

	A	B	C	D
=				
1	a		2	
A1	a			

$$y = 2 \csc\left(2x + \frac{3\pi}{4}\right) + 0$$

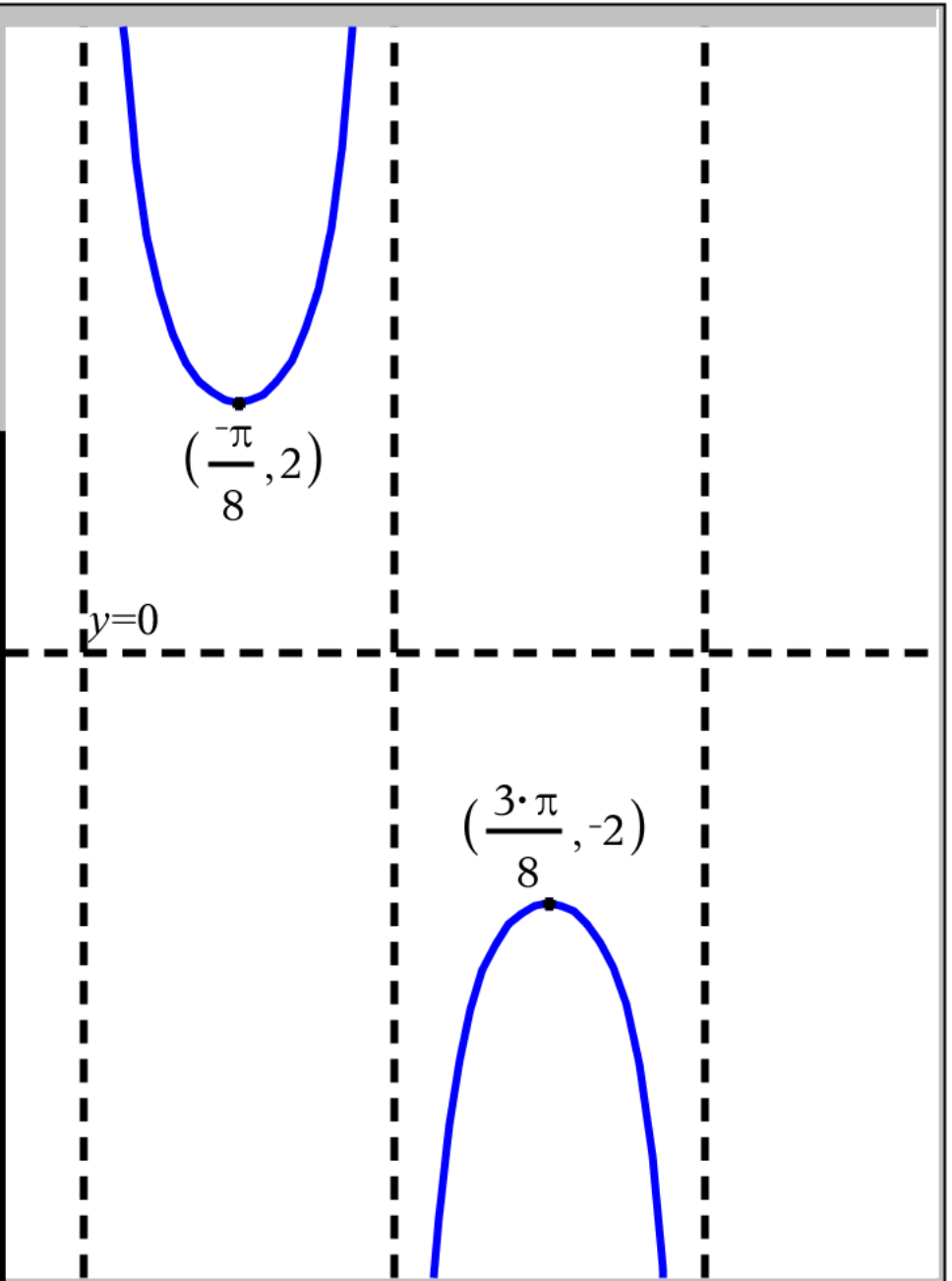
$$y = 2 \csc\left(2\left(x + \frac{3\pi}{8}\right)\right) +$$

$$\text{Period} \left(\frac{-3\pi}{8}, \frac{5\pi}{8}\right)$$

Amplitude 2 Equation of Midline  $y = 0$

$$\text{Equation of Asymptotes } x = \frac{-3\pi}{8} \quad x = \frac{\pi}{8} \quad x = \frac{5\pi}{8}$$

$$\text{Equation of All Asymptotes } x = \frac{-3\pi}{8} + \frac{n\pi}{2} \text{ with } n \in \mathbb{Z}$$



Finding b and horizontal shift

Shift to left because start =  $-\frac{3 \cdot \pi}{8}$

We know  $y = a \csc(b(x + \frac{3 \cdot \pi}{8})) + d$

Period of this function  $(\frac{-3 \cdot \pi}{8}, \frac{5 \cdot \pi}{8})$

This gives us a period length of  $\frac{5 \cdot \pi}{8} - \frac{-3 \cdot \pi}{8} = \pi$

$$|b| = 2 \cdot \pi \div \pi = 2 \cdot \pi \cdot \frac{1}{\pi} = 2$$

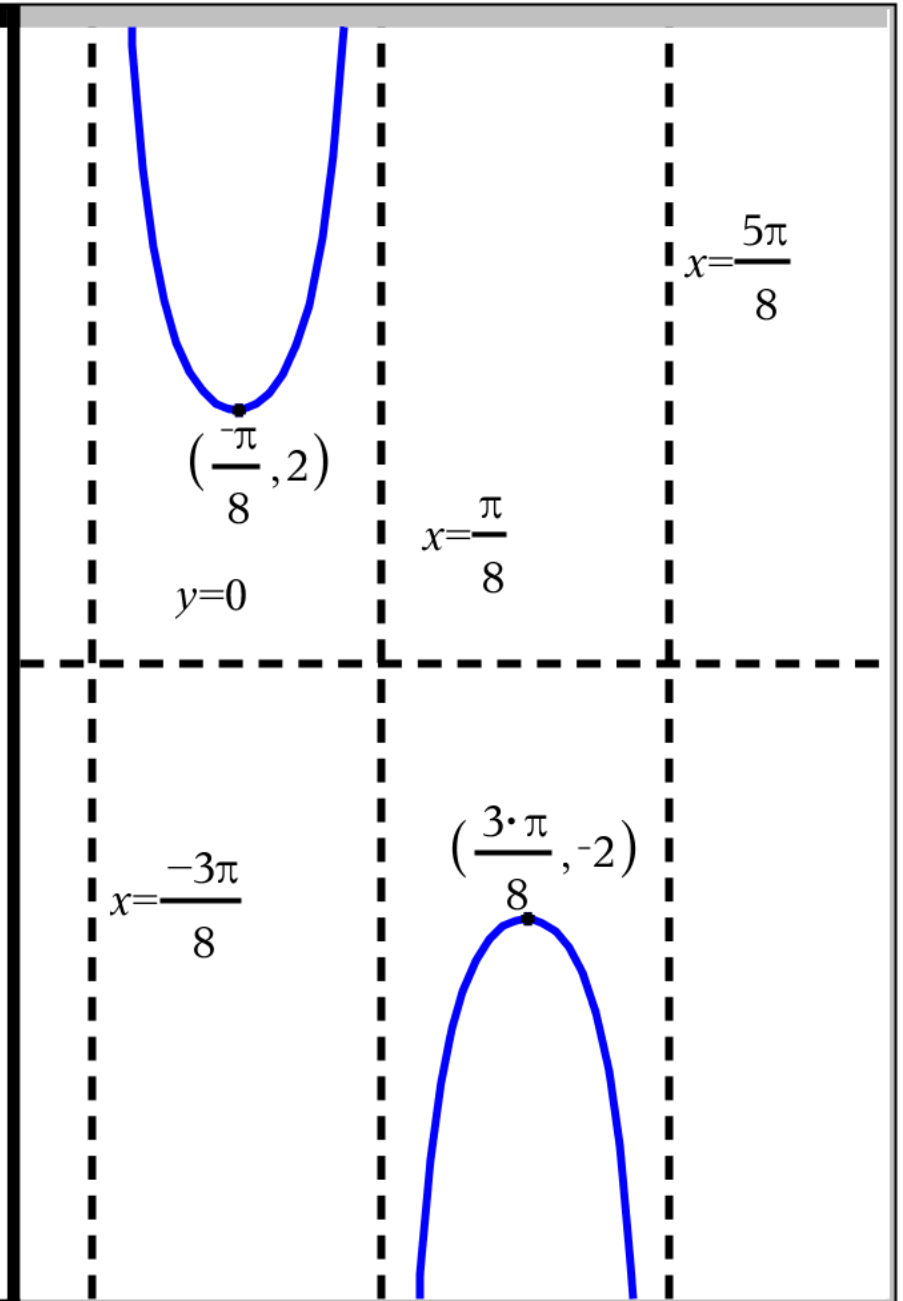
Since this IS NOT a reflection,

both a and b are positive

This means  $y = a \csc(2(x + \frac{3 \cdot \pi}{8})) + d$

Since the midline is  $y=0$ , this means

$$y = a \csc(2(x + \frac{3 \cdot \pi}{8})) + 0$$



Finding a this is the distance from the midline to the local extremes

$$|-2 - 0| = 2$$

$$|0 - 2| = 2$$

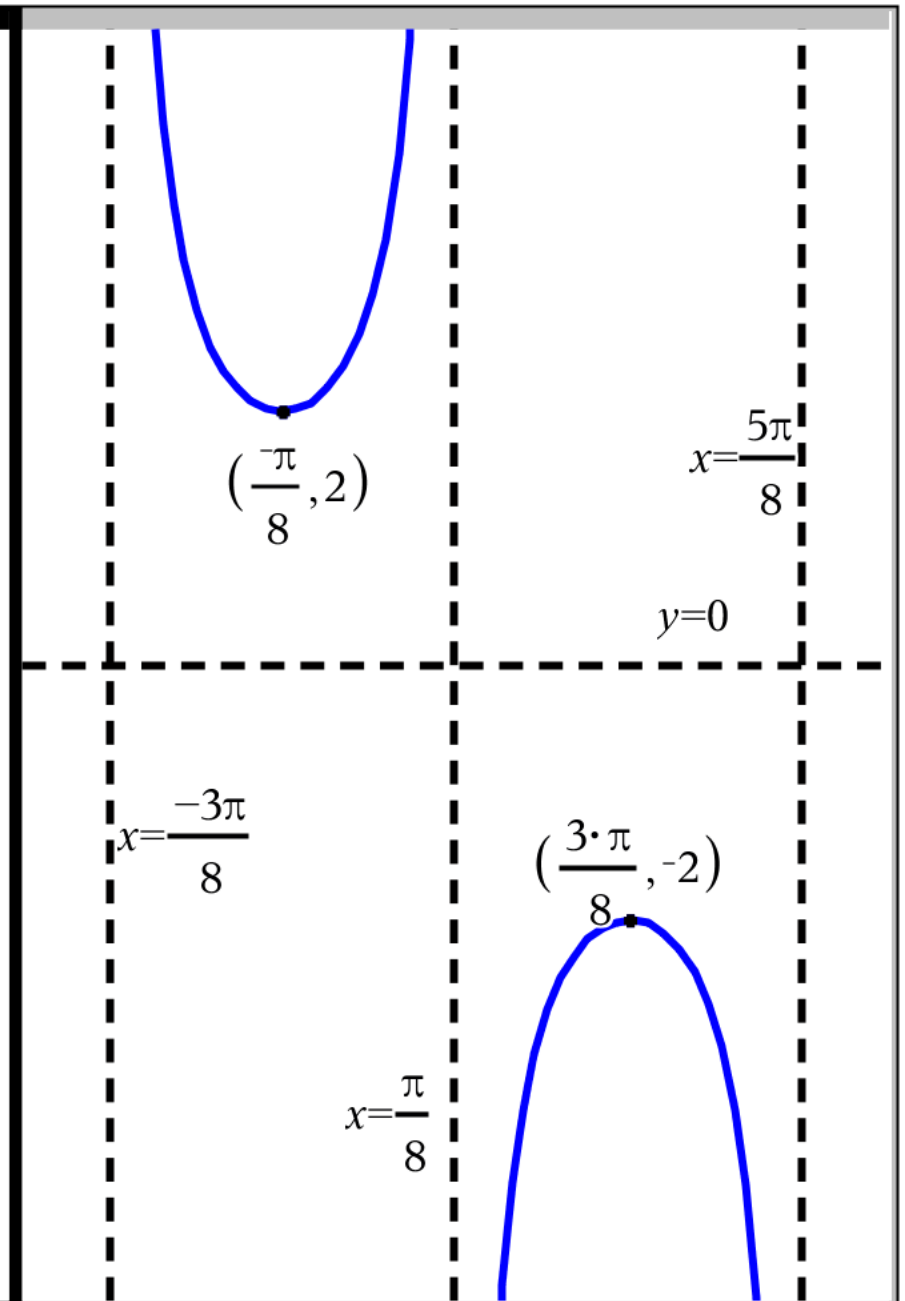
This is NOT a VERTICAL reflection

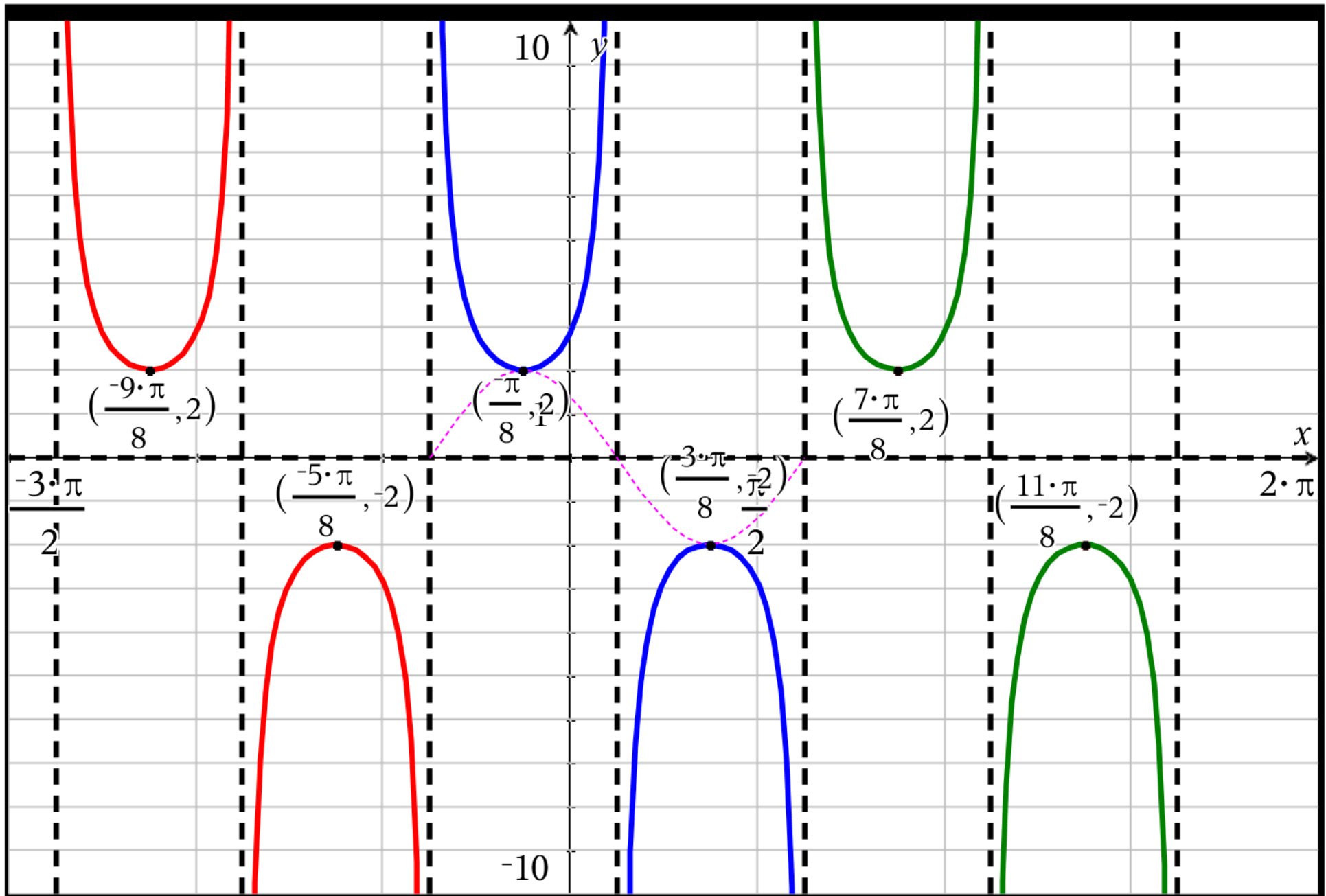
So  $a=2$

So we know that

$$y = 2 \csc\left(2\left(x + \frac{3 \cdot \pi}{8}\right)\right) + 0$$

$$y = 2 \csc\left(2x + \frac{3 \cdot \pi}{4}\right) + 0$$





sec 1

	A	B	C	D
=				
1	a		-3	
A1	a			

$$y = -3 \sec\left(\frac{1}{2}x + \frac{\pi}{2}\right) + 1$$

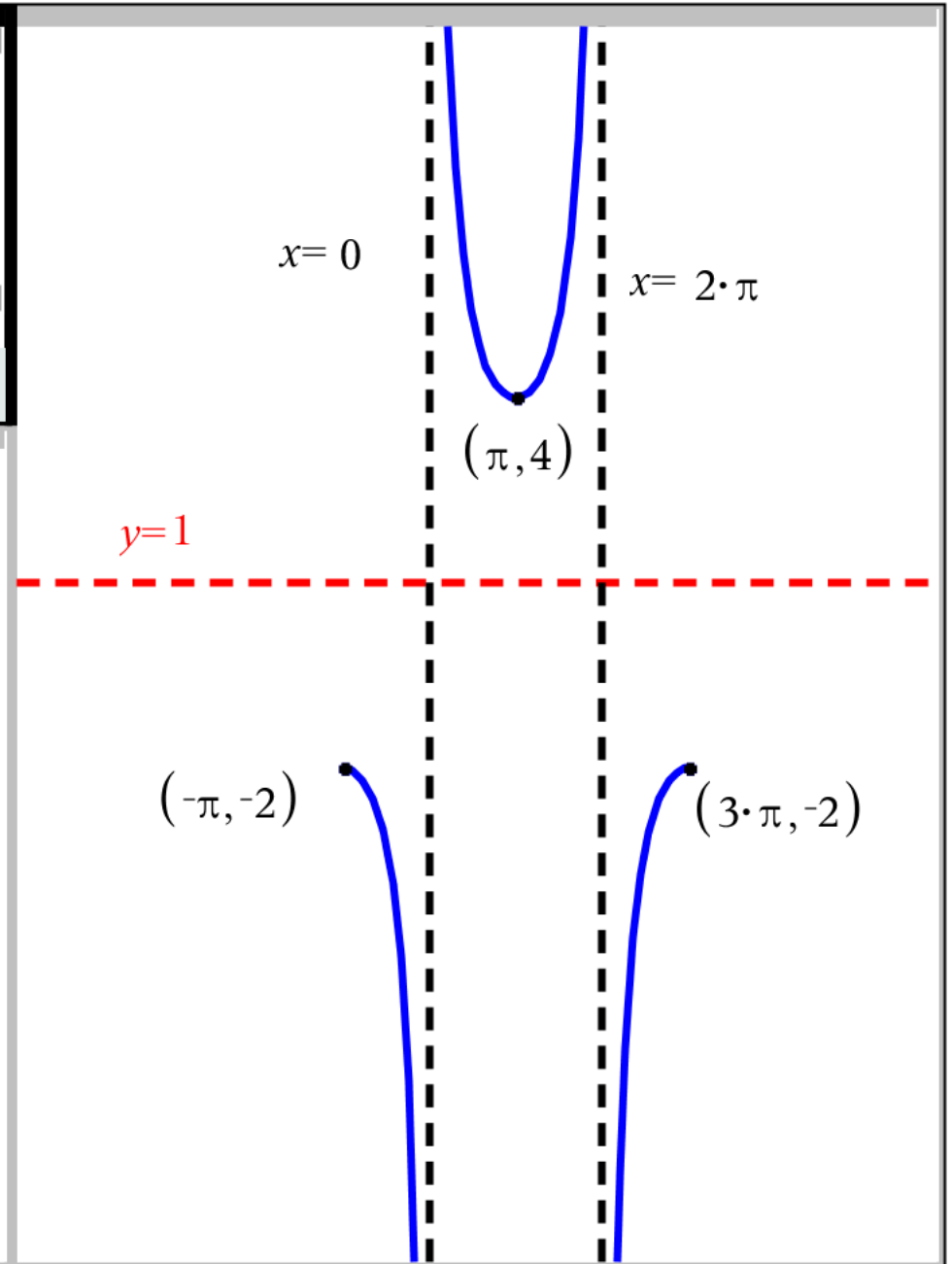
$$y = -3 \sec\left(\frac{1}{2}(x + \pi)\right) + 1$$

Period  $[-\pi, 3\pi)$

Amplitude 3 Equation of Midline  $y = 1$

Equation of Asymptotes  $x = 0$   $x = 2\pi$

Equation of All Asymptotes  $x = 0 + 2 \cdot n \cdot \pi$  with  $n \in \mathbb{Z}$



Finding b and horizontal shift

Shift to left because start  $= -\pi$

We know  $y = a \sec(b(x + \pi)) + d$

Period of this function  $[-\pi, 3\pi)$

This gives us a period length of  $3\pi - (-\pi) = 4\pi$

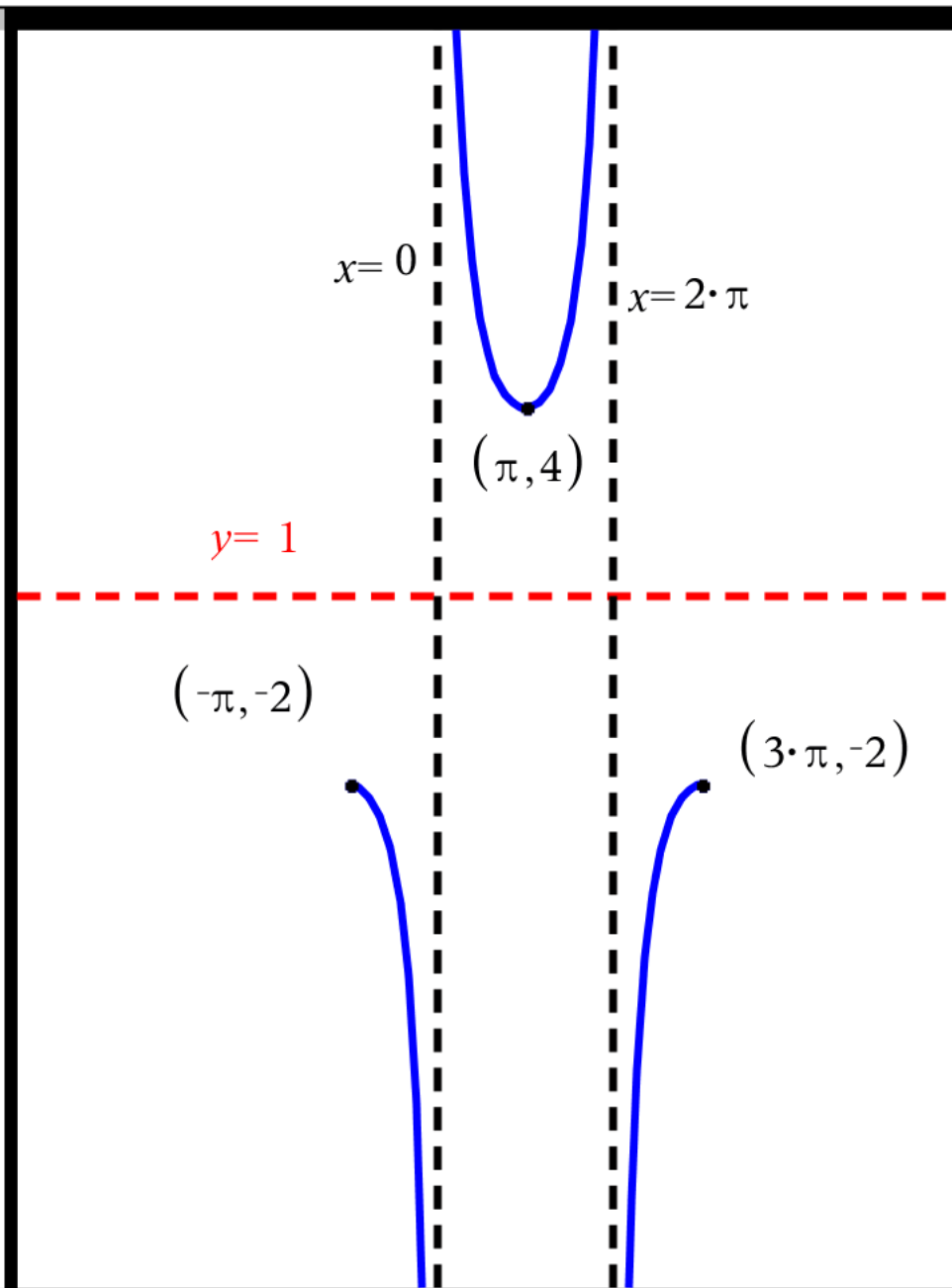
$$|b| = 2\pi \div 4\pi = 2\pi \cdot \frac{1}{4\pi} = \frac{1}{2}$$

Since this IS NOT a reflection,  
both a and b are positive

This means  $y = a \sec\left(\frac{1}{2}(x + \pi)\right) + d$

Since the midline is  $y=1$ , this means

$$y = a \sec\left(\frac{1}{2}(x + \pi)\right) + 1$$



Finding a this is the distance from the  
midline to the local extremes

$$|-2 - 1| = 3$$

$$|1 - 4| = 3$$

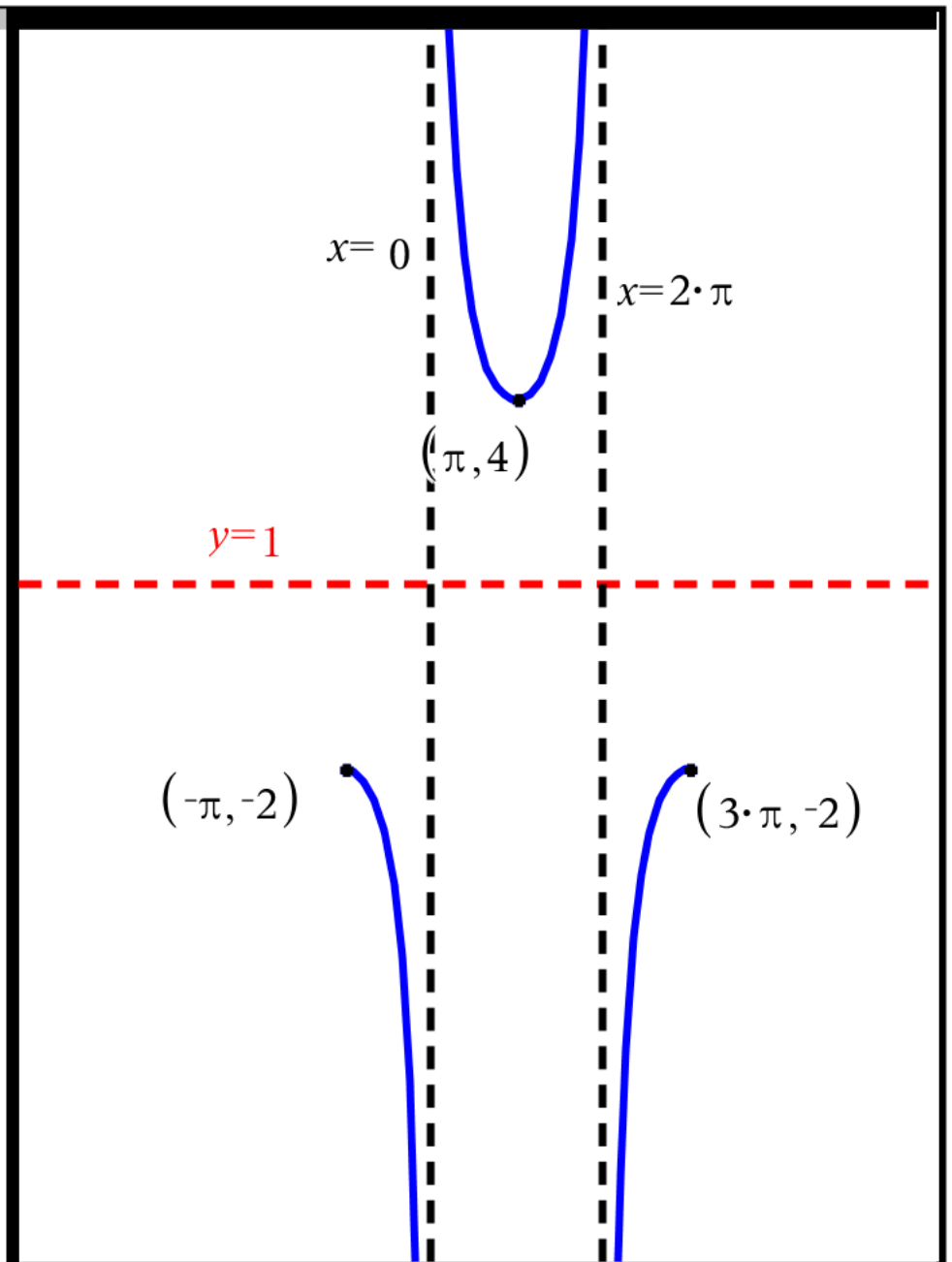
This is a VERTICAL reflection

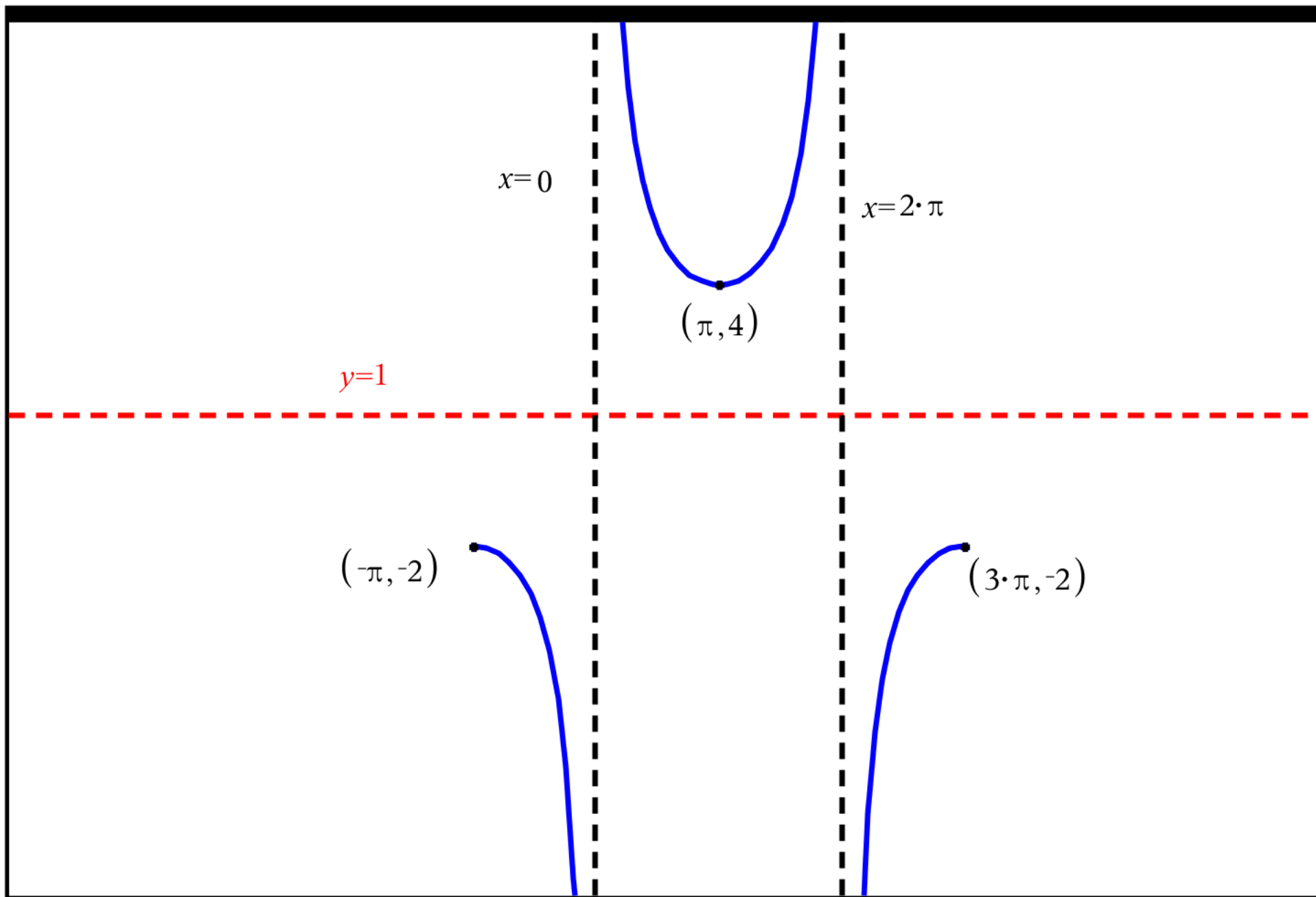
So  $a = -3$

So we know that

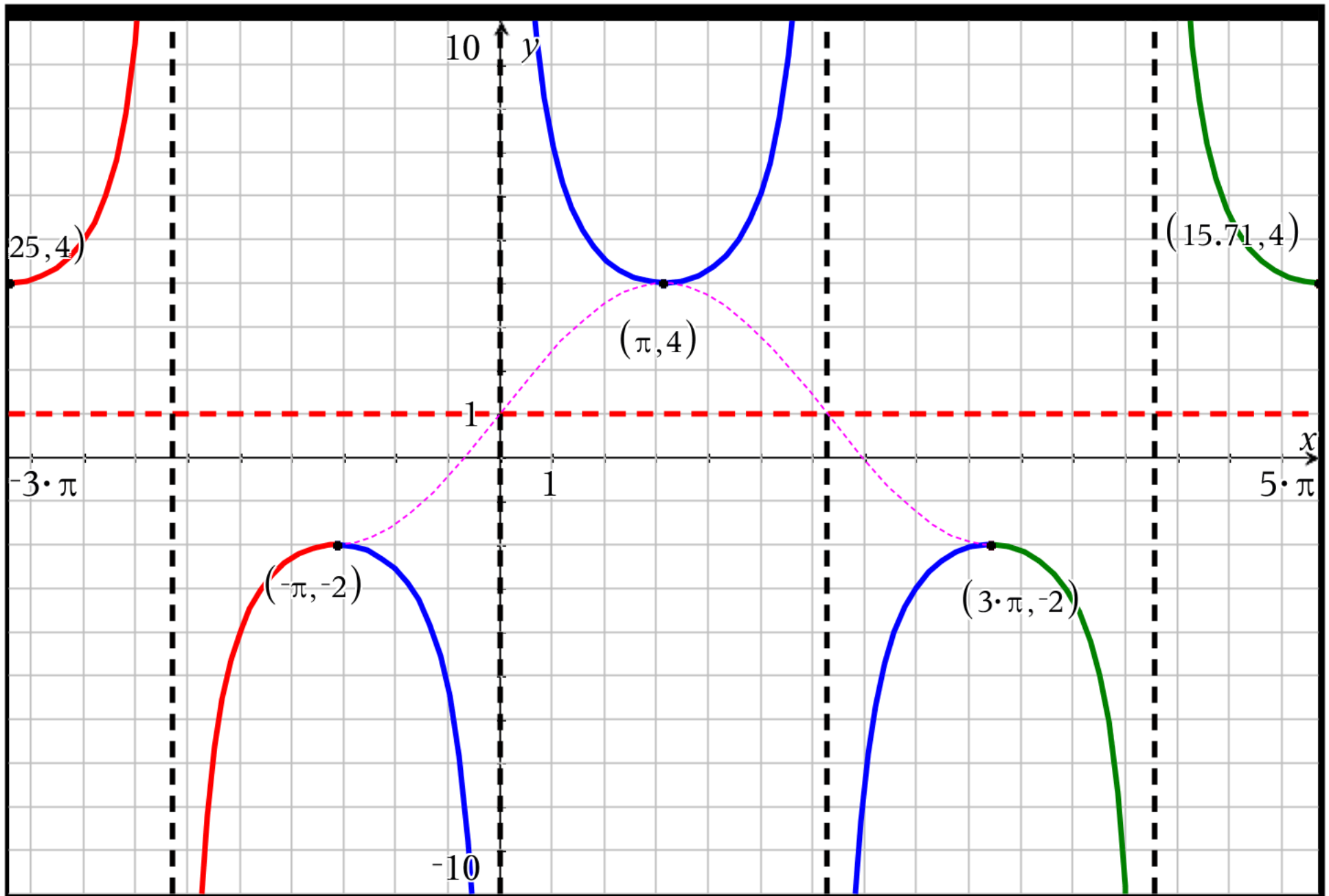
$$y = -3 \sec\left(\frac{1}{2}(x + \pi)\right) + 1$$

$$y = -3 \sec\left(\frac{1}{2}x + \frac{\pi}{2}\right) + 1$$

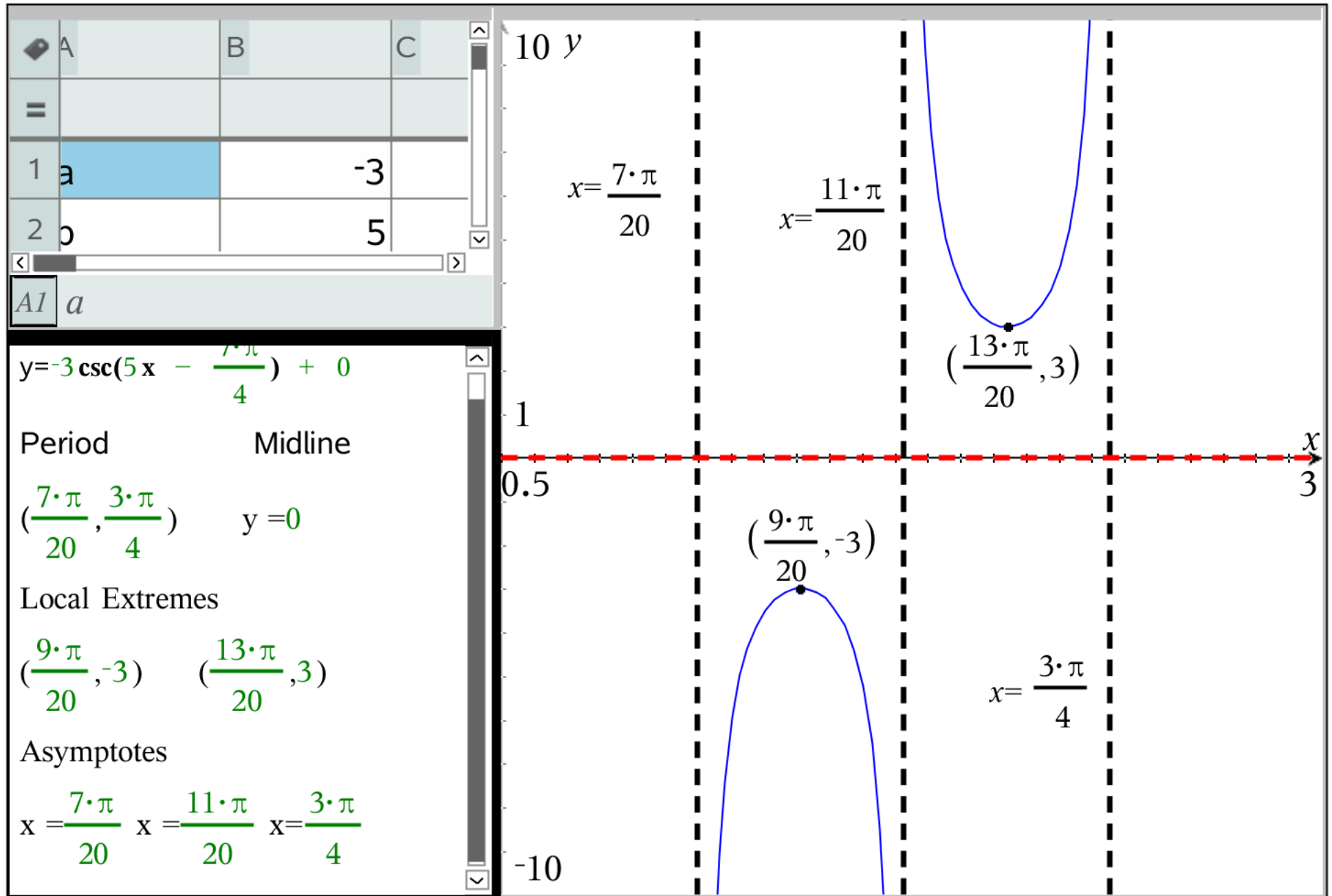








Problem 3



Step 1 What are the a, b, c d of the transformation?

$$a=-3 \quad b=5 \quad c = \frac{-7 \cdot \pi}{4} \quad d=0$$

Step 2: Factor INTERIOR function

$$y = -3 \csc\left(5x - \frac{7 \cdot \pi}{4}\right) + 0 \quad \text{NOTE: This asking you to do this } \frac{-7 \cdot \pi}{4} \div 5 = \frac{-7 \cdot \pi}{4} \cdot \frac{1}{5} = \frac{-7 \cdot \pi}{20}$$

$$y = -3 \csc\left(5\left(x - \frac{7 \cdot \pi}{20}\right)\right) + 0$$

Note: Since  $\frac{c}{b} = \frac{-7 \cdot \pi}{20}$  we know that the horizontal shift is **SHIFT RIGHT**

Note: Since  $b = 5$ , We know that there is a horizontal **compression**

Note: Since we know there is **SHIFT RIGHT**

and we have factored form, A period begins at  $x = \frac{7 \cdot \pi}{20}$

Step 3: Determine the length of period using period  $= \frac{2 \cdot \pi}{|b|}$

**NOTE: This asking you to do this**  $2 \cdot \pi \div 5 = 2 \cdot \pi \cdot \frac{1}{5} = \frac{2 \cdot \pi}{5}$  So we know the period is  $\frac{2 \cdot \pi}{5}$  LONG

$$a=-3 \quad b=5 \quad c = \frac{-7 \cdot \pi}{4} \quad d=0 \quad y = -3 \csc\left(5x - \frac{7 \cdot \pi}{4}\right) + 0 \quad y = -3 \csc\left(5\left(x - \frac{7 \cdot \pi}{20}\right)\right) + 0$$

period length is  $\frac{2 \cdot \pi}{5}$  and ONE PERIOD starts at  $\frac{7 \cdot \pi}{20}$

Step 4: Determine the FIVE IMPORTANT x values

$$\text{start} = \frac{7 \cdot \pi}{20} \quad (\text{this is the first asymptote of cosecant})$$

$$\text{start} + \frac{1 \cdot \text{period}}{4} = \frac{7 \cdot \pi}{20} + \frac{\pi}{10} = \frac{9 \cdot \pi}{20} \quad (\text{this is where one of the local extremes occurs})$$

$$\text{start} + \frac{2 \cdot \text{period}}{4} = \frac{7 \cdot \pi}{20} + \frac{\pi}{5} = \frac{11 \cdot \pi}{20} \quad (\text{this is the second asymptote of cosecant})$$

$$\text{start} + \frac{3 \cdot \text{period}}{4} = \frac{7 \cdot \pi}{20} + \frac{3 \cdot \pi}{10} = \frac{13 \cdot \pi}{20} \quad (\text{this is where one of the local extremes occurs})$$

$$\text{start} + \text{period} = \frac{7 \cdot \pi}{20} + \frac{2 \cdot \pi}{5} = \frac{3 \cdot \pi}{4} \quad (\text{this is the third asymptote of cosecant})$$

$$a=-3 \quad b=5 \quad c = \frac{-7 \cdot \pi}{4} \quad d = 0 \quad y = -3 \csc\left(5x - \frac{7 \cdot \pi}{4}\right) + 0 \quad y = -3 \csc\left(5\left(x - \frac{7 \cdot \pi}{20}\right)\right) + 0$$

period length is  $\frac{2 \cdot \pi}{5}$  and ONE PERIOD starts at  $\frac{7 \cdot \pi}{20}$

"cool stuff" happens at  $x = \left\{ \frac{7 \cdot \pi}{20}, \frac{9 \cdot \pi}{20}, \frac{11 \cdot \pi}{20}, \frac{13 \cdot \pi}{20}, \frac{3 \cdot \pi}{4} \right\}$

Step 5: Use d to determine midline  $y = 0$

Step 6: Use a and b to determine IF and WHAT KIND of reflections have occurred

$$a = -3 \quad b = 5$$

Since  $a < 0$  and  $b > 0$ ,

we know that **THERE IS** vertical reflection and **THERE IS NO** horizontal reflection

Step 7: Find y coordinates of the local extremes using a and d

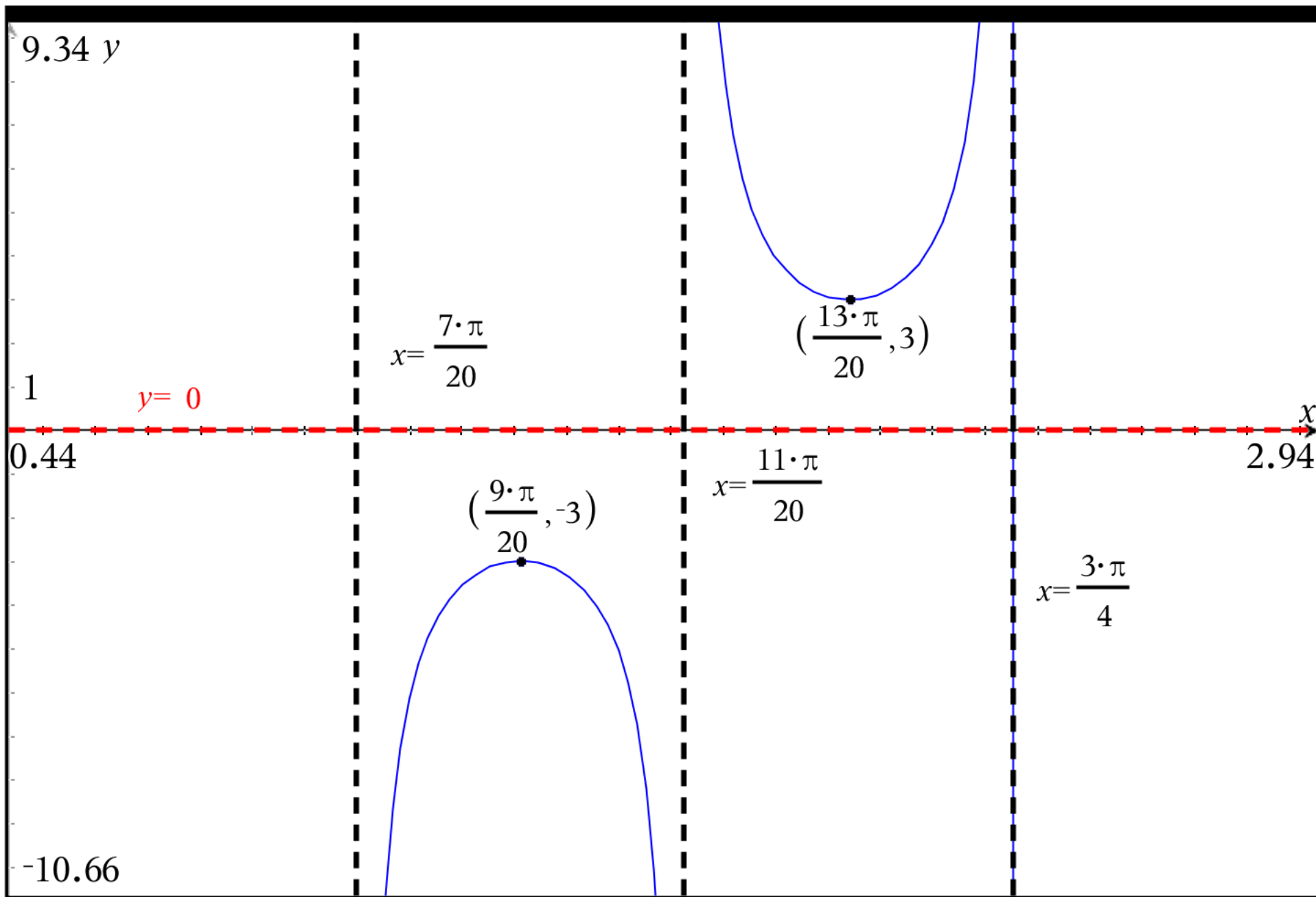
$$\text{Since } a = -3 \text{ and } d = 0$$

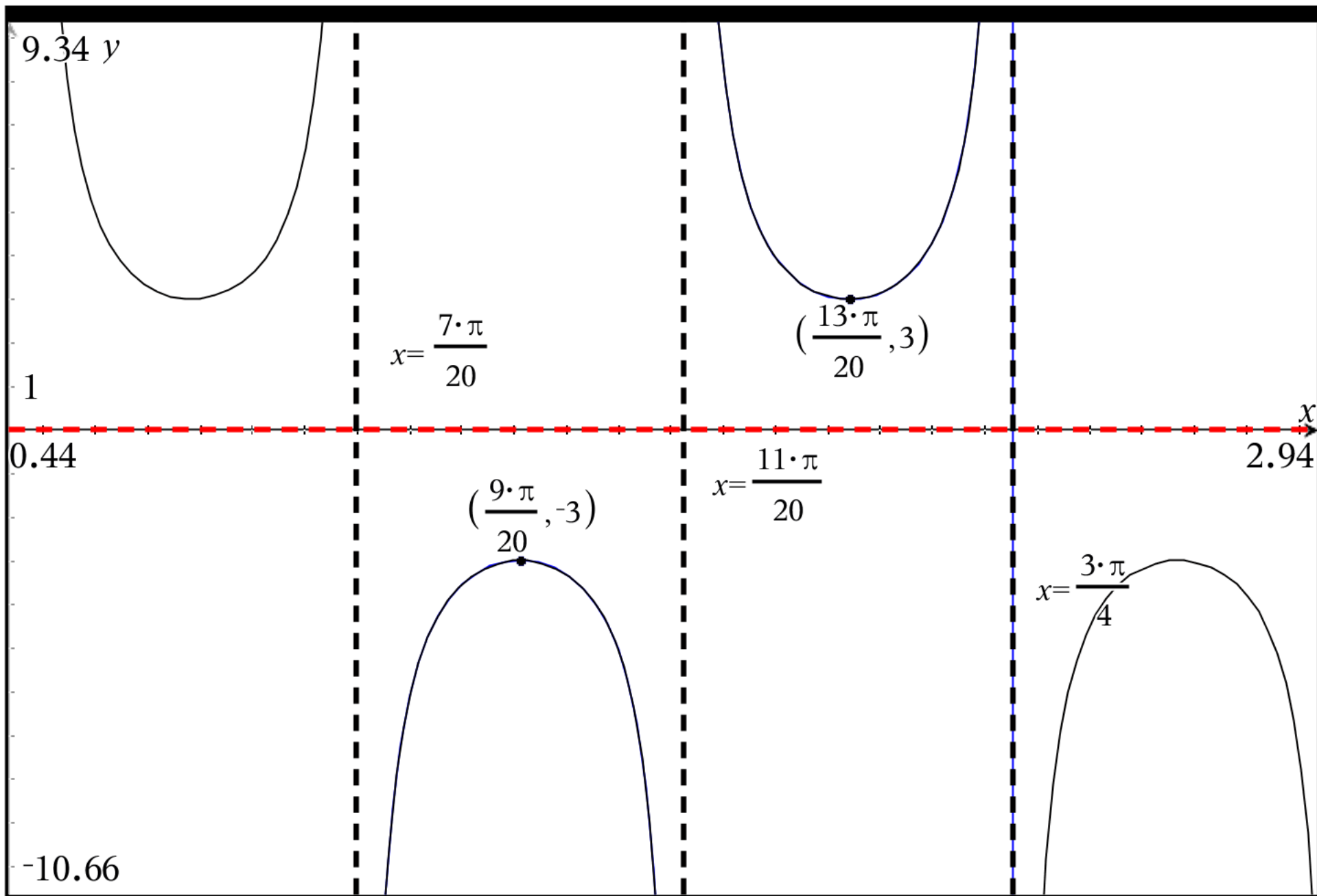
local extremes (top and bottom of "U" shape) occur at  $y = d + |a| = 0 + 3 = 3$  (bottom of a "U")

$$y = d - |a| = 0 - 3 = -3 \text{ (top of a "U")}$$

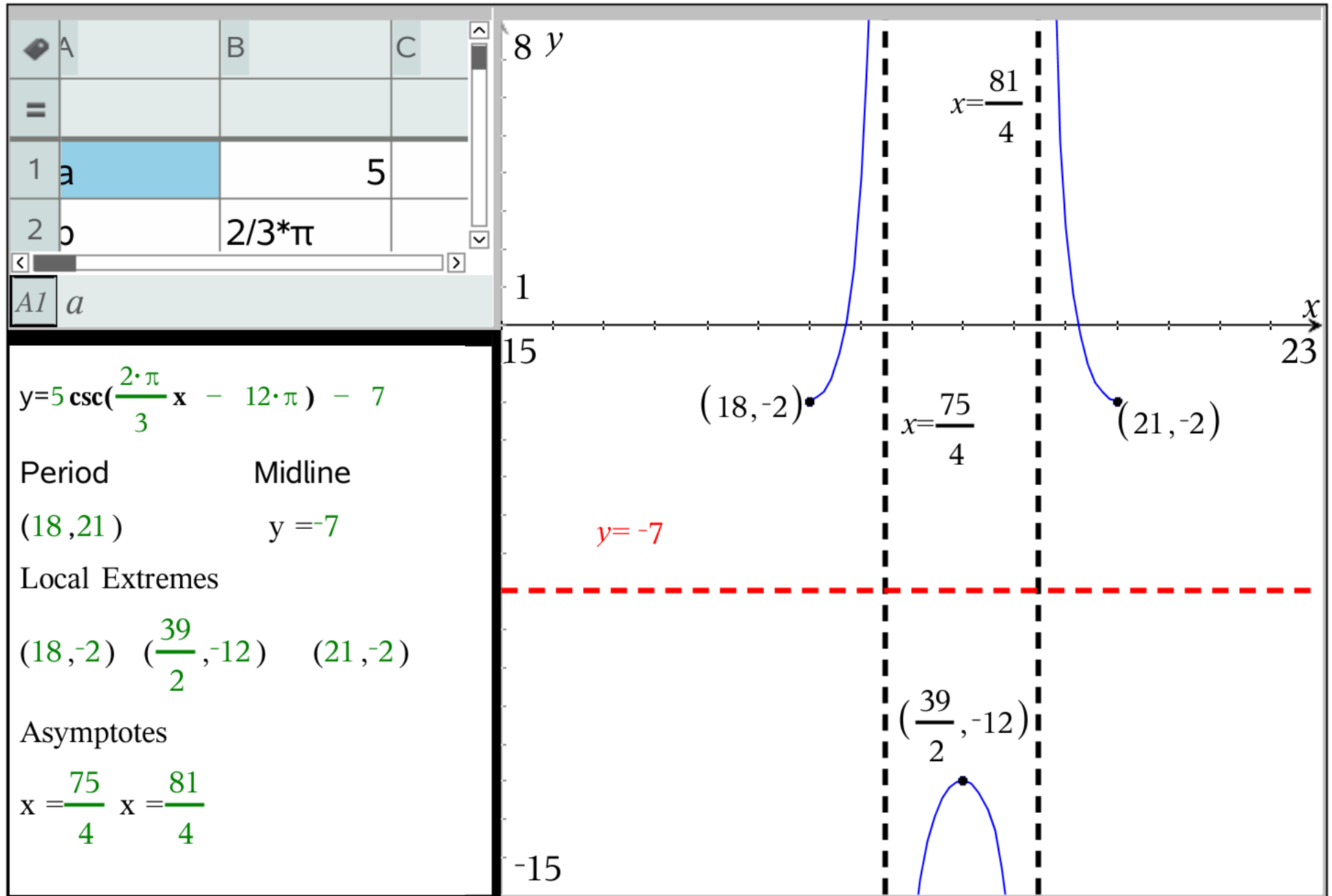
Step 8: Build local extremes

$$\left(\frac{9 \cdot \pi}{20}, 3\right) \text{ and } \left(\frac{13 \cdot \pi}{20}, -3\right)$$





Problem 4





Step 1 What are the a, b, c d of the transformation?

$$a=5 \quad b=\frac{2\cdot\pi}{3} \quad c=-12\cdot\pi \quad d=-7$$

Step 2: Factor INTERIOR function

$$y=5 \sec\left(\frac{2\cdot\pi}{3}x - 12\cdot\pi\right) - 7 \quad \text{NOTE: This asking you to do this } -12\cdot\pi \div \frac{2\cdot\pi}{3} = -12\cdot\pi \cdot \frac{3}{2\cdot\pi} = -18$$

$$y=5 \csc\left(\frac{2\cdot\pi}{3}(x - 18)\right) - 7$$

Note: Since  $\frac{c}{b} = -18$  we know that the horizontal shift is **SHIFT RIGHT**

Note: Since  $b = \frac{2\cdot\pi}{3}$ , We know that there is a horizontal **compression**

Note: Since we know there is **SHIFT RIGHT**

and we have factored form, A period begins at  $x = 18$

Step 3: Determine the length of period using period =  $\frac{2\cdot\pi}{|b|}$

**NOTE: This asking you to do this**  $2\cdot\pi \div \frac{2\cdot\pi}{3} = 2\cdot\pi \cdot \frac{3}{2\cdot\pi} = 3$  So we know the period is **3 LONG**

$$a=5 \quad b=\frac{2\cdot\pi}{3} \quad c=-12\cdot\pi \quad d=-7 \quad y=5 \sec\left(\frac{2\cdot\pi}{3}x - 12\cdot\pi\right) - 7 \quad y=5 \sec\left(\frac{2\cdot\pi}{3}(x - 18)\right) - 7$$

period length is 3 and ONE PERIOD starts at 18

Step 4: Determine the FIVE IMPORTANT x values

start =18 (this is the first local extreme of secant)

$$\text{start} + \frac{1 \cdot \text{period}}{4} = 18 + \frac{3}{4} = \frac{75}{4} \quad (\text{this is where first asymptote occurs})$$

$$\text{start} + \frac{2 \cdot \text{period}}{4} = 18 + \frac{3}{2} = \frac{39}{2} \quad (\text{this is the second local extreme of secant})$$

$$\text{start} + \frac{3 \cdot \text{period}}{4} = 18 + \frac{9}{4} = \frac{81}{4} \quad (\text{this is the second asymptote of secant})$$

$$\text{start} + \text{period} = 18 + 3 = 21 \quad (\text{this is the last local extreme of secant})$$

$$a=5 \quad b=\frac{2\cdot\pi}{3} \quad c=-12\cdot\pi \quad d=-7 \quad y=5\sec\left(\frac{2\cdot\pi}{3}x - 12\cdot\pi\right) - 7 \quad y=5\sec\left(\frac{2\cdot\pi}{3}(x - 18)\right) - 7$$

period length is 3 and ONE PERIOD starts at 18

"cool stuff" happens at  $x = \left\{18, \frac{75}{4}, \frac{39}{2}, \frac{81}{4}, 21\right\}$

Step 5: Use d to determine midline  $y = -7$

Step 6: Use a and b to determine IF and WHAT KIND of reflections have occurred

$$a = 5 \quad b = \frac{2\cdot\pi}{3}$$

Since  $a > 0$  and  $b > 0$ ,

we know that **NO** vertical reflection and **THERE IS NO** horizontal reflection

Step 7: Find y coordinates of the local extremes using a and d

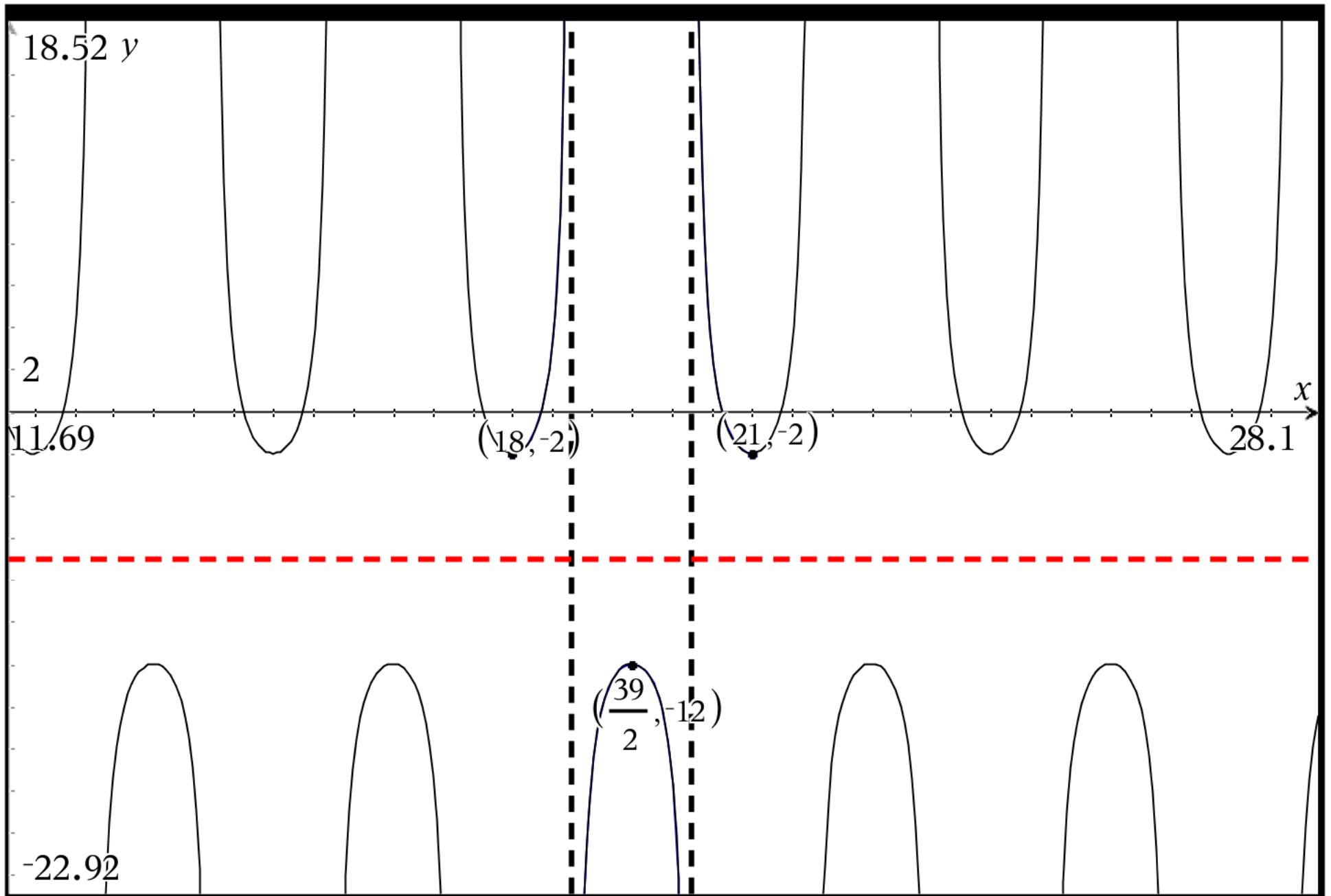
Since  $a = 5$  and  $d = -7$

local extremes (top and bottom of "U" shape) occur at  $y = d + |a| = -7 + 5 = -2$  (bottom of a "U")

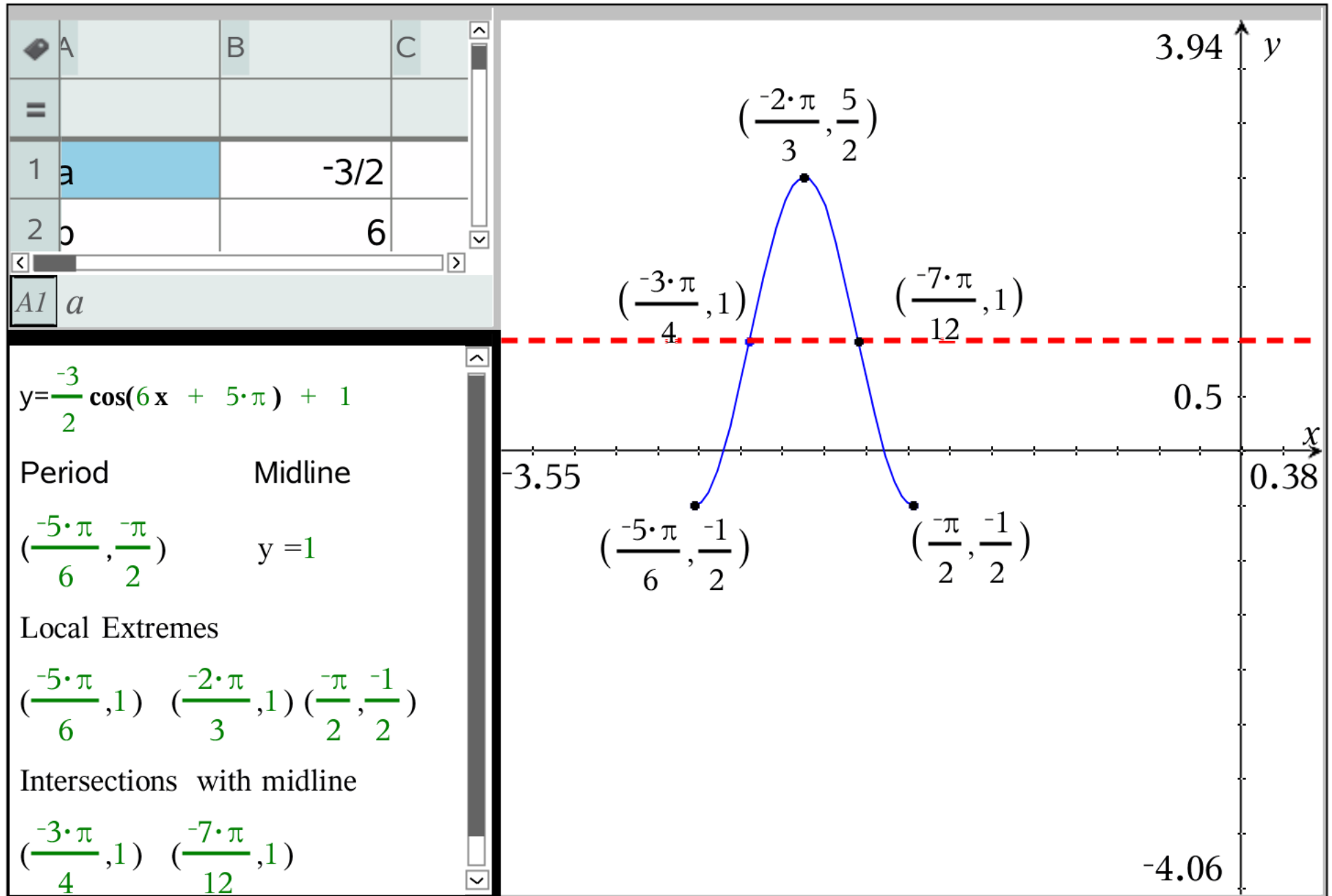
$$y = d - |a| = -7 - 5 = -12 \text{ (top of a "U")}$$

Step 8: Build local extremes

$$(18, -12) \text{ and } \left(\frac{39}{2}, -2\right) \text{ and } (21, -12)$$



Problem 5



Step 1 What are the a, b, c d of the transformation?

$$a = \frac{-3}{2} \quad b = 6 \quad c = 5 \cdot \pi \quad d = 1$$

Step 2: Factor INTERIOR function

$$y = \frac{-3}{2} \cos(6x + 5 \cdot \pi) + 1 \quad \text{NOTE: This asking you to do this } 5 \cdot \pi \div 6 = 5 \cdot \pi \cdot \frac{1}{6} = \frac{5 \cdot \pi}{6}$$

$$y = \frac{-3}{2} \csc\left(6\left(x + \frac{5 \cdot \pi}{6}\right)\right) + 1$$

Note: Since  $\frac{c}{b} = \frac{5 \cdot \pi}{6}$  we know that the horizontal shift is **SHIFT LEFT**

Note: Since  $b = 6$ , We know that there is a horizontal **compression**

Note: Since we know there is **SHIFT LEFT**

and we have factored form, A period begins at  $x = \frac{-5 \cdot \pi}{6}$

Step 3: Determine the length of period using period =  $\frac{2 \cdot \pi}{|b|}$

**NOTE: This asking you to do this**  $2 \cdot \pi \div 6 = 2 \cdot \pi \cdot \frac{1}{6} = \frac{\pi}{3}$  So we know the period is  $\frac{\pi}{3}$  LONG

$$a = \frac{-3}{2} \quad b = 6 \quad c = 5 \cdot \pi \quad d = 1 \quad y = \frac{-3}{2} \cos(6x + 5 \cdot \pi) + 1 \quad y = \frac{-3}{2} \cos\left(6\left(x + \frac{5 \cdot \pi}{6}\right)\right) + 1$$

period length is  $\frac{\pi}{3}$  and ONE PERIOD starts at  $\frac{-5 \cdot \pi}{6}$

Step 4: Determine the FIVE IMPORTANT x values

$$\text{start} = \frac{-5 \cdot \pi}{6} \quad (\text{this is the first local extreme of cosine})$$

$$\text{start} + \frac{1 \cdot \text{period}}{4} = \frac{-5 \cdot \pi}{6} + \frac{\pi}{12} = \frac{-3 \cdot \pi}{4} \quad (\text{this is where first intersection of midline occurs})$$

$$\text{start} + \frac{2 \cdot \text{period}}{4} = \frac{-5 \cdot \pi}{6} + \frac{\pi}{6} = \frac{-2 \cdot \pi}{3} \quad (\text{this is the second local extreme of cosine})$$

$$\text{start} + \frac{3 \cdot \text{period}}{4} = \frac{-5 \cdot \pi}{6} + \frac{\pi}{4} = \frac{-7 \cdot \pi}{12} \quad (\text{this is where first intersection of midline occurs})$$

$$\text{start} + \text{period} = \frac{-5 \cdot \pi}{6} + \frac{\pi}{3} = \frac{-\pi}{2} \quad (\text{this is the last local extreme of cosine})$$

$$a = \frac{-3}{2} \quad b = 6 \quad c = 5 \cdot \pi \quad d = 1 \quad y = \frac{-3}{2} \cos(6x + 5 \cdot \pi) + 1 \quad y = \frac{-3}{2} \cos\left(6\left(x + \frac{5 \cdot \pi}{6}\right)\right) + 1$$

period length is  $\frac{\pi}{3}$  and ONE PERIOD starts at  $\frac{-5 \cdot \pi}{6}$

"cool stuff" happens at  $x = \left\{ \frac{-5 \cdot \pi}{6}, \frac{-3 \cdot \pi}{4}, \frac{-2 \cdot \pi}{3}, \frac{-7 \cdot \pi}{12}, \frac{-\pi}{2} \right\}$

Step 5: Use d to determine midline  $y = 1$

Step 6: Use a and b to determine IF and WHAT KIND of reflections have occurred  $a = \frac{-3}{2}$   $b = 6$

Since  $a < 0$  and  $b > 0$ ,

we know that **THERE IS A** vertical reflection and **THERE IS NO** horizontal reflection

Step 7: Find y coordinates of the local extremes using a and d

Since  $a = \frac{-3}{2}$  and  $d = 1$

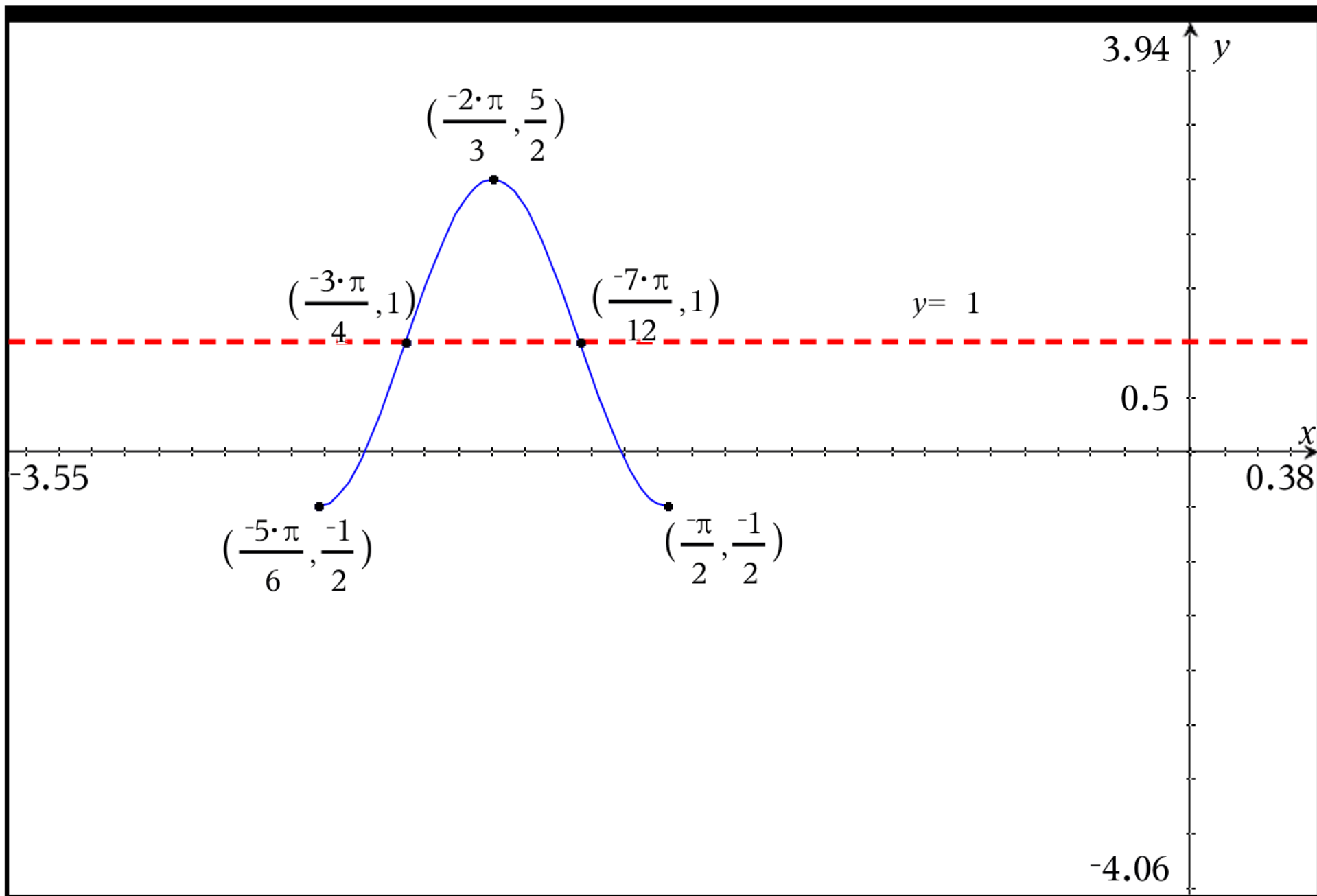
local extremes (top and bottom of "U" shape) occur at  $y = d + |a| = 1 + \frac{3}{2} = \frac{5}{2}$  (TOP of a "U")

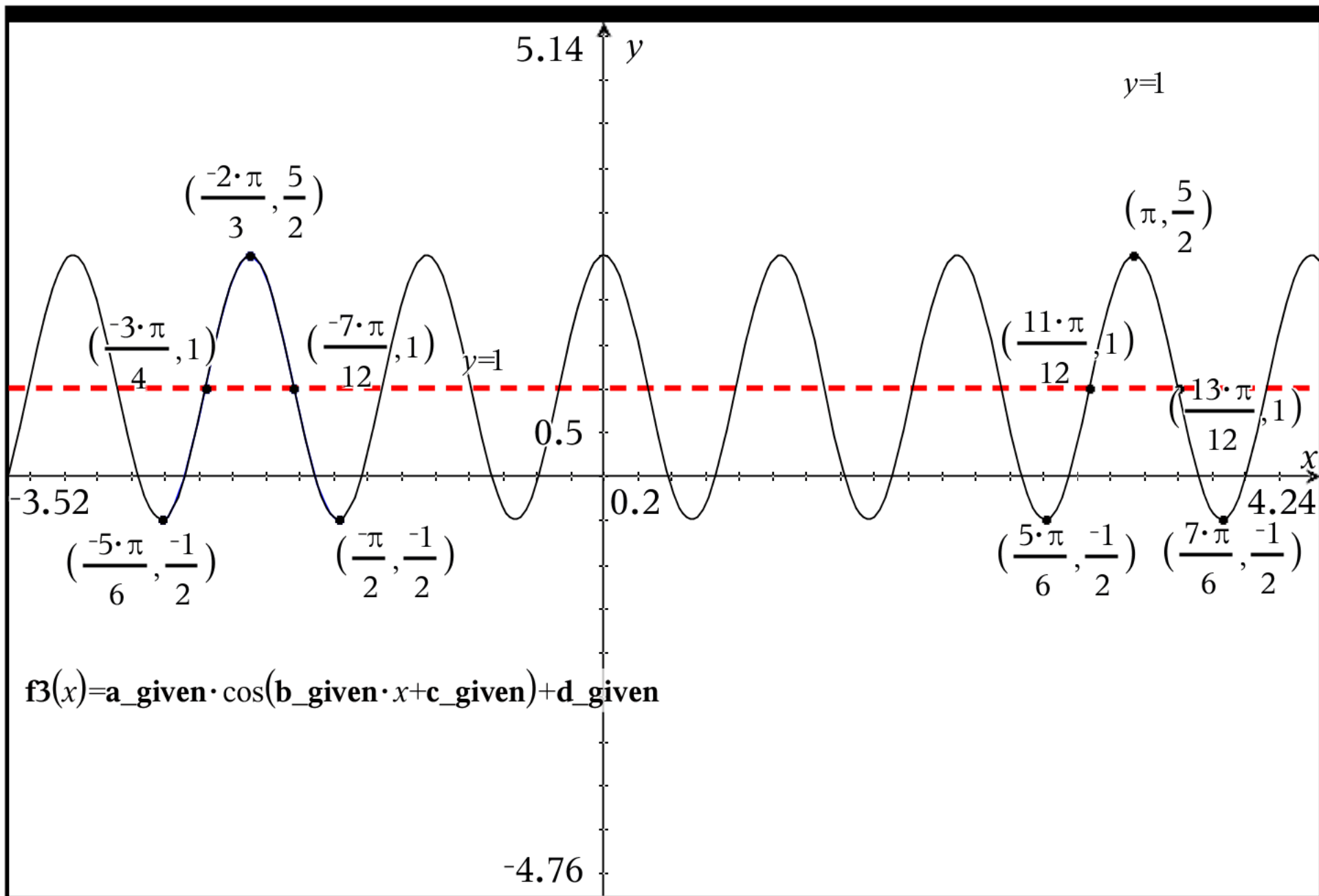
$$y = d - |a| = 1 - \frac{3}{2} = \frac{-1}{2} \quad (\text{BOTTOM of a "U"})$$

Step 8: Build local extremes

$$\left(\frac{-5 \cdot \pi}{6}, \frac{-1}{2}\right) \text{ and } \left(\frac{-2 \cdot \pi}{3}, \frac{5}{2}\right) \text{ and } \left(\frac{-\pi}{2}, \frac{-1}{2}\right)$$







Problem 6

A
=
1 SHIFT LEFT
2 SHIFT DOWN
3 Horizontal REFLECTIO
4 STRETCH period
5 Vertically STRETCH
6
7
8
9
10
11
< >
A1 "SHIFT LEFT"

Fact 1: Shift LEFT implies  $(x + \frac{18 \cdot \pi}{5})$

inside the secant function

So we know  $y = a \sec (b(x + \frac{18 \cdot \pi}{5})) + d$

start =  $\frac{-18 \cdot \pi}{5}$

Fact 2: Shift DOWN implies  $d = -8$

outside the secant function

So we know  $y = a \sec (b(x + \frac{18 \cdot \pi}{5})) - 8$

Fact 3: There is a HORIZONTAL reflection

This means  $b < 0$

Fact 4: There is a stretch of period to  $48 \cdot \pi$

$0 < |b| < 1$

Use  $|b| = \frac{2 \cdot \pi}{\text{period}} = 2 \cdot \pi \div 48 \cdot \pi = 2 \cdot \pi \cdot \frac{1}{48 \cdot \pi} = \frac{1}{24}$

Fact 3 and Fact 4 tell us  $|b| = \frac{1}{24}$  that  $b = \frac{-1}{24}$  because of the horizontal reflection

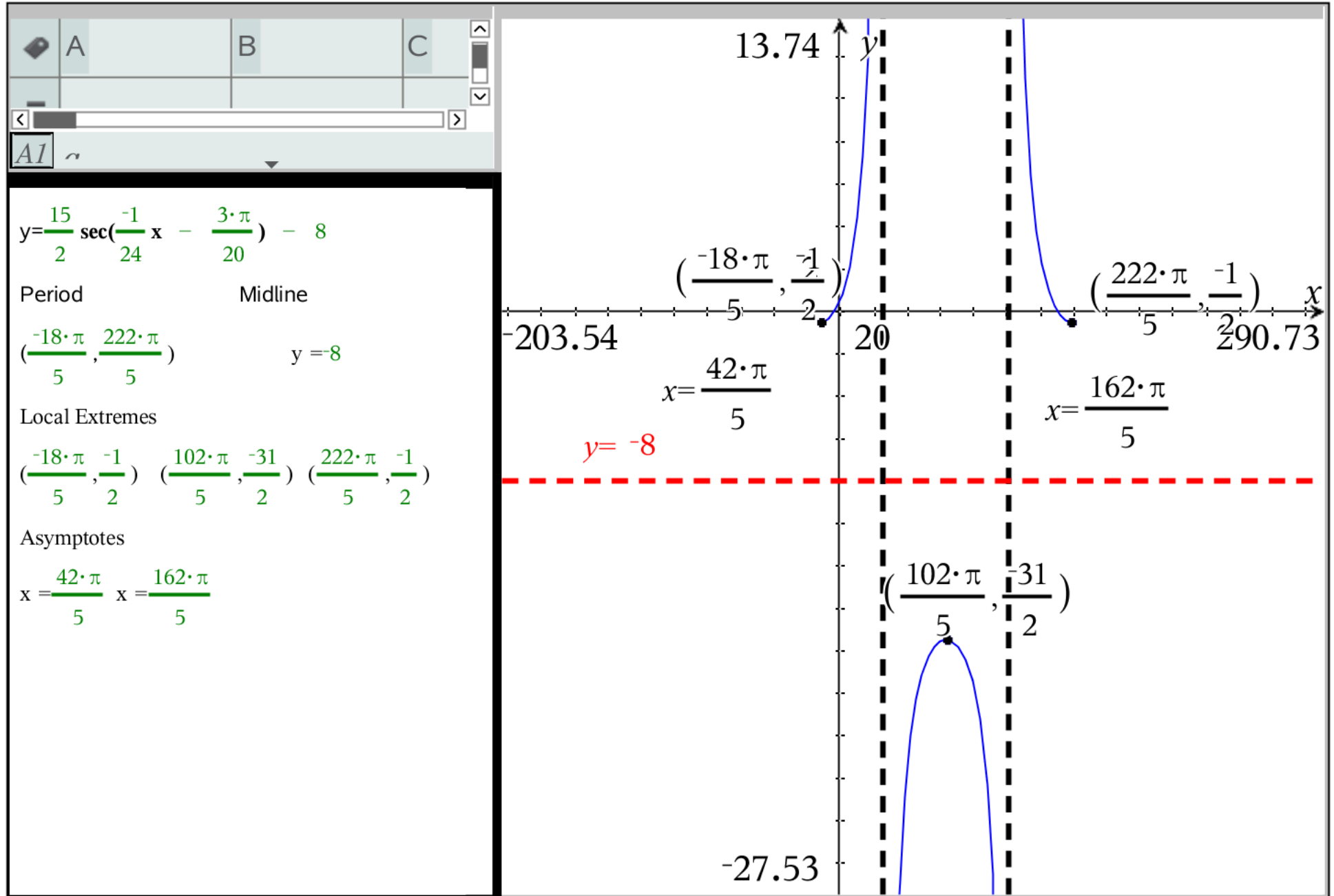
We now know  $y = a \sec\left(\frac{-1}{24}\left(x + \frac{18 \cdot \pi}{5}\right)\right) - 8$

Fact 5: Vertically Stretch by a factor of  $\frac{15}{2}$

and the fact that NO vertical reflection has occurred we know  $a = \frac{15}{2}$

We now know  $y = \frac{15}{2} \sec\left(\frac{-1}{24}\left(x + \frac{18 \cdot \pi}{5}\right)\right) - 8$

Problem 7



Step 1 What are the a, b, c, d of the transformation?

$$a = \frac{15}{2} \quad b = \frac{-1}{24} \quad c = \frac{-3 \cdot \pi}{20} \quad d = -8$$

Step 2: Factor INTERIOR function

$$y = \frac{15}{2} \sec\left(\frac{-1}{24}x - \frac{3 \cdot \pi}{20}\right) - 8 \quad \text{NOTE: This asking you to do this } \frac{-3 \cdot \pi}{20} \div \frac{-1}{24} = \frac{-3 \cdot \pi}{20} \cdot -24 = \frac{18 \cdot \pi}{5}$$

$$y = \frac{15}{2} \sec\left(\frac{-1}{24}\left(x + \frac{18 \cdot \pi}{5}\right)\right) - 8$$

Note: Since  $\frac{c}{b} = \frac{18 \cdot \pi}{5}$  we know that the horizontal shift is **SHIFT LEFT**

Note: Since  $b = \frac{-1}{24}$ , We know that there is a horizontal **stretch**

Note: Since we know there is **SHIFT LEFT**

and we have factored form, A period begins at  $x = \frac{-18 \cdot \pi}{5}$

Step 3: Determine the length of period using period =  $\frac{2 \cdot \pi}{|b|}$

NOTE: This asking you to do this  $2 \cdot \pi \div \frac{-1}{24} = 2 \cdot \pi \cdot 24 = 48 \cdot \pi$  So we know the period is **48 · π LONG**

$$a = \frac{15}{2} \quad b = \frac{-1}{24} \quad c = \frac{-3 \cdot \pi}{20} \quad d = -8 \quad y = \frac{15}{2} \sec\left(\frac{-1}{24}x - \frac{3 \cdot \pi}{20}\right) - 8 \quad y = \frac{15}{2} \sec\left(\frac{-1}{24}\left(x + \frac{18 \cdot \pi}{5}\right)\right) - 8$$

period length is  $48 \cdot \pi$  and ONE PERIOD starts at  $\frac{-18 \cdot \pi}{5}$

Step 4: Determine the FIVE IMPORTANT x values

$$\text{start} = \frac{-18 \cdot \pi}{5} \quad (\text{this is the first local extreme of secant})$$

$$\text{start} + \frac{1 \cdot \text{period}}{4} = \frac{-18 \cdot \pi}{5} + 12 \cdot \pi = \frac{42 \cdot \pi}{5} \quad (\text{this is where first asymptote occurs})$$

$$\text{start} + \frac{2 \cdot \text{period}}{4} = \frac{-18 \cdot \pi}{5} + 24 \cdot \pi = \frac{102 \cdot \pi}{5} \quad (\text{this is the second local extreme of secant})$$

$$\text{start} + \frac{3 \cdot \text{period}}{4} = \frac{-18 \cdot \pi}{5} + 36 \cdot \pi = \frac{162 \cdot \pi}{5} \quad (\text{this is the second asymptote of secant})$$

$$\text{start} + \text{period} = \frac{-18 \cdot \pi}{5} + 48 \cdot \pi = \frac{222 \cdot \pi}{5} \quad (\text{this is the last local extreme of secant})$$

$$a = \frac{15}{2} \quad b = \frac{-1}{24} \quad c = \frac{-3 \cdot \pi}{20} \quad d = -8 \quad y = \frac{15}{2} \sec\left(\frac{-1}{24}x - \frac{3 \cdot \pi}{20}\right) - 8 \quad y = \frac{15}{2} \sec\left(\frac{-1}{24}(x + \quad)\right) - 8$$

period length is  $48 \cdot \pi$  and ONE PERIOD starts at  $\frac{-18 \cdot \pi}{5}$

"cool stuff" happens at  $x = \left\{ \frac{-18 \cdot \pi}{5}, \frac{42 \cdot \pi}{5}, \frac{102 \cdot \pi}{5}, \frac{162 \cdot \pi}{5}, \frac{222 \cdot \pi}{5} \right\}$

Step 5: Use d to determine midline  $y = -8$

Step 6: Use a and b to determine IF and WHAT KIND of reflections have occurred  $a = \frac{15}{2}$   $b = \frac{-1}{24}$

Since  $a > 0$  and  $b < 0$ ,

we know that NO vertical reflection and THERE IS horizontal reflection

Step 7: Find y coordinates of the local extremes using a and d

Since  $a = \frac{15}{2}$  and  $d = -8$

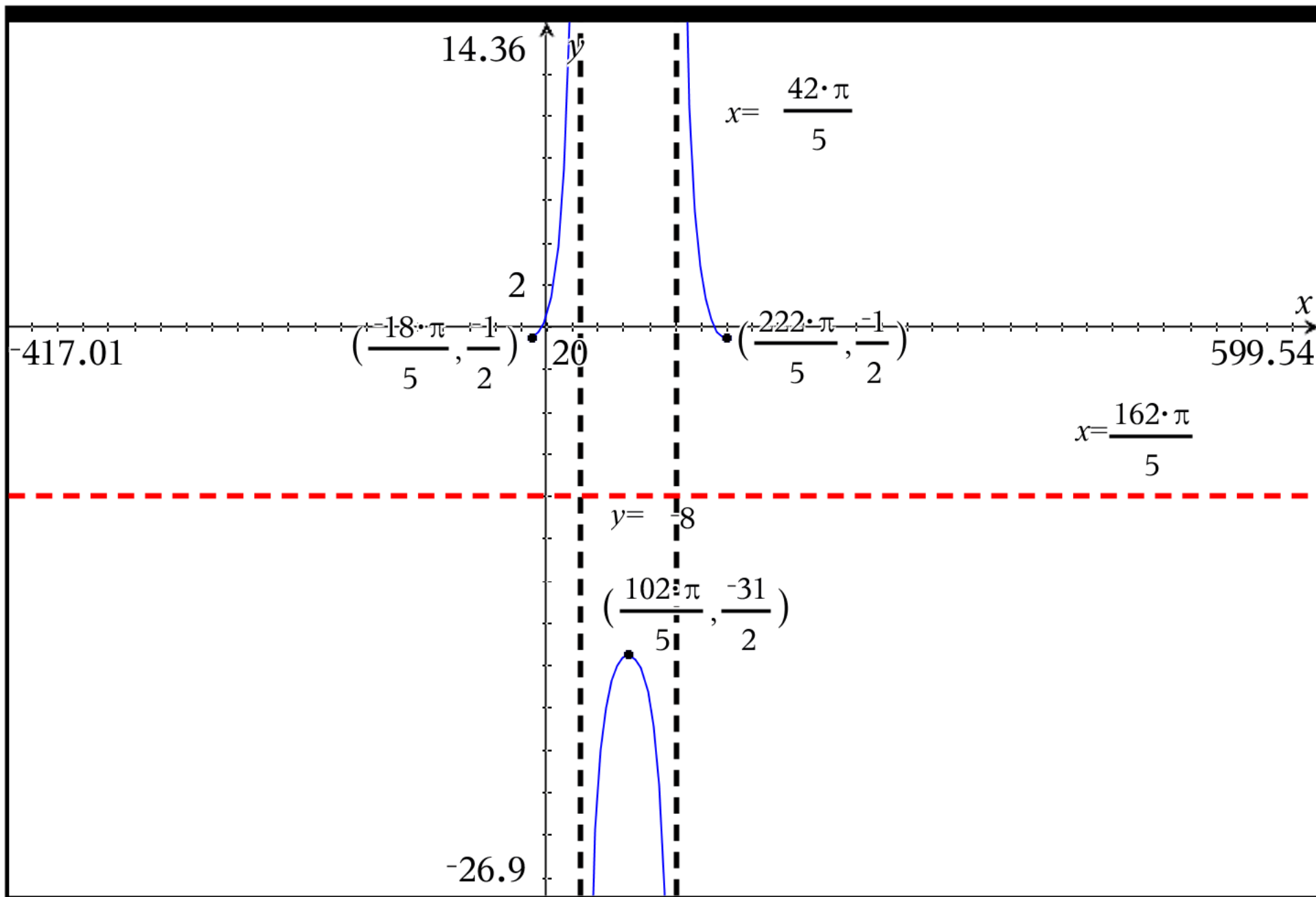
local extremes (top and bottom of "U" shape) occur at  $y = d + |a| = -8 + \frac{15}{2} = \frac{-1}{2}$  (bottom of a "U")

$$y = d - |a| = -8 - \frac{15}{2} = \frac{-31}{2} \text{ (top of a "U")}$$

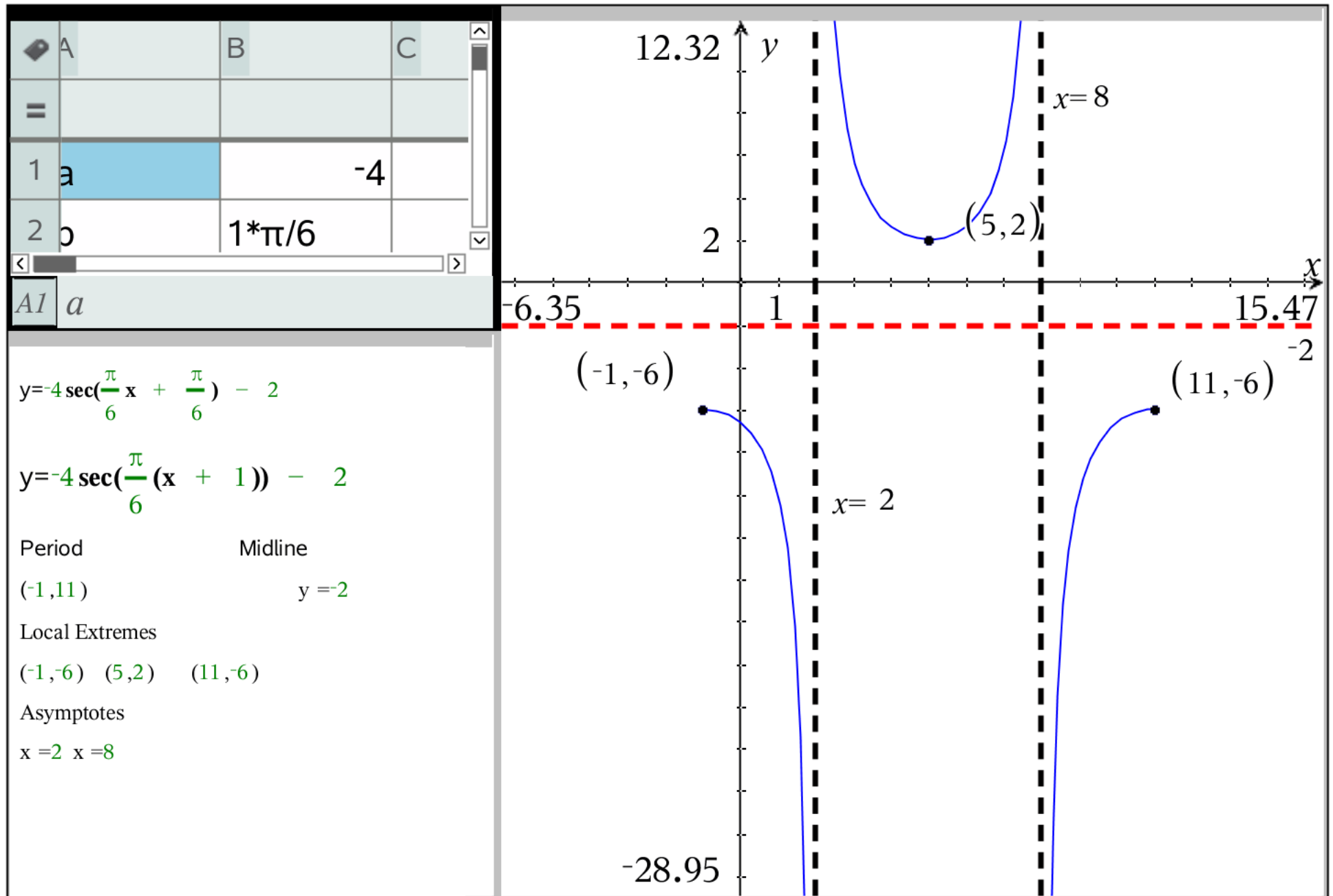
Step 8: Build local extremes

$$\left(\frac{-18 \cdot \pi}{5}, \frac{-31}{2}\right) \text{ and } \left(\frac{102 \cdot \pi}{5}, \frac{-1}{2}\right) \text{ and } \left(\frac{222 \cdot \pi}{5}, \frac{-31}{2}\right)$$





Problem 8



Step 1 What are the a, b, c d of the transformation?

$$a=-4 \quad b=\frac{\pi}{6} \quad c=\frac{\pi}{6} \quad d=-2$$

Step 2: Factor INTERIOR function

$$y=-4 \sec\left(\frac{\pi}{6}x + \frac{\pi}{6}\right) - 2 \quad \text{NOTE: This asking you to do this } \frac{\pi}{6} \div \frac{\pi}{6} = \frac{\pi}{6} \cdot \frac{6}{\pi} = 1$$

$$y=-4 \csc\left(\frac{\pi}{6}(x + 1)\right) - 2$$

Note: Since  $\frac{c}{b} = 1$  we know that the horizontal shift is **SHIFT LEFT**

Note: Since  $b = \frac{\pi}{6}$ , We know that there is a horizontal **stretch**

Note: Since we know there is **SHIFT LEFT**

and we have factored form, A period begins at  $x = -1$

Step 3: Determine the length of period using period  $= \frac{2 \cdot \pi}{|b|}$

NOTE: This asking you to do this  $2 \cdot \pi \div \frac{\pi}{6} = 2 \cdot \pi \cdot \frac{6}{\pi} = 12$  So we know the period is **12 LONG**

$$a=-4 \quad b=\frac{\pi}{6} \quad c=\frac{\pi}{6} \quad d=-2 \quad y=-4\sec\left(\frac{\pi}{6}x + \frac{\pi}{6}\right) - 2 \quad y=-4\sec\left(\frac{\pi}{6}(x + 1)\right) - 2$$

period length is 12 and ONE PERIOD starts at -1

Step 4: Determine the FIVE IMPORTANT x values

start = -1 (this is the first local extreme of secant)

start +  $\frac{1 \cdot \text{period}}{4} = -1 + 3 = 2$  (this is where first asymptote occurs)

start +  $\frac{2 \cdot \text{period}}{4} = -1 + 6 = 5$  (this is the second local extreme of secant)

start +  $\frac{3 \cdot \text{period}}{4} = -1 + 9 = 8$  (this is the second asymptote of secant)

start +  $\text{period} = -1 + 12 = 11$  (this is the last local extreme of secant)

$$a=-4 \quad b=\frac{\pi}{6} \quad c=\frac{\pi}{6} \quad d=-2 \quad y=-4 \sec\left(\frac{\pi}{6}x + \frac{\pi}{6}\right) - 2 \quad y=-4 \sec\left(\frac{\pi}{6}(x + 1)\right) - 2$$

period length is 12 and ONE PERIOD starts at -1

"cool stuff" happens at  $x = \{-1, 2, 5, 8, 11\}$

Step 5: Use d to determine midline  $y = -2$

Step 6: Use a and b to determine IF and WHAT KIND of reflections have occurred  $a = -4$   $b = \frac{\pi}{6}$

Since  $a < 0$  and  $b > 0$ ,

we know that **THERE IS A** vertical reflection and **THERE IS NO** horizontal reflection

Step 7: Find y coordinates of the local extremes using a and d

Since  $a = -4$  and  $d = -2$

local extremes (top and bottom of "U" shape) occur at  $y = d + |a| = -2 + 4 = 2$  (bottom of a "U")

$$y = d - |a| = -2 - 4 = -6 \text{ (top of a "U")}$$

Step 8: Build local extremes

$(-1, -6)$  and  $(5, 2)$  and  $(11, -6)$

