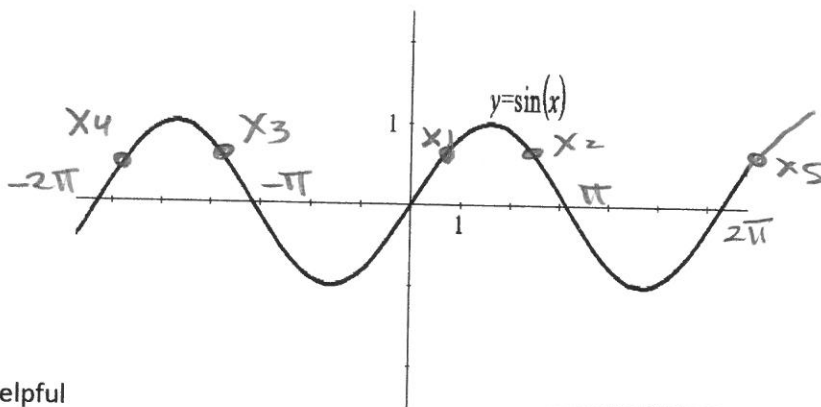1. Solve the following trigonometric equation $\dfrac{-5}{4}\sin(x)+2=\dfrac{3}{2}$

   Give three EXACT solutions and their approximations

$y=\sin(x)$

This may be helpful

Show necessary work here

(A) $-\dfrac{5}{4}\sin x + 2 = \dfrac{3}{2} = \dfrac{3}{2}$

$\phantom{-\dfrac{5}{4}\sin x}-2 \quad -2 \quad -\dfrac{4}{2}$

(B) $-\dfrac{5}{4}\sin x = -\dfrac{1}{2}$

$-\dfrac{4}{5}\left(-\dfrac{5}{4}\sin x\right) = -\dfrac{4}{5}\left(-\dfrac{1}{2}\right)$

$\sin x = \dfrac{4}{10}$

(C) $\boxed{\sin x = \dfrac{2}{5}}$

$\boxed{X_1 = \sin^{-1}\left(\dfrac{2}{5}\right) \approx 0.412 \text{ radians}}$

$\boxed{X_2 = \pi - \sin^{-1}\left(\dfrac{2}{5}\right)}$

$\approx 2.730 \text{ radians}$

$\boxed{X_3 = -\pi - \sin^{-1}\left(\dfrac{2}{5}\right)}$

$\approx -3.553 \text{ radians}$

$\boxed{X_4 = -2\pi + \sin^{-1}\left(\dfrac{2}{5}\right)}$
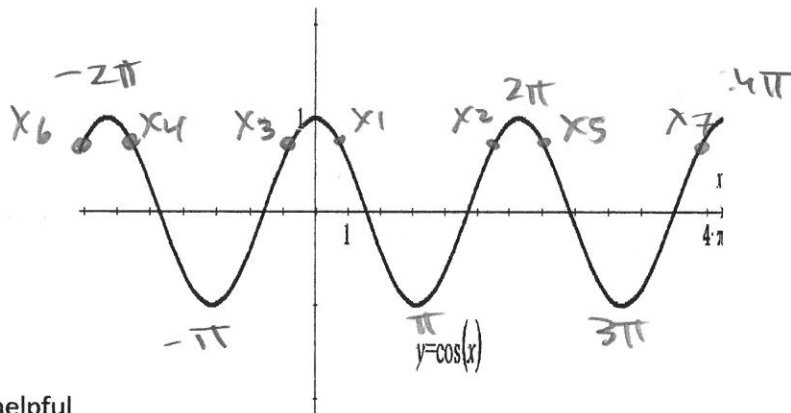
$\approx -5.872 \text{ radians}$

$\boxed{X_5 = 2\pi + \sin^{-1}\left(\dfrac{2}{5}\right)}$

$\approx 6.695 \text{ radians}$

Exact Solution #1 $\sin^{-1}\left(\dfrac{2}{5}\right)$ Approximation (round to three decimals) 0.412

Exact Solution #2 $\pi - \sin^{-1}\left(\dfrac{2}{5}\right)$ Approximation (round to three decimals) 2.730

Exact Solution #3 $-\pi - \sin^{-1}\left(\dfrac{2}{5}\right)$ Approximation (round to three decimals) -5.872

2. Solve the following trigonometric equation $7\cos(x) - 2 = \dfrac{7}{2}$

Give three EXACT solutions and their approximations

$y = \cos(x)$

This may be helpful

Show necessary work here

(A) $7\cos x - 2 = \dfrac{7}{2} = \dfrac{7}{2}$

$ +2 \quad +2 \quad +\dfrac{4}{2}$

(B) $7\cos x = \dfrac{11}{2}$

$\dfrac{7\cos x}{7} = \dfrac{11}{2} \cdot \dfrac{1}{7}$

(C) $\boxed{\cos x = \dfrac{11}{14}}$

$\boxed{X_1 = \cos^{-1}\left(\dfrac{11}{14}\right) \approx 0.667 \text{ radians}}$

$\boxed{X_2 = 2\pi - \cos^{-1}\left(\dfrac{11}{14}\right) \approx 5.616 \text{ radians}}$

$\boxed{X_3 = -\cos^{-1}\left(\dfrac{11}{14}\right) \approx -0.667 \text{ radians}}$

$\boxed{X_4 = -2\pi + \cos^{-1}\left(\dfrac{11}{14}\right) \approx -5.616 \text{ radians}}$

$\boxed{X_5 = 2\pi + \cos^{-1}\left(\dfrac{11}{14}\right) \approx 6.950 \text{ radians}}$

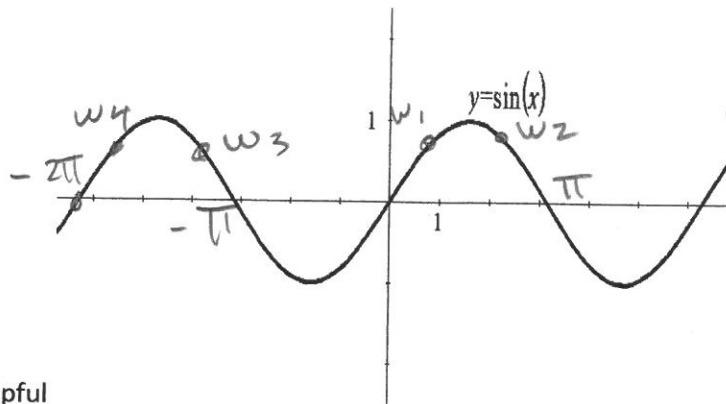$\boxed{X_6 = -2\pi - \cos^{-1}\left(\dfrac{11}{14}\right) \approx -6.950 \text{ radians}}$

$\boxed{X_7 = 4\pi - \cos^{-1}\left(\dfrac{11}{14}\right) \approx 11.899 \text{ radians}}$

Exact Solution #1 $\quad \cos^{-1}\left(\dfrac{11}{14}\right) \quad$ Approximation (round to three decimals) $\quad 0.667$

Exact Solution #2 $\quad 2\pi - \cos^{-1}\left(\dfrac{11}{14}\right) \quad$ Approximation (round to three decimals) $\quad 5.616$

Exact Solution #3 $\quad -\cos^{-1}\left(\dfrac{11}{14}\right) \quad$ Approximation (round to three decimals) $\quad -0.667$

3. Solve the following trigonometric equation $4\sin\left(\dfrac{1}{2}x - \dfrac{\pi}{4}\right) - 2 = \dfrac{1}{2}$

   Give three EXACT solutions and their approximations

This may be helpful

Show necessary work here

let $w = \dfrac{1}{2}x - \dfrac{\pi}{4}$

(A) $4\sin w - 2 = \dfrac{1}{2} = \dfrac{1}{2}$

$\qquad \dfrac{+2 \quad +2 \quad +\dfrac{4}{2}}{}$

(B) $4\sin w = \dfrac{5}{2}$

(C) $\dfrac{4\sin w}{4} = \dfrac{5}{2} \cdot \dfrac{1}{4}$

(D) $\boxed{\sin w = \dfrac{5}{8}}$

(E) $\dfrac{1}{2}x - \dfrac{\pi}{4} = \sin^{-1}\left(\dfrac{5}{8}\right)$

$\qquad \dfrac{+\dfrac{\pi}{4} \quad +\dfrac{\pi}{4}}{}$

$\dfrac{1}{2}x = \dfrac{\pi}{4} + \sin^{-1}\left(\dfrac{5}{8}\right)$

$2\left(\dfrac{1}{2}x\right) = 2\left(\dfrac{\pi}{4} + \sin^{-1}\left(\dfrac{5}{8}\right)\right)$

$x = \dfrac{2\pi}{4} + 2\sin^{-1}\left(\dfrac{5}{8}\right)$

$\boxed{x_1 = \dfrac{\pi}{2} + 2\sin^{-1}\left(\dfrac{5}{8}\right) \approx 2.921 \text{ radians}}$

(E) Exact Solution #1 _____

Approximation (round to three decimals) __2.921__

Exact Solution #2 __$\dfrac{5\pi}{2} - 2\sin^{-1}\left(\dfrac{5}{8}\right)$__

Approximation (round to three decimals) __6.504__

Exact Solution #3 __$\dfrac{-3\pi}{2} - 2\sin^{-1}\left(\dfrac{5}{8}\right)$__

Approximation (round to three decimals) __-6.063__

(G) $\dfrac{1}{2}x_3 - \dfrac{\pi}{4} = -\pi - \sin^{-1}\left(\dfrac{5}{8}\right)$

$\qquad \dfrac{+\dfrac{\pi}{4} \quad +\dfrac{\pi}{4}}{}$

$\dfrac{1}{2}x_3 = \dfrac{-3\pi}{4} - \sin^{-1}\left(\dfrac{5}{8}\right)$

$2\left(\dfrac{1}{2}x_3\right) = \left(\dfrac{-3\pi}{4} - \sin^{-1}\left(\dfrac{5}{8}\right)\right)2$

$\boxed{x_3 = \dfrac{-6\pi}{4} - 2\sin^{-1}\left(\dfrac{5}{8}\right) = \dfrac{-3\pi}{2} - 2\sin^{-1}\left(\dfrac{5}{8}\right) \approx -6.063 \text{ radians}}$

(E) $w_1 = \sin^{-1}\left(\dfrac{5}{8}\right)$

(F) $w_2 = \pi - \sin^{-1}\left(\dfrac{5}{8}\right)$

(G) $w_3 = -\pi - \sin^{-1}\left(\dfrac{5}{8}\right)$

(H) $w_4 = -2\pi + \sin^{-1}\left(\dfrac{5}{8}\right)$

Now we need to put $w = \dfrac{1}{2}x - \dfrac{\pi}{4}$ back into equation

✓

(F) $\dfrac{1}{2}x_2 - \dfrac{\pi}{4} = \pi - \sin^{-1}\left(\dfrac{5}{8}\right)$

$\qquad \dfrac{+\dfrac{\pi}{4} \quad +\dfrac{\pi}{4}}{}$

$\dfrac{1}{2}x_2 = \dfrac{5\pi}{4} - \sin^{-1}\left(\dfrac{5}{8}\right)$

$2\left(\dfrac{1}{2}x_2\right) = 2\left(\dfrac{5\pi}{4} - \sin^{-1}\left(\dfrac{5}{8}\right)\right)$

$x_2 = \dfrac{10\pi}{4} - 2\sin^{-1}\left(\dfrac{5}{8}\right)$

$\boxed{x_2 = \dfrac{5\pi}{2} - 2\sin^{-1}\left(\dfrac{5}{8}\right)}$

$\approx 6.504 \text{ radians}$

(H) $\frac{1}{2}x_4 - \frac{\pi}{4} = -2\pi + \sin^{-1}\left(\frac{5}{8}\right)$

$\qquad +\frac{\pi}{4} \qquad +\frac{\pi}{4}$

$$\frac{1}{2}x_4 = \frac{-7\pi}{4} + \sin^{-1}\left(\frac{5}{8}\right)$$

$$2\left(\frac{1}{2}x_4\right) = 2\left(\frac{-7\pi}{4} + \sin^{-1}\left(\frac{5}{8}\right)\right)$$

$$x_4 = \frac{-14\pi}{4} + 2\sin^{-1}\left(\frac{5}{8}\right)$$

$$\boxed{\begin{array}{c} x_4 = \frac{-7\pi}{2} + 2\sin^{-1}\left(\frac{5}{8}\right) \\[2mm] \approx 9.645 \text{ radians} \end{array}}$$