
1 a		-2	
A1 a			

$$y = -2 \sec\left(\frac{2}{3}x + \frac{\pi}{2}\right) + 2$$

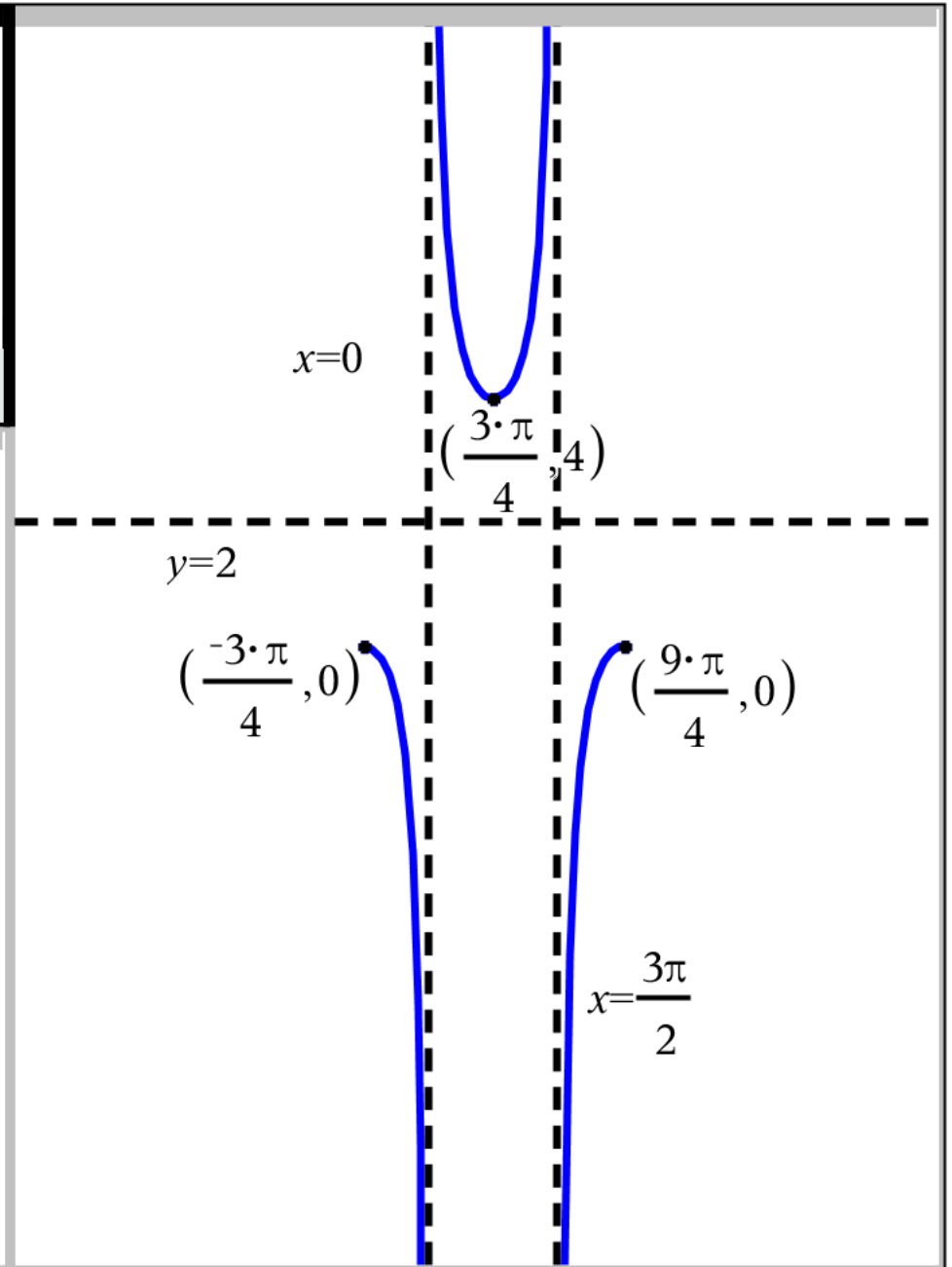
$$y = -2 \sec\left(\frac{2}{3}\left(x + \frac{3 \cdot \pi}{4}\right)\right) + 2$$

Period $\left[\frac{-3 \cdot \pi}{4}, \frac{9 \cdot \pi}{4}\right)$

Amplitude 2 Equation of Midline $y = 2$

Equation of Asymptotes $x = 0$ $x = \frac{3 \cdot \pi}{2}$

Equation of All Asymptotes $x = 0 + \frac{3 \cdot n \cdot \pi}{2}$ with $n \in \mathbb{Z}$



Finding b and horizontal shift

Shift to left because start $= \frac{-3 \cdot \pi}{4}$

We know $y = a \sec(b(x + \frac{3 \cdot \pi}{4})) + d$

Period of this function $[\frac{-3 \cdot \pi}{4}, \frac{9 \cdot \pi}{4})$

This gives us a period length of $\frac{9 \cdot \pi}{4} - \frac{-3 \cdot \pi}{4} = 3 \cdot \pi$

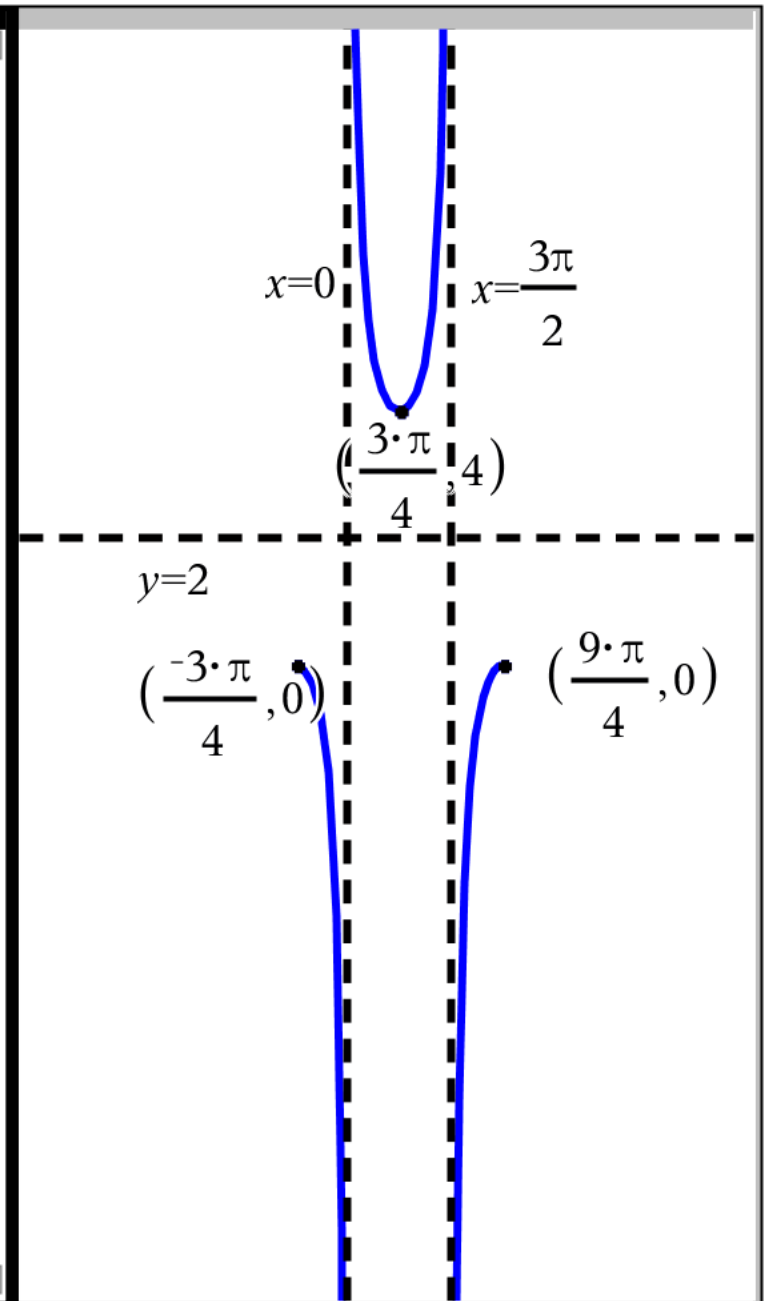
$$|b| = 2 \cdot \pi \div 3 \cdot \pi = 2 \cdot \pi \cdot \frac{1}{3 \cdot \pi} = \frac{2}{3}$$

Since this IS NOT a reflection,
both a and b are positive

This means $y = a \sec(\frac{2}{3}(x + \frac{3 \cdot \pi}{4})) + d$

Since the midline is $y=2$, this means

$$y = a \sec(\frac{2}{3}(x + \frac{3 \cdot \pi}{4})) + 2$$



Finding a this is the distance from the midline
to the local extremes

$$|0 - 2| = 2$$

$$|2 - 4| = 2$$

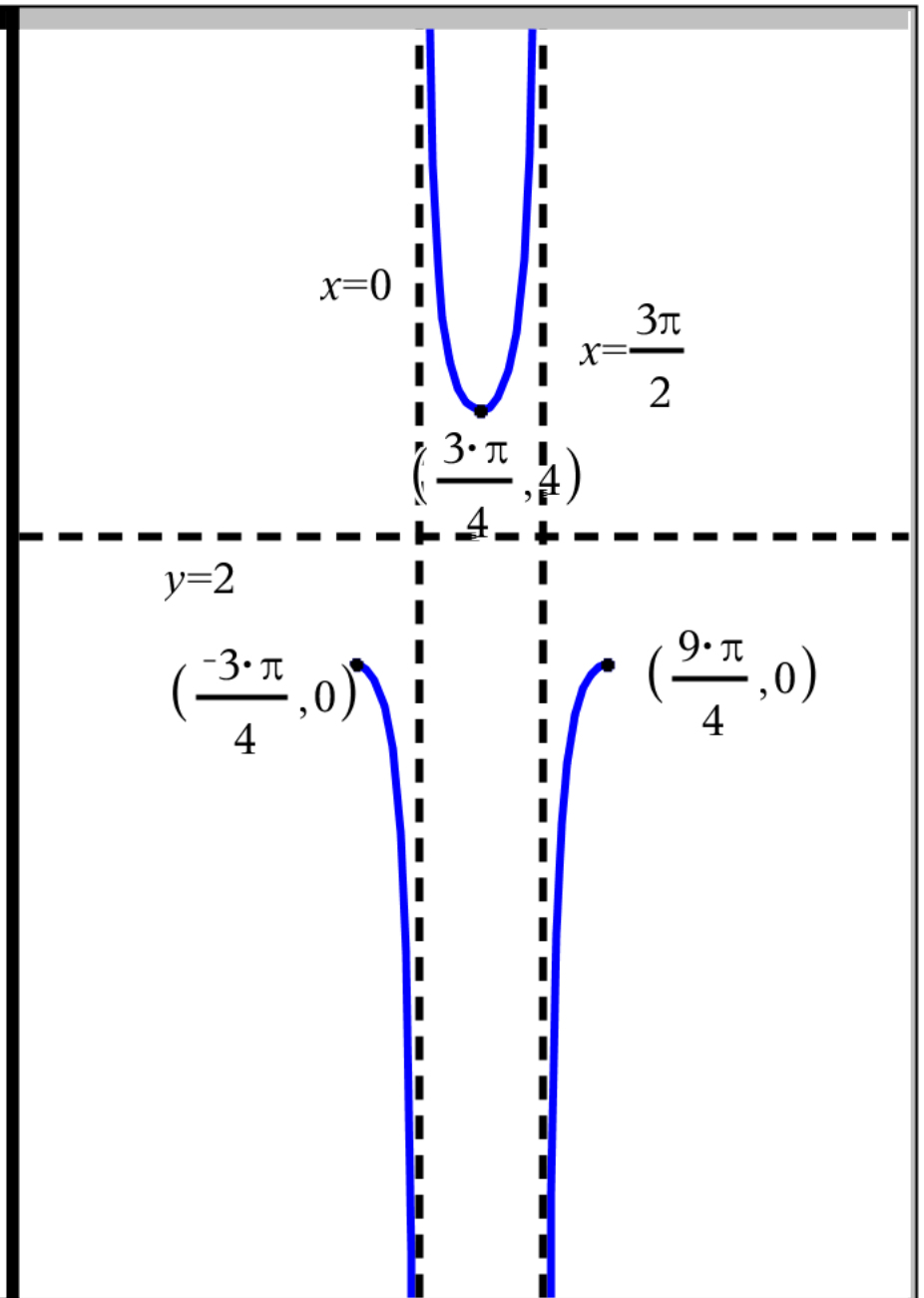
This is a VERTICAL reflection

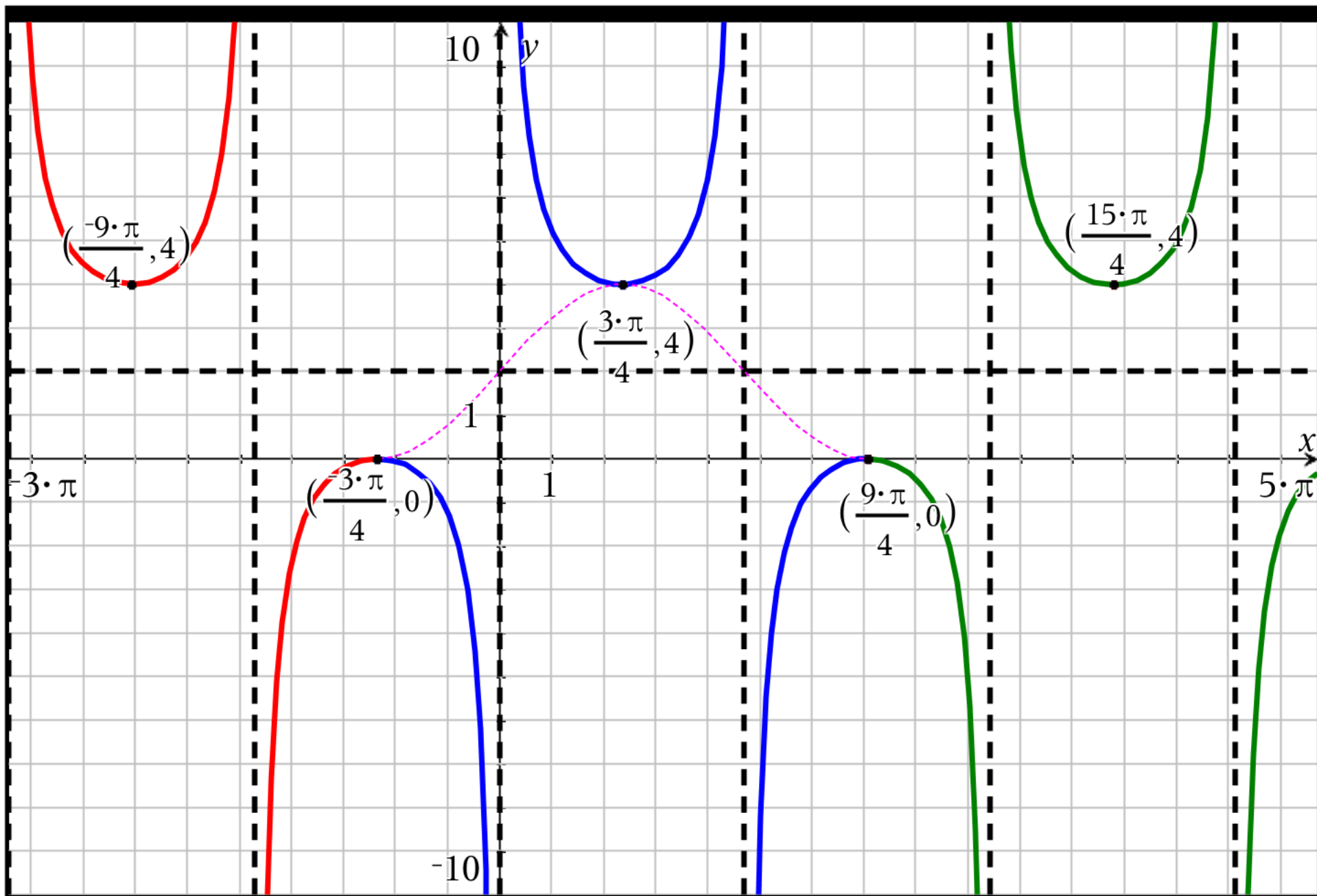
So $a = -2$

So we know that

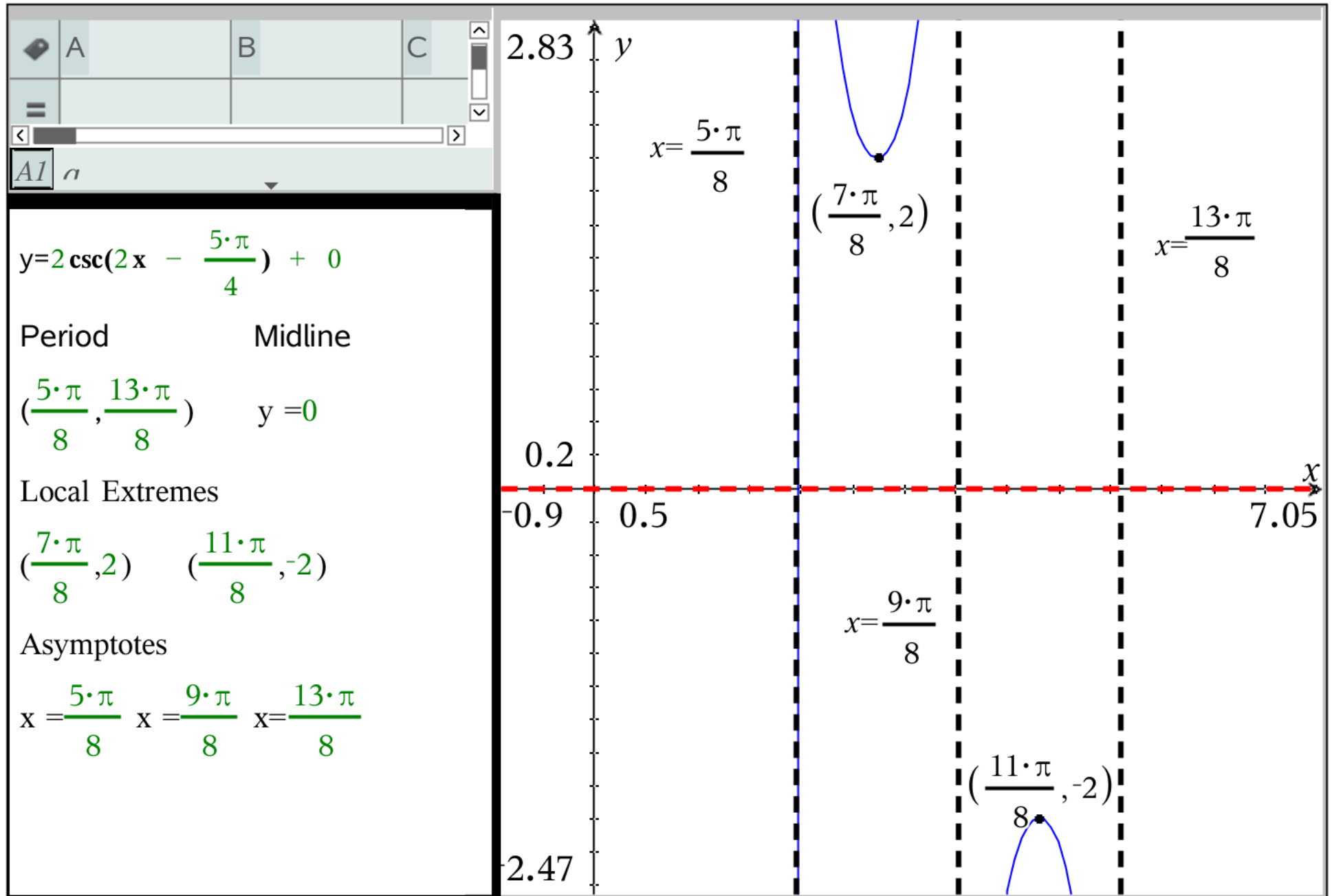
$$y = -2 \sec\left(\frac{2}{3}\left(x + \frac{3 \cdot \pi}{4}\right)\right) + 2$$

$$y = -2 \sec\left(\frac{2}{3}x + \frac{\pi}{2}\right) + 2$$





Problem 3



Step 1 What are the a, b, c d of the transformation?

$$a=2 \quad b=2 \quad c = \frac{-5 \cdot \pi}{4} \quad d = 0$$

Step 2: Factor INTERIOR function

$$y = 2 \csc\left(2x - \frac{5 \cdot \pi}{4}\right) + 0 \quad \text{NOTE: This asking you to do this } \frac{-5 \cdot \pi}{4} \div 2 = \frac{-5 \cdot \pi}{4} \cdot \frac{1}{2} = \frac{-5 \cdot \pi}{8}$$

$$y = 2 \csc\left(2\left(x - \frac{5 \cdot \pi}{8}\right)\right) + 0$$

Note: Since $\frac{c}{b} = \frac{-5 \cdot \pi}{8}$ we know that the horizontal shift is **SHIFT RIGHT**

Note: Since $b = 2$, We know that there is a horizontal **compression**

Note: Since we know there is **SHIFT RIGHT**

and we have factored form, A period begins at $x = \frac{5 \cdot \pi}{8}$

Step 3: Determine the length of period using period = $\frac{2 \cdot \pi}{|b|}$

NOTE: This asking you to do this $2 \cdot \pi \div 2 = 2 \cdot \pi \cdot \frac{1}{2} = \pi$ So we know the period is **π LONG**

$$a=2 \quad b=2 \quad c = \frac{-5 \cdot \pi}{4} \quad d=0 \quad y=2 \csc\left(2x - \frac{5 \cdot \pi}{4}\right) + 0 \quad y=2 \csc\left(2\left(x - \frac{5 \cdot \pi}{8}\right)\right) + 0$$

period length is π and ONE PERIOD starts at $\frac{5 \cdot \pi}{8}$

Step 4: Determine the FIVE IMPORTANT x values

$$\text{start} = \frac{5 \cdot \pi}{8} \quad (\text{this is the first asymptote of cosecant})$$

$$\text{start} + \frac{1 \cdot \text{period}}{4} = \frac{5 \cdot \pi}{8} + \frac{\pi}{4} = \frac{7 \cdot \pi}{8} \quad (\text{this is where one of the local extremes occurs})$$

$$\text{start} + \frac{2 \cdot \text{period}}{4} = \frac{5 \cdot \pi}{8} + \frac{\pi}{2} = \frac{9 \cdot \pi}{8} \quad (\text{this is the second asymptote of cosecant})$$

$$\text{start} + \frac{3 \cdot \text{period}}{4} = \frac{5 \cdot \pi}{8} + \frac{3 \cdot \pi}{4} = \frac{11 \cdot \pi}{8} \quad (\text{this is where one of the local extremes occurs})$$

$$\text{start} + \text{period} = \frac{5 \cdot \pi}{8} + \pi = \frac{13 \cdot \pi}{8} \quad (\text{this is the third asymptote of cosecant})$$

$$a=2 \quad b=2 \quad c = \frac{-5 \cdot \pi}{4} \quad d=0 \quad y=2 \csc\left(2x - \frac{5 \cdot \pi}{4}\right) + 0 \quad y=2 \csc\left(2\left(x - \frac{5 \cdot \pi}{8}\right)\right) + 0$$

period length is π and ONE PERIOD starts at $\frac{5 \cdot \pi}{8}$

"cool stuff" happens at $x = \left\{ \frac{5 \cdot \pi}{8}, \frac{7 \cdot \pi}{8}, \frac{9 \cdot \pi}{8}, \frac{11 \cdot \pi}{8}, \frac{13 \cdot \pi}{8} \right\}$

Step 5: Use d to determine midline $y = 0$

Step 6: Use a and b to determine IF and WHAT KIND of reflections have occurred

$$a = 2 \quad b = 2$$

Since $a > 0$ and $b > 0$,

we know that **THERE IS NO** vertical reflection and **THERE IS NO** horizontal reflection

Step 7: Find y coordinates of the local extremes using a and d

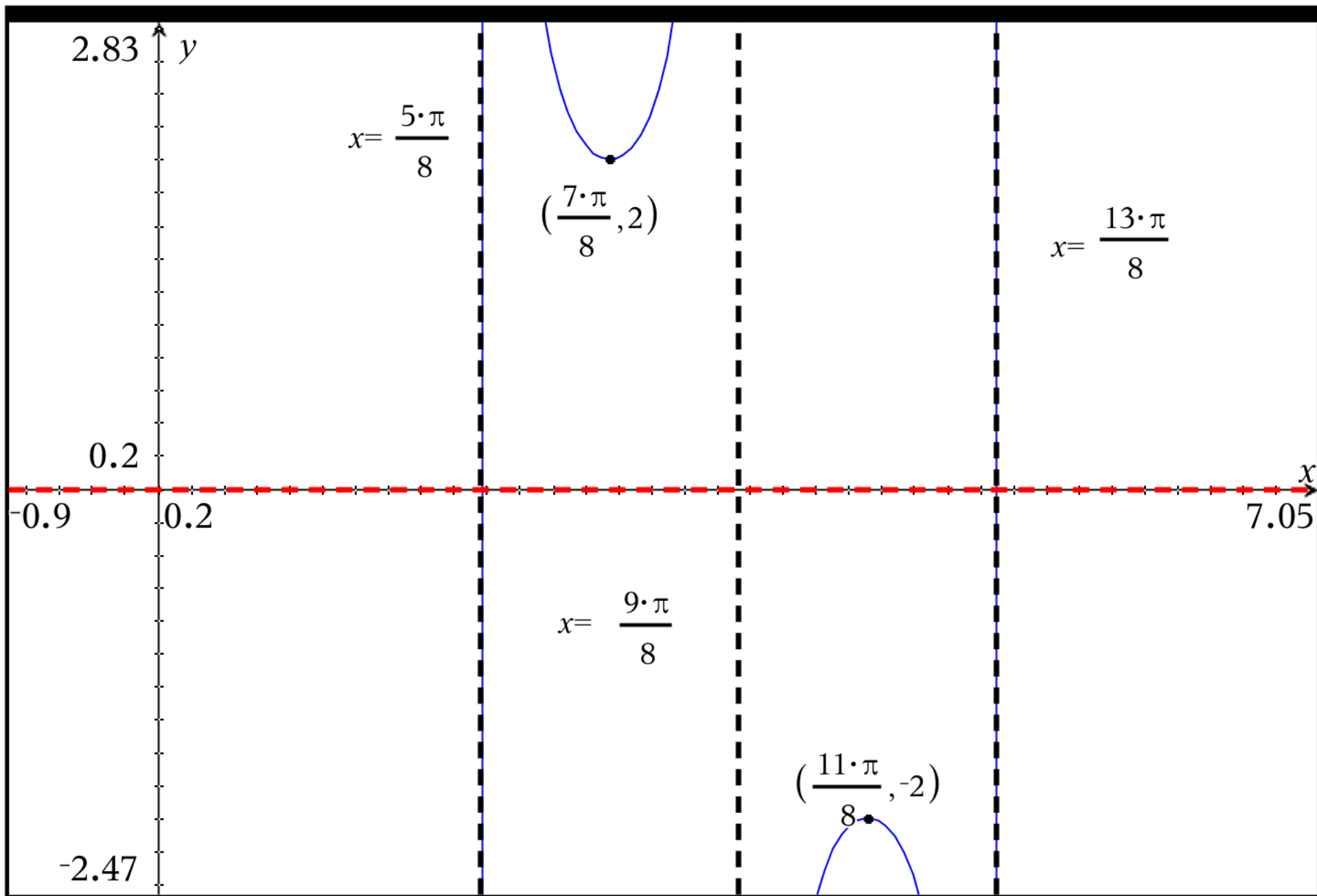
Since $a = 2$ and $d = 0$

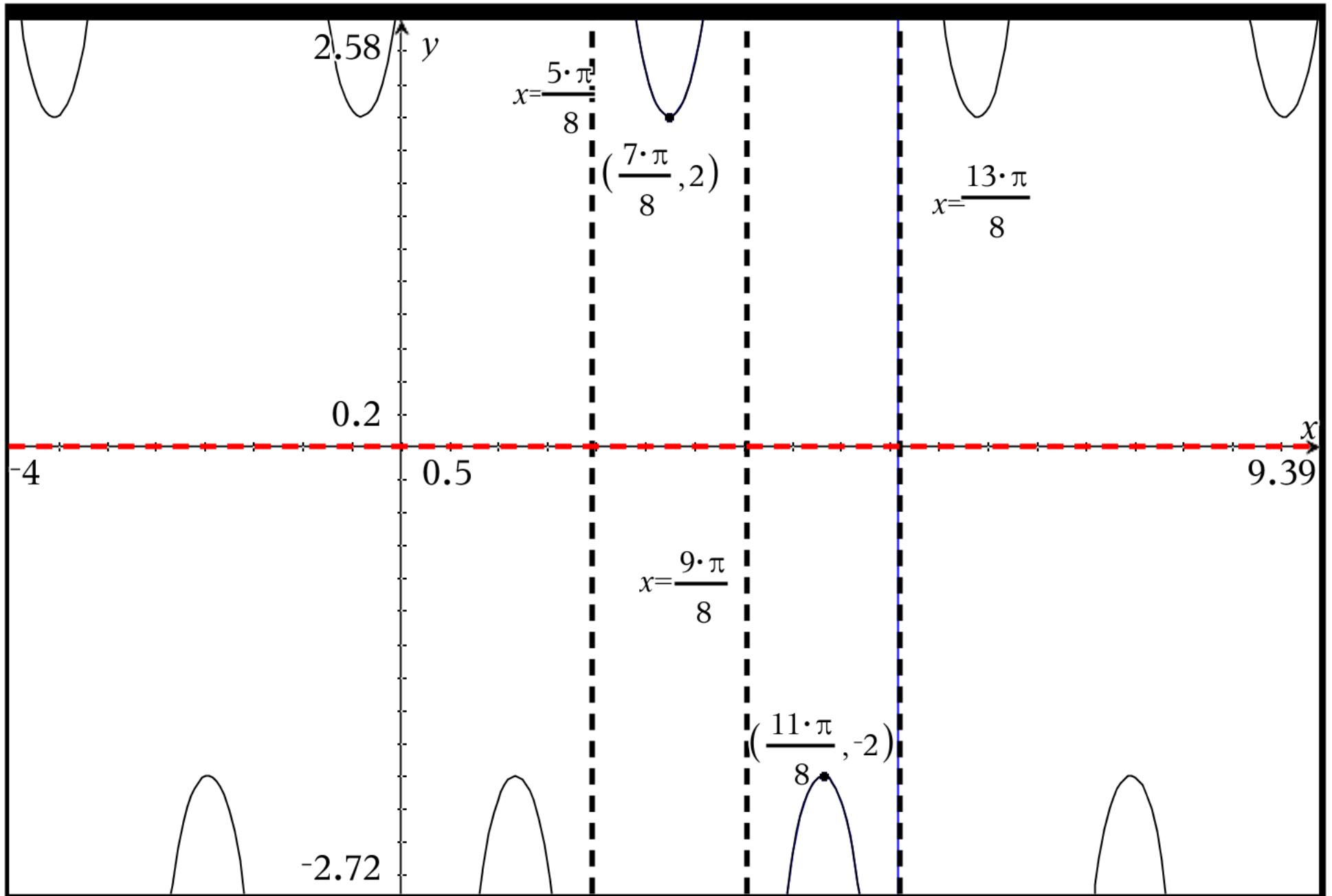
local extremes (top and bottom of "U" shape) occur at $y = d + |a| = 0 + 2 = 2$ (bottom of a "U")

$$y = d - |a| = 0 - 2 = -2 \text{ (top of a "U")}$$

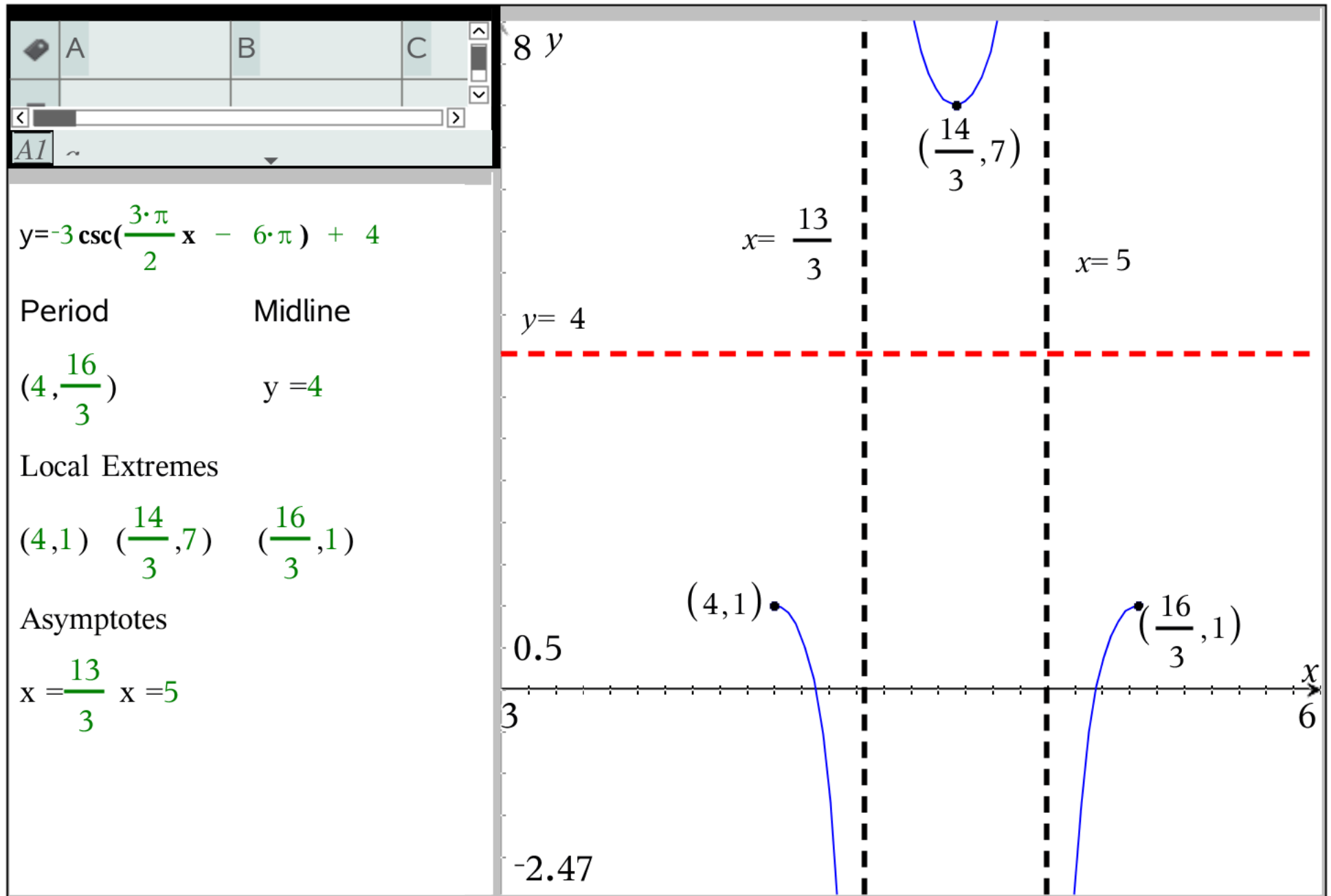
Step 8: Build local extremes

$$\left(\frac{7 \cdot \pi}{8}, 2\right) \text{ and } \left(\frac{11 \cdot \pi}{8}, -2\right)$$





Problem 4



Step 1 What are the a, b, c d of the transformation?

$$a=-3 \quad b=\frac{3\cdot\pi}{2} \quad c=-6\cdot\pi \quad d=4$$

Step 2: Factor INTERIOR function

$$y=-3 \sec\left(\frac{3\cdot\pi}{2}x - 6\cdot\pi\right) + 4 \quad \text{NOTE: This asking you to do this } -6\cdot\pi \div \frac{3\cdot\pi}{2} = -6\cdot\pi \cdot \frac{2}{3\cdot\pi} = -4$$

$$y=-3 \csc\left(\frac{3\cdot\pi}{2}(x - 4)\right) + 4$$

Note: Since $\frac{c}{b} = -4$ we know that the horizontal shift is **SHIFT RIGHT**

Note: Since $b = \frac{3\cdot\pi}{2}$, We know that there is a horizontal **compression**

Note: Since we know there is **SHIFT RIGHT**

and we have factored form, A period begins at $x = 4$

Step 3: Determine the length of period using period $= \frac{2\cdot\pi}{|b|}$

NOTE: This asking you to do this $2\cdot\pi \div \frac{3\cdot\pi}{2} = 2\cdot\pi \cdot \frac{2}{3\cdot\pi} = \frac{4}{3}$ So we know the period is $\frac{4}{3}$ LONG

$$a=-3 \quad b=\frac{3\cdot\pi}{2} \quad c=-6\cdot\pi \quad d=4 \quad y=-3\sec\left(\frac{3\cdot\pi}{2}x - 6\cdot\pi\right) + 4 \quad y=-3\sec\left(\frac{3\cdot\pi}{2}(x - 4)\right) + 4$$

period length is $\frac{4}{3}$ and ONE PERIOD starts at 4

Step 4: Determine the FIVE IMPORTANT x values

start = 4 (this is the first local extreme of secant)

$$\text{start} + \frac{1 \cdot \text{period}}{4} = 4 + \frac{1}{3} = \frac{13}{3} \quad (\text{this is where first asymptote occurs})$$

$$\text{start} + \frac{2 \cdot \text{period}}{4} = 4 + \frac{2}{3} = \frac{14}{3} \quad (\text{this is the second local extreme of secant})$$

$$\text{start} + \frac{3 \cdot \text{period}}{4} = 4 + 1 = 5 \quad (\text{this is the second asymptote of secant})$$

$$\text{start} + \text{period} = 4 + \frac{4}{3} = \frac{16}{3} \quad (\text{this is the last local extreme of secant})$$

period length is $\frac{4}{3}$ and ONE PERIOD starts at 4

"cool stuff" happens at $x = \{4, \frac{13}{3}, \frac{14}{3}, 5, \frac{16}{3}\}$

Step 5: Use d to determine midline $y = 4$

Step 6: Use a and b to determine IF and WHAT KIND of reflections have occurred

$$a = -3 \quad b = \frac{3 \cdot \pi}{2}$$

Since $a < 0$ and $b > 0$,

we know that **THERE IS A** vertical reflection and **THERE IS NO** horizontal reflection

Step 7: Find y coordinates of the local extremes using a and d

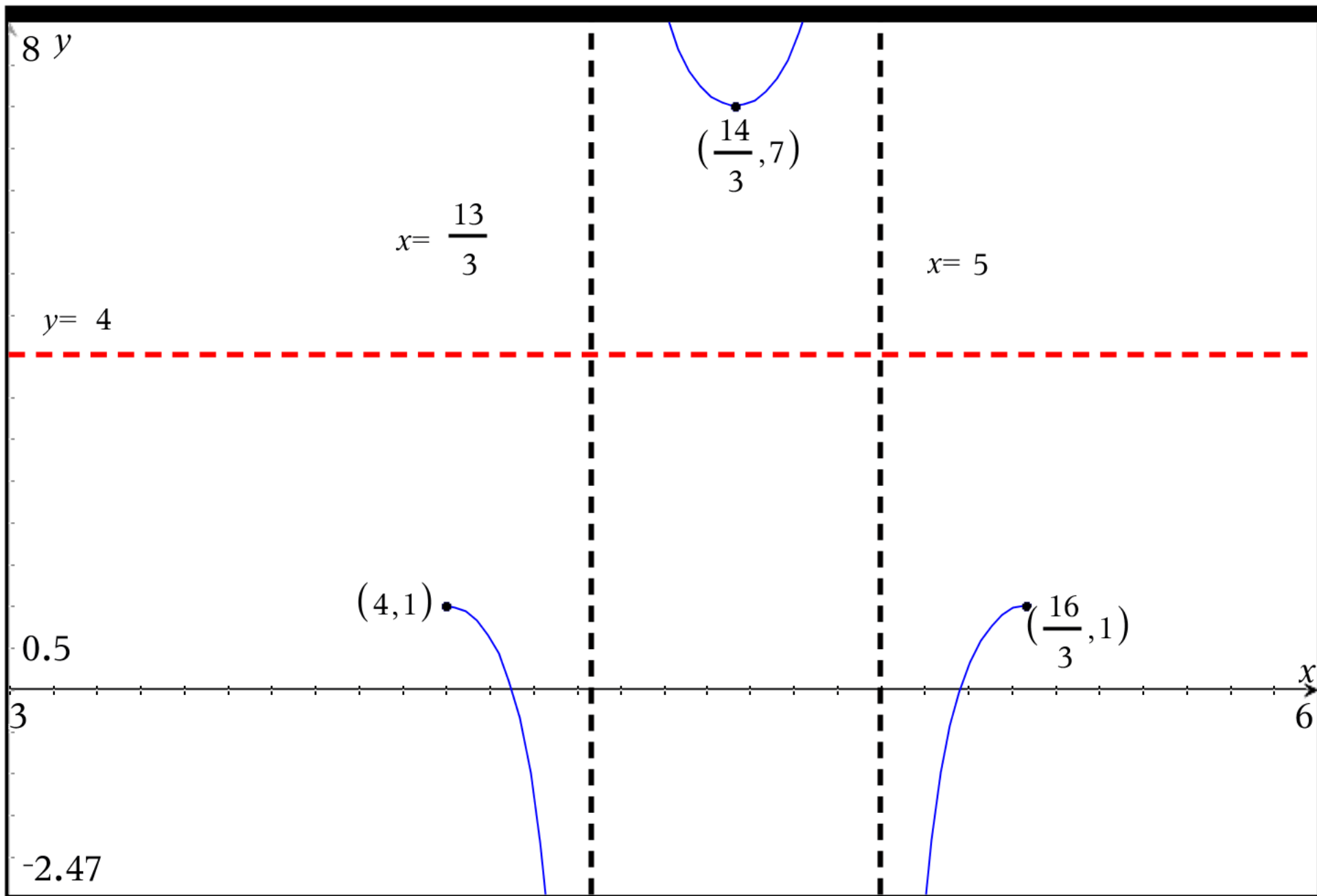
Since $a = -3$ and $d = 4$

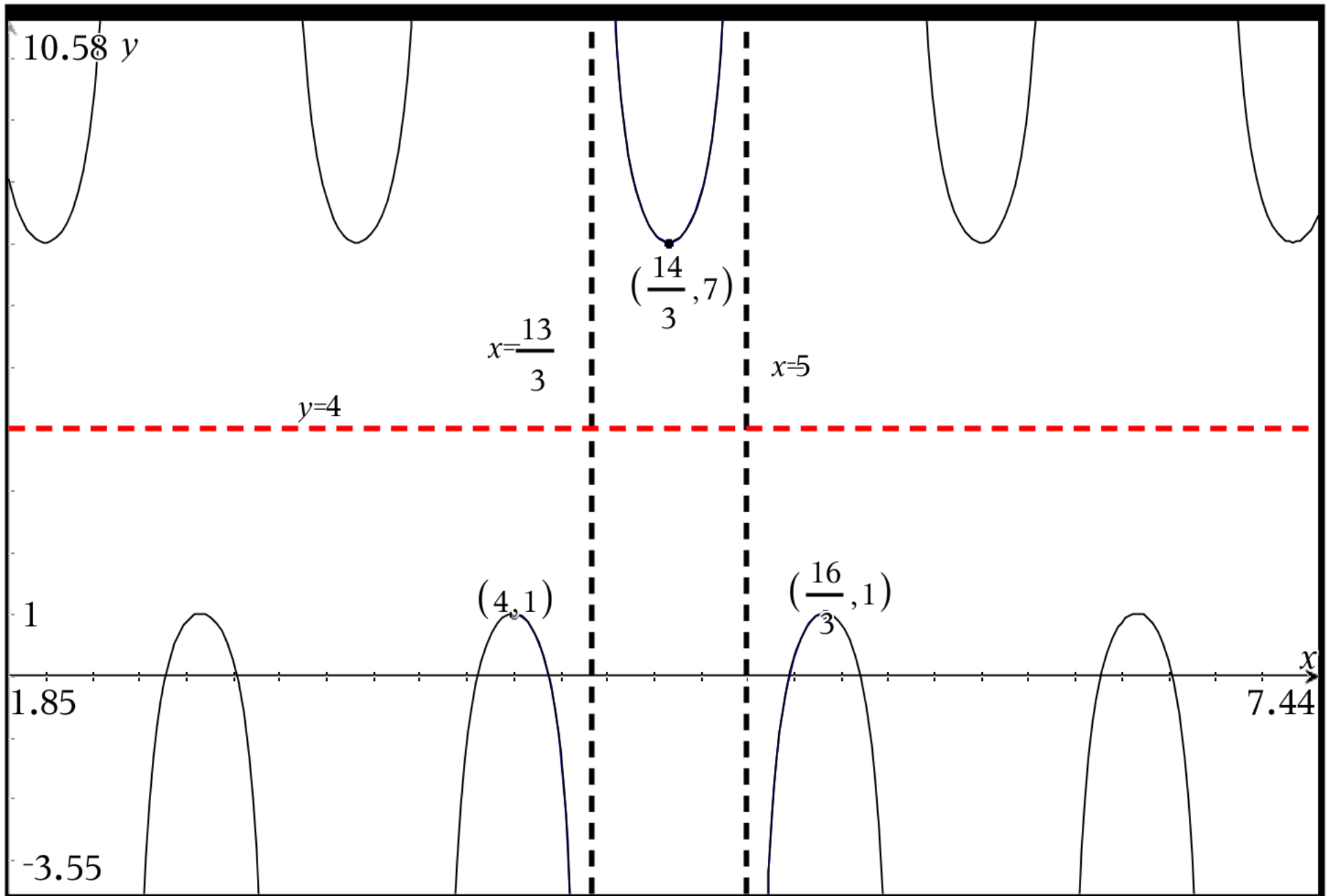
local extremes (top and bottom of "U" shape) occur at $y = d + |a| = 4 + 3 = 7$ (bottom of a "U")

$$y = d - |a| = 4 - 3 = 1 \text{ (top of a "U")}$$

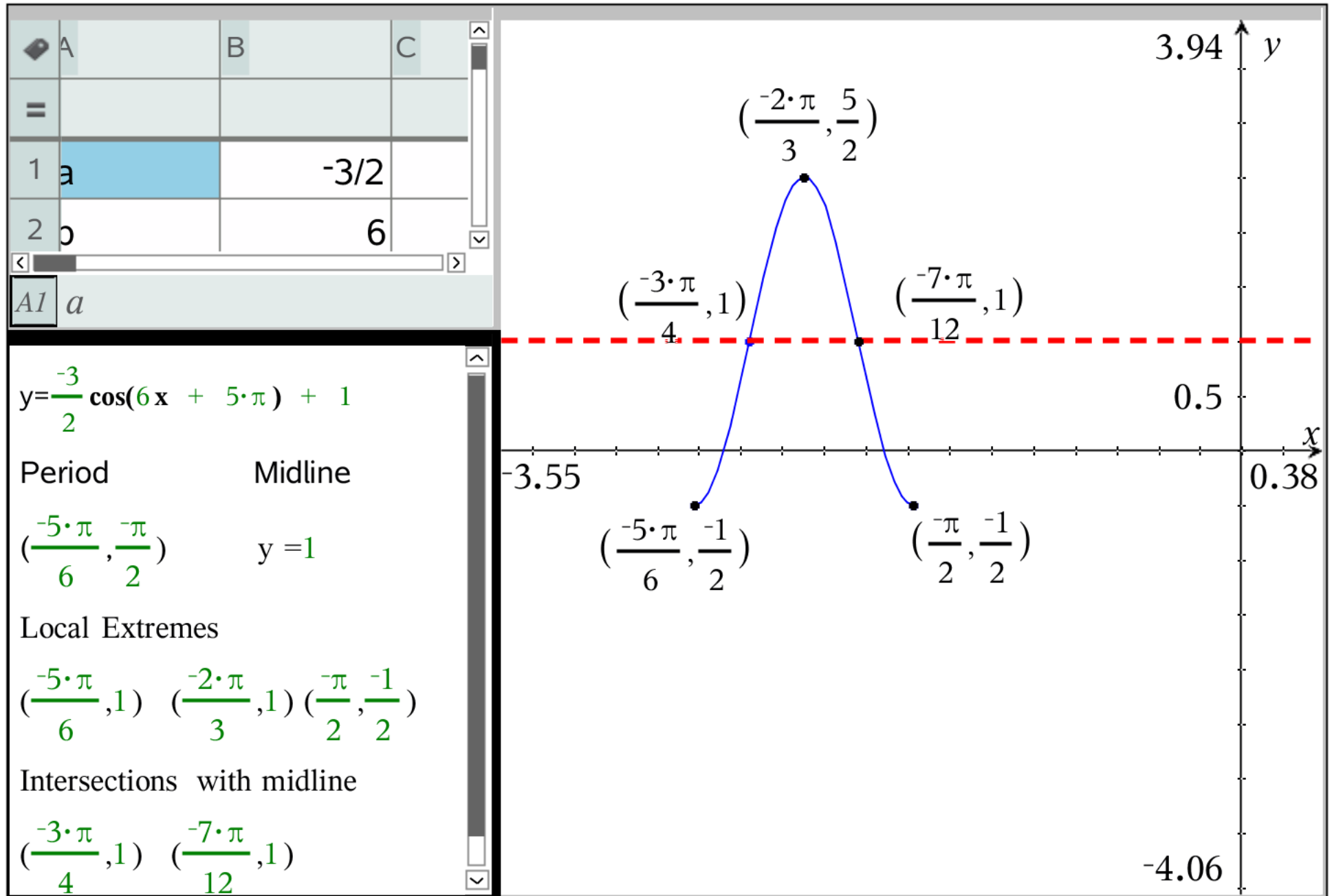
Step 8: Build local extremes

$$(4, 1) \text{ and } (\frac{14}{3}, 7) \text{ and } (\frac{16}{3}, 1)$$





Problem 5



Step 1 What are the a, b, c d of the transformation?

$$a = \frac{-3}{2} \quad b = 6 \quad c = 5 \cdot \pi \quad d = 1$$

Step 2: Factor INTERIOR function

$$y = \frac{-3}{2} \cos(6x + 5 \cdot \pi) + 1 \quad \text{NOTE: This asking you to do this } 5 \cdot \pi \div 6 = 5 \cdot \pi \cdot \frac{1}{6} = \frac{5 \cdot \pi}{6}$$

$$y = \frac{-3}{2} \csc\left(6\left(x + \frac{5 \cdot \pi}{6}\right)\right) + 1$$

Note: Since $\frac{c}{b} = \frac{5 \cdot \pi}{6}$ we know that the horizontal shift is **SHIFT LEFT**

Note: Since $b = 6$, We know that there is a horizontal **compression**

Note: Since we know there is **SHIFT LEFT**

and we have factored form, A period begins at $x = \frac{-5 \cdot \pi}{6}$

Step 3: Determine the length of period using period = $\frac{2 \cdot \pi}{|b|}$

NOTE: This asking you to do this $2 \cdot \pi \div 6 = 2 \cdot \pi \cdot \frac{1}{6} = \frac{\pi}{3}$ So we know the period is $\frac{\pi}{3}$ LONG

$$a = \frac{-3}{2} \quad b = 6 \quad c = 5 \cdot \pi \quad d = 1 \quad y = \frac{-3}{2} \cos(6x + 5 \cdot \pi) + 1 \quad y = \frac{-3}{2} \cos\left(6\left(x + \frac{5 \cdot \pi}{6}\right)\right) + 1$$

period length is $\frac{\pi}{3}$ and ONE PERIOD starts at $\frac{-5 \cdot \pi}{6}$

Step 4: Determine the FIVE IMPORTANT x values

$$\text{start} = \frac{-5 \cdot \pi}{6} \quad (\text{this is the first local extreme of cosine})$$

$$\text{start} + \frac{1 \cdot \text{period}}{4} = \frac{-5 \cdot \pi}{6} + \frac{\pi}{12} = \frac{-3 \cdot \pi}{4} \quad (\text{this is where first intersection of midline occurs})$$

$$\text{start} + \frac{2 \cdot \text{period}}{4} = \frac{-5 \cdot \pi}{6} + \frac{\pi}{6} = \frac{-2 \cdot \pi}{3} \quad (\text{this is the second local extreme of cosine})$$

$$\text{start} + \frac{3 \cdot \text{period}}{4} = \frac{-5 \cdot \pi}{6} + \frac{\pi}{4} = \frac{-7 \cdot \pi}{12} \quad (\text{this is where first intersection of midline occurs})$$

$$\text{start} + \text{period} = \frac{-5 \cdot \pi}{6} + \frac{\pi}{3} = \frac{-\pi}{2} \quad (\text{this is the last local extreme of cosine})$$

$$a = \frac{-3}{2} \quad b = 6 \quad c = -5\pi \quad d = 1 \quad y = \frac{-3}{2} \cos\left(6x + 5\pi\right) + 1 \quad y = \frac{-3}{2} \cos\left(6\left(x + \frac{5\pi}{6}\right)\right) + 1$$

period length is $\frac{\pi}{3}$ and ONE PERIOD starts at $\frac{-5\pi}{6}$

"cool stuff" happens at $x = \left\{ \frac{-5\pi}{6}, \frac{-3\pi}{4}, \frac{-2\pi}{3}, \frac{-7\pi}{12}, \frac{-\pi}{2} \right\}$

Step 5: Use d to determine midline $y = 1$

Step 6: Use a and b to determine IF and WHAT KIND of reflections have occurred $a = \frac{-3}{2}$ $b = 6$

Since $a < 0$ and $b > 0$,

we know that **THERE IS A** vertical reflection and **THERE IS NO** horizontal reflection

Step 7: Find y coordinates of the local extremes using a and d

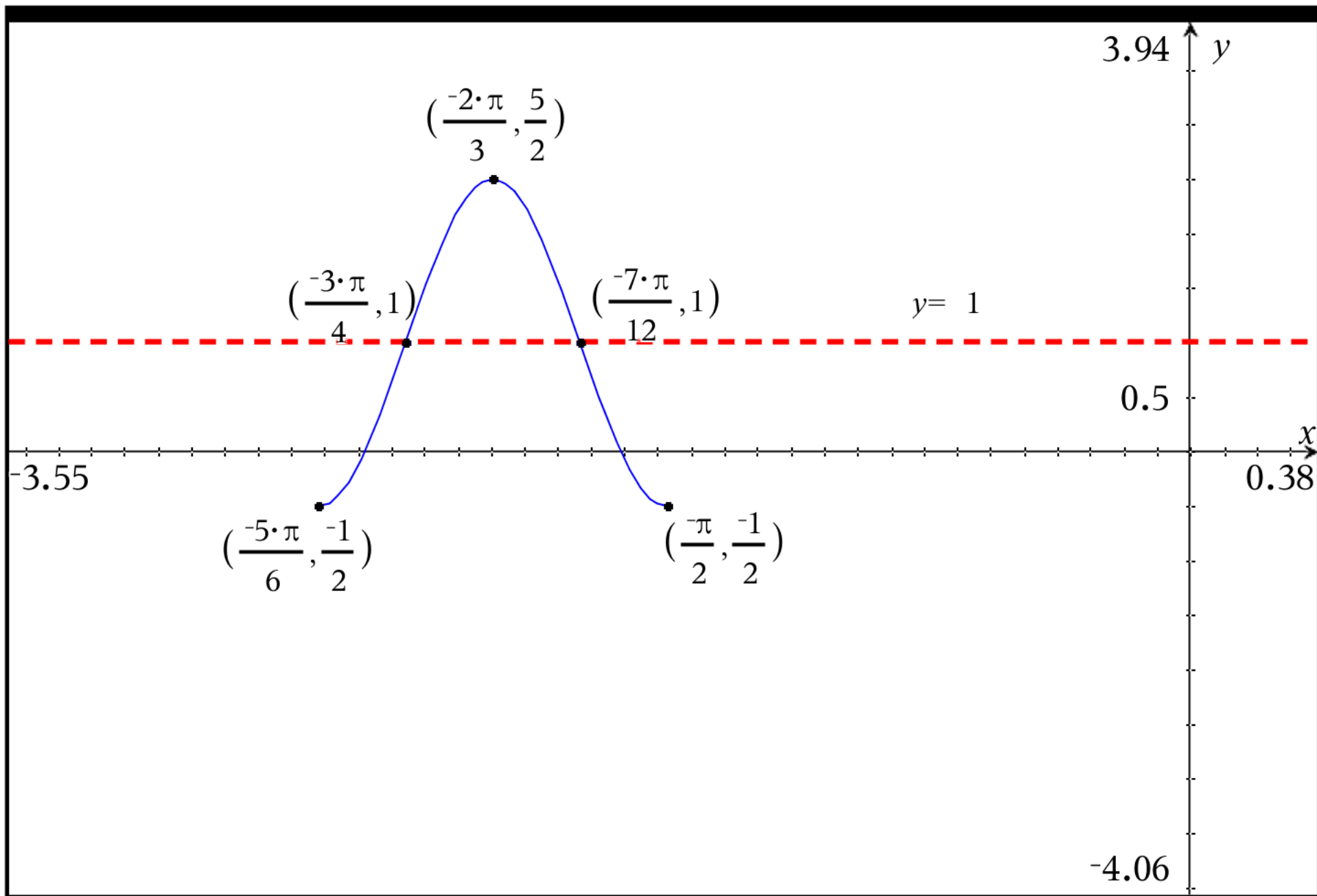
Since $a = \frac{-3}{2}$ and $d = 1$

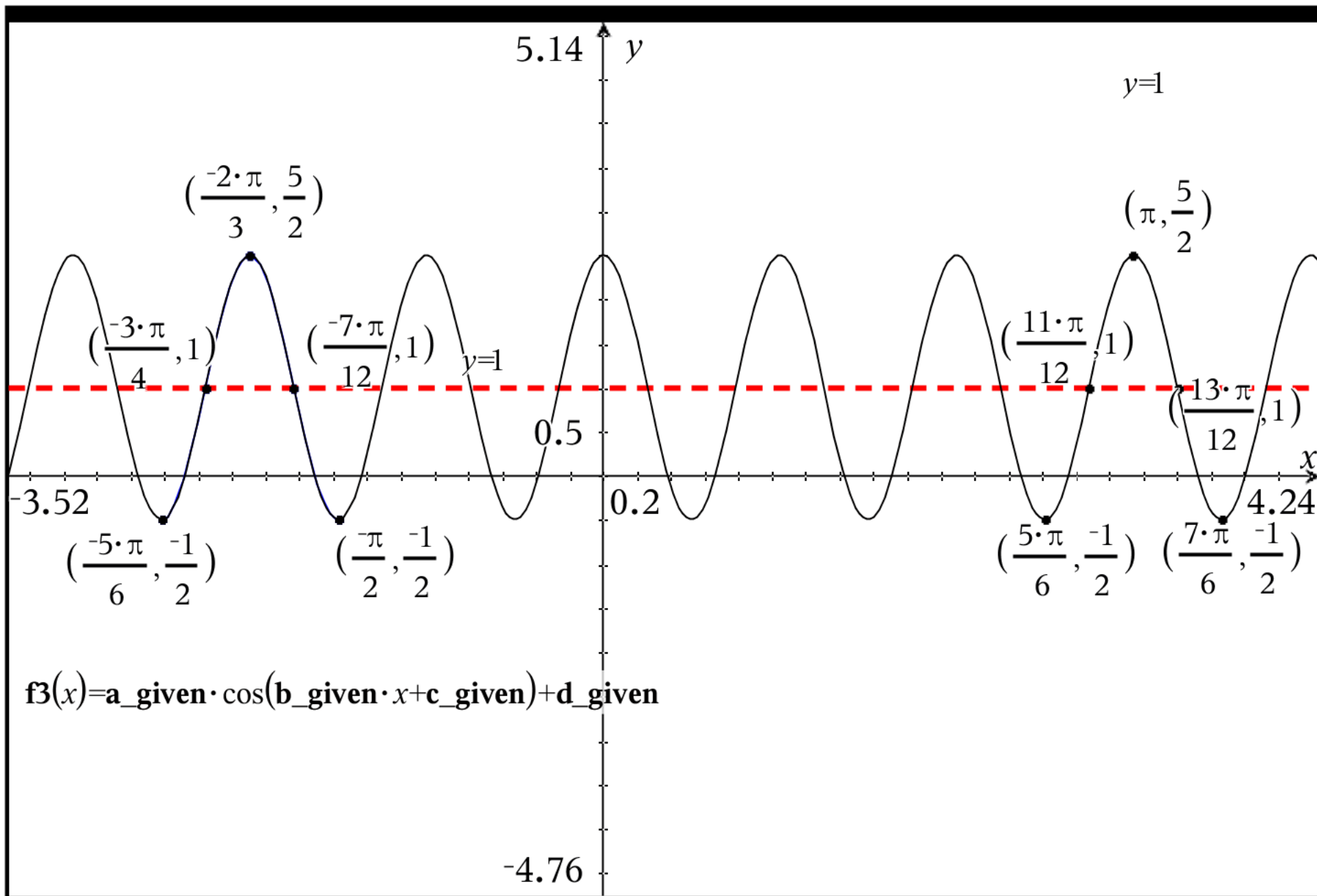
local extremes (top and bottom of "U" shape) occur at $y = d + |a| = 1 + \frac{3}{2} = \frac{5}{2}$ (TOP of a "U")

$$y = d - |a| = 1 - \frac{3}{2} = \frac{-1}{2} \quad (\text{BOTTOM of a "U"})$$

Step 8: Build local extremes

$$\left(\frac{-5\pi}{6}, \frac{-1}{2}\right) \text{ and } \left(\frac{-2\pi}{3}, \frac{5}{2}\right) \text{ and } \left(\frac{-\pi}{2}, \frac{-1}{2}\right)$$





Problem 6

A
=
1 SHIFT LEFT
2 SHIFT DOWN
3 Horizontal REFLECTIO
4 STRETCH period
5 Vertically STRETCH
6
7
8
9
10
11
A1 "SHIFT LEFT"

Fact 1: Shift LEFT implies $(x + \frac{18 \cdot \pi}{5})$

inside the secant function

So we know $y = a \sec (b(x + \frac{18 \cdot \pi}{5})) + d$

$$\text{start} = \frac{-18 \cdot \pi}{5}$$

Fact 2: Shift DOWN implies $d = -8$

outside the secant function

So we know $y = a \sec (b(x + \frac{18 \cdot \pi}{5})) - 8$

Fact 3: There is a HORIZONTAL reflection

This means $b < 0$

Fact 4: There is a stretch of period to $48 \cdot \pi$

$$0 < |b| < 1$$

$$\text{Use } |b| = \frac{2 \cdot \pi}{\text{period}} = 2 \cdot \pi \div 48 \cdot \pi = 2 \cdot \pi \cdot \frac{1}{48 \cdot \pi} = \frac{1}{24}$$

Fact 3 and Fact 4 tell us $|b| = \frac{1}{24}$ that $b = \frac{-1}{24}$ because of the horizontal reflection

We now know $y = a \sec\left(\frac{-1}{24}\left(x + \mathbf{h_shift} + \frac{18 \cdot \pi}{5}\right)\right) - 8$

Fact 5: Vertically Stretch by a factor of $\frac{15}{2}$

and the fact that NO vertical reflection has occurred we know $a = \frac{15}{2}$

We now know $y = \frac{15}{2} \sec\left(\frac{-1}{24}\left(x + \frac{18 \cdot \pi}{5}\right)\right) - 8$

Problem 7

A	B	C
=		
1 a		15/2
2 b		-1/24

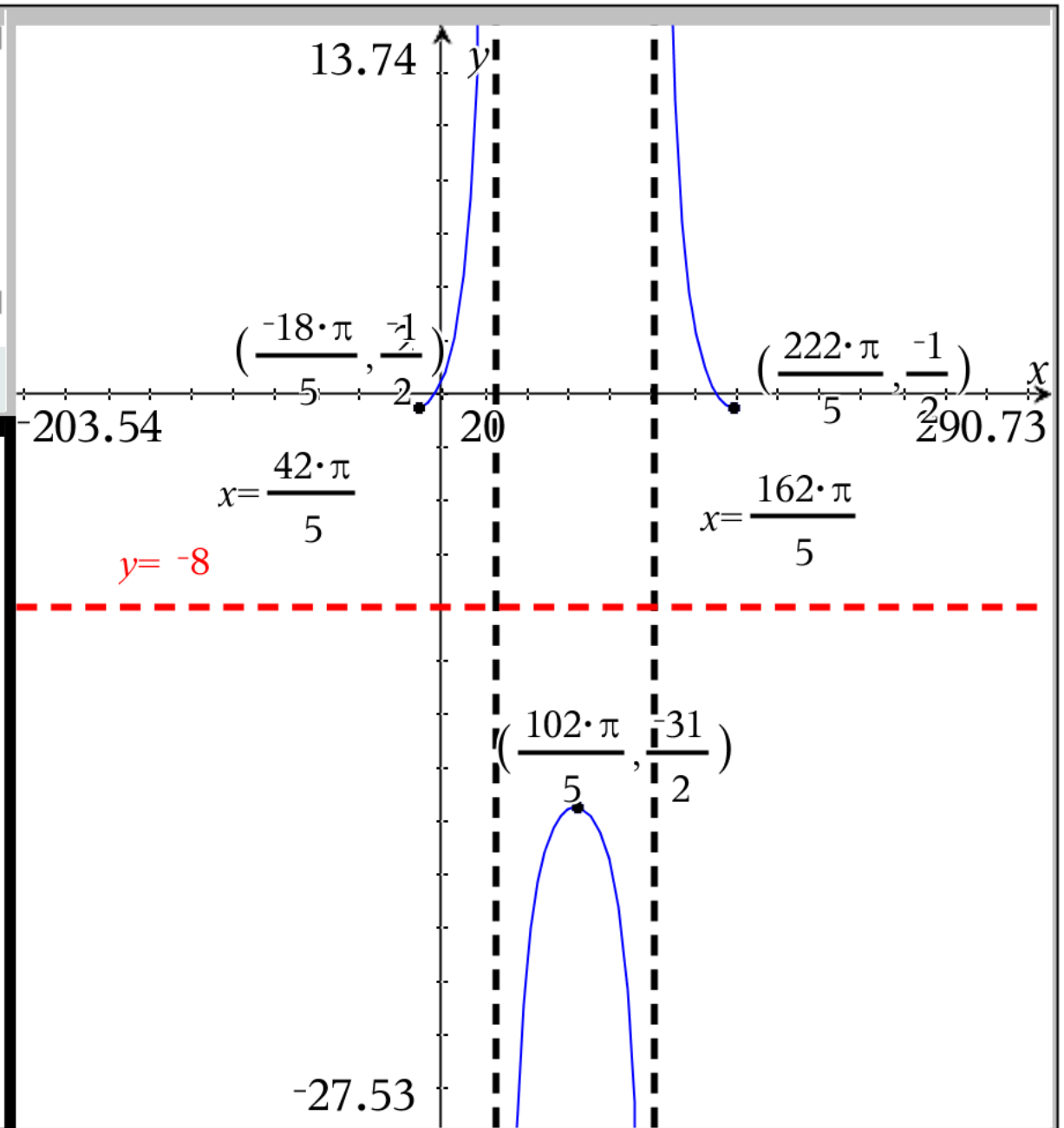
A1 a

$$y = \frac{15}{2} \sec\left(\frac{-1}{24}x - \frac{3 \cdot \pi}{20}\right) - 8$$

Period $\left(\frac{-18 \cdot \pi}{5}, \frac{222 \cdot \pi}{5}\right)$ Midline $y = -8$

Local Extremes $\left(\frac{-18 \cdot \pi}{5}, \frac{-1}{2}\right)$ $\left(\frac{102 \cdot \pi}{5}, \frac{-31}{2}\right)$ $\left(\frac{222 \cdot \pi}{5}, \frac{-1}{2}\right)$

Asymptotes $x = \frac{42 \cdot \pi}{5}$ $x = \frac{162 \cdot \pi}{5}$



Step 1 What are the a, b, c, d of the transformation?

$$a = \frac{15}{2} \quad b = \frac{-1}{24} \quad c = \frac{-3 \cdot \pi}{20} \quad d = -8$$

Step 2: Factor INTERIOR function

$$y = \frac{15}{2} \sec\left(\frac{-1}{24}x - \frac{3 \cdot \pi}{20}\right) - 8 \quad \text{NOTE: This asking you to do this } \frac{-3 \cdot \pi}{20} \div \frac{-1}{24} = \frac{-3 \cdot \pi}{20} \cdot -24 = \frac{18 \cdot \pi}{5}$$

$$y = \frac{15}{2} \sec\left(\frac{-1}{24}\left(x + \frac{18 \cdot \pi}{5}\right)\right) - 8$$

Note: Since $\frac{c}{b} = \frac{18 \cdot \pi}{5}$ we know that the horizontal shift is **SHIFT LEFT**

Note: Since $b = \frac{-1}{24}$, We know that there is a horizontal **stretch**

Note: Since we know there is **SHIFT LEFT**

and we have factored form, A period begins at $x = \frac{-18 \cdot \pi}{5}$

Step 3: Determine the length of period using period = $\frac{2 \cdot \pi}{|b|}$

NOTE: This asking you to do this $2 \cdot \pi \div \frac{-1}{24} = 2 \cdot \pi \cdot 24 = 48 \cdot \pi$ So we know the period is **48 · π LONG**

$$a = \frac{15}{2} \quad b = \frac{-1}{24} \quad c = \frac{-3 \cdot \pi}{20} \quad d = -8 \quad y = \frac{15}{2} \sec\left(\frac{-1}{24}x - \frac{3 \cdot \pi}{20}\right) - 8 \quad y = \frac{15}{2} \sec\left(\frac{-1}{24}\left(x + \frac{18 \cdot \pi}{5}\right)\right) - 8$$

period length is $48 \cdot \pi$ and ONE PERIOD starts at $\frac{-18 \cdot \pi}{5}$

Step 4: Determine the FIVE IMPORTANT x values

$$\text{start} = \frac{-18 \cdot \pi}{5} \quad (\text{this is the first local extreme of secant})$$

$$\text{start} + \frac{1 \cdot \text{period}}{4} = \frac{-18 \cdot \pi}{5} + 12 \cdot \pi = \frac{42 \cdot \pi}{5} \quad (\text{this is where first asymptote occurs})$$

$$\text{start} + \frac{2 \cdot \text{period}}{4} = \frac{-18 \cdot \pi}{5} + 24 \cdot \pi = \frac{102 \cdot \pi}{5} \quad (\text{this is the second local extreme of secant})$$

$$\text{start} + \frac{3 \cdot \text{period}}{4} = \frac{-18 \cdot \pi}{5} + 36 \cdot \pi = \frac{162 \cdot \pi}{5} \quad (\text{this is the second asymptote of secant})$$

$$\text{start} + \text{period} = \frac{-18 \cdot \pi}{5} + 48 \cdot \pi = \frac{222 \cdot \pi}{5} \quad (\text{this is the last local extreme of secant})$$

$$a = \frac{15}{2} \quad b = \frac{1}{24} \quad c = \frac{1}{20} \quad d = -8 \quad y = \frac{15}{2} \sec\left(\frac{1}{24}x - \frac{1}{20}\right) - 8 \quad y = \frac{15}{2} \sec\left(\frac{1}{24}(x + \dots)\right) - 8$$

period length is $48 \cdot \pi$ and ONE PERIOD starts at $\frac{-18 \cdot \pi}{5}$

"cool stuff" happens at $x = \left\{ \frac{-18 \cdot \pi}{5}, \frac{42 \cdot \pi}{5}, \frac{102 \cdot \pi}{5}, \frac{162 \cdot \pi}{5}, \frac{222 \cdot \pi}{5} \right\}$

Step 5: Use d to determine midline $y = d$ given

Step 6: Use a and b to determine IF and WHAT KIND of reflections have occurred $a = \frac{15}{2}$ $b = \frac{-1}{24}$

Since $a > 0$ and $b < 0$,

we know that NO vertical reflection and THERE IS horizontal reflection

Step 7: Find y coordinates of the local extremes using a and d

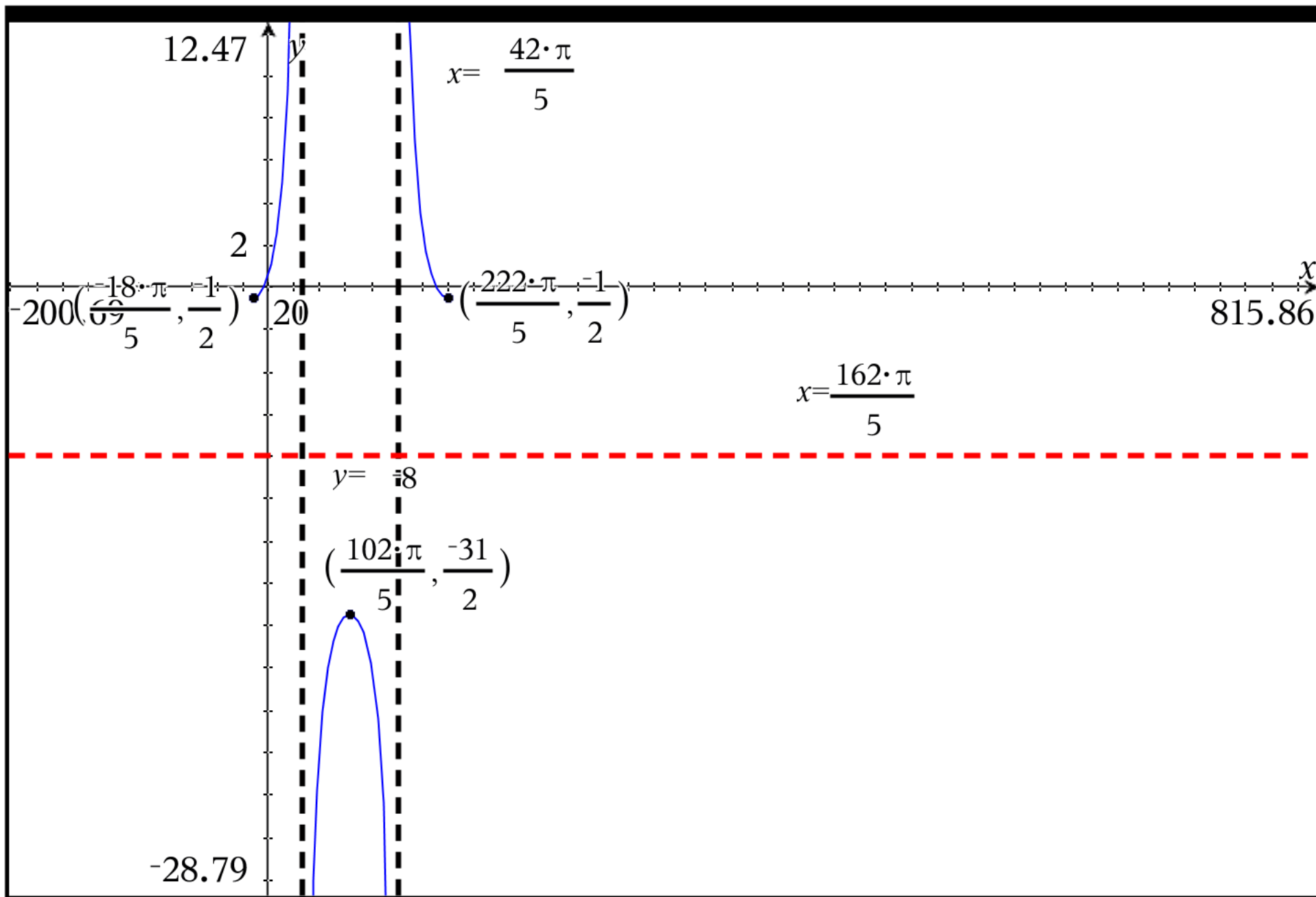
Since $a = \frac{15}{2}$ and $d = -8$

local extremes (top and bottom of "U" shape) occur at $y = d + |a| = -8 + \frac{15}{2} = \frac{-1}{2}$ (bottom of a "U")

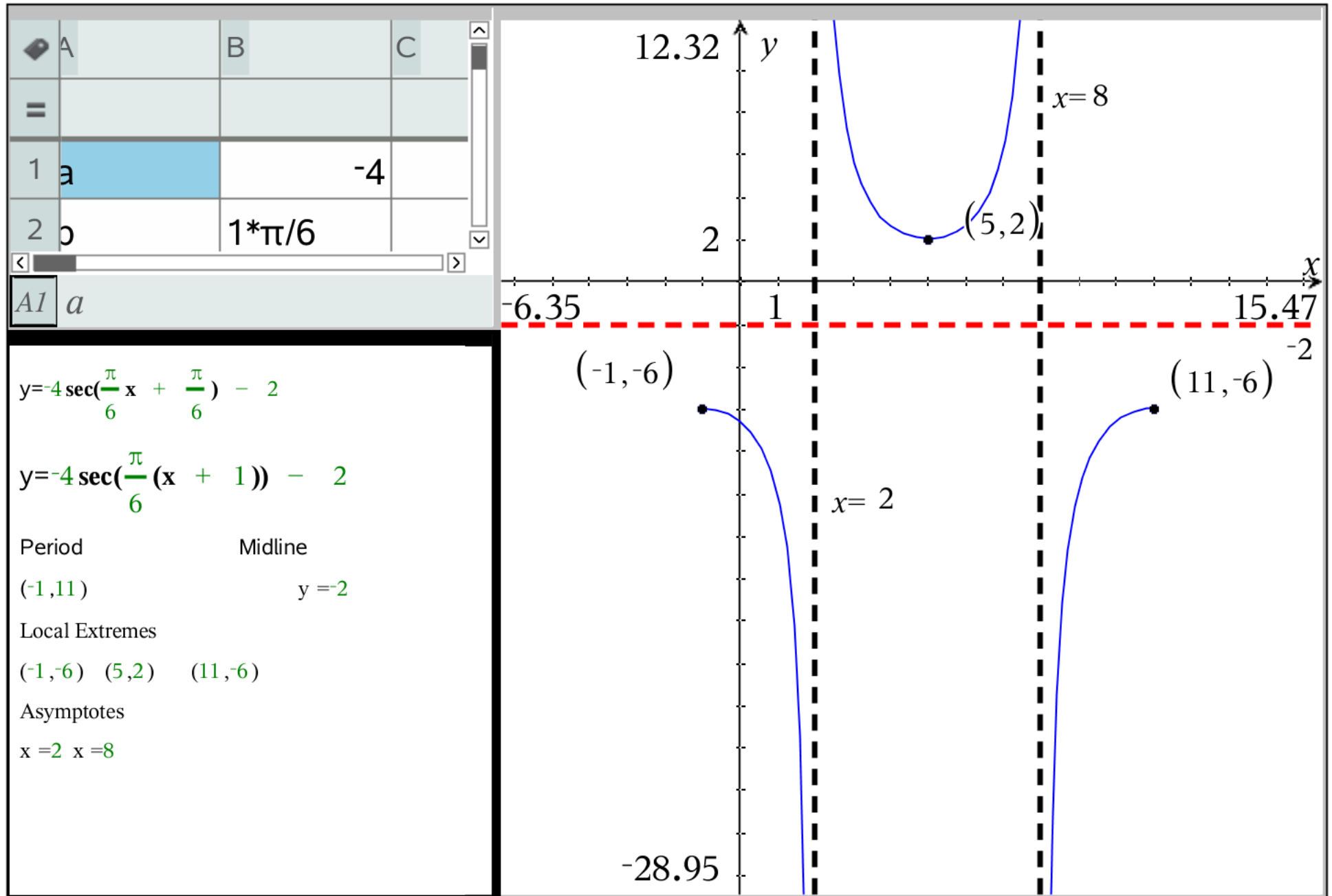
$$y = d - |a| = -8 - \frac{15}{2} = \frac{-31}{2} \text{ (top of a "U")}$$

Step 8: Build local extremes

$$\left(\frac{-18 \cdot \pi}{5}, \frac{-31}{2} \right) \text{ and } \left(\frac{102 \cdot \pi}{5}, \frac{-1}{2} \right) \text{ and } \left(\frac{222 \cdot \pi}{5}, \frac{-31}{2} \right)$$



Problem 8



Step 1 What are the a, b, c d of the transformation?

$$a=-4 \quad b=\frac{\pi}{6} \quad c=\frac{\pi}{6} \quad d=-2$$

Step 2: Factor INTERIOR function

$$y=-4 \sec\left(\frac{\pi}{6}x + \frac{\pi}{6}\right) - 2 \quad \text{NOTE: This asking you to do this } \frac{\pi}{6} \div \frac{\pi}{6} = \frac{\pi}{6} \cdot \frac{6}{\pi} = 1$$

$$y=-4 \csc\left(\frac{\pi}{6}(x + 1)\right) - 2$$

Note: Since $\frac{c}{b} = 1$ we know that the horizontal shift is **SHIFT LEFT**

Note: Since $b = \frac{\pi}{6}$, We know that there is a horizontal **stretch**

Note: Since we know there is **SHIFT LEFT**

and we have factored form, A period begins at $x = -1$

Step 3: Determine the length of period using period = $\frac{2 \cdot \pi}{|b|}$

NOTE: This asking you to do this $2 \cdot \pi \div \frac{\pi}{6} = 2 \cdot \pi \cdot \frac{6}{\pi} = 12$ So we know the period is **12 LONG**

$$a=-4 \quad b=\frac{\pi}{6} \quad c=\frac{\pi}{6} \quad d=-2 \quad y=-4\sec\left(\frac{\pi}{6}x + \frac{\pi}{6}\right) - 2 \quad y=-4\sec\left(\frac{\pi}{6}(x + 1)\right) - 2$$

period length is 12 and ONE PERIOD starts at -1

Step 4: Determine the FIVE IMPORTANT x values

start = -1 (this is the first local extreme of secant)

start + $\frac{1 \cdot \text{period}}{4} = -1 + 3 = 2$ (this is where first asymptote occurs)

start + $\frac{2 \cdot \text{period}}{4} = -1 + 6 = 5$ (this is the second local extreme of secant)

start + $\frac{3 \cdot \text{period}}{4} = -1 + 9 = 8$ (this is the second asymptote of secant)

start + **period** = -1 + 12 = 11 (this is the last local extreme of secant)

$$a=-4 \quad b=\frac{\pi}{6} \quad c=\frac{\pi}{6} \quad d=-2 \quad y=-4 \sec\left(\frac{\pi}{6}x + \frac{\pi}{6}\right) - 2 \quad y=-4 \sec\left(\frac{\pi}{6}(x + 1)\right) - 2$$

period length is 12 and ONE PERIOD starts at -1

"cool stuff" happens at $x = \{-1, 2, 5, 8, 11\}$

Step 5: Use d to determine midline $y = -2$

Step 6: Use a and b to determine IF and WHAT KIND of reflections have occurred $a = -4$ $b = \frac{\pi}{6}$

Since $a < 0$ and $b > 0$,

we know that **THERE IS A** vertical reflection and **THERE IS NO** horizontal reflection

Step 7: Find y coordinates of the local extremes using a and d

Since $a = -4$ and $d = -2$

local extremes (top and bottom of "U" shape) occur at $y = d + |a| = -2 + 4 = 2$ (bottom of a "U")

$$y = d - |a| = -2 - 4 = -6 \text{ (top of a "U")}$$

Step 8: Build local extremes

$(-1, -6)$ and $(5, 2)$ and $(11, -6)$

