p543 #4

Solution to p543

#4 $\cot^2 x(\sec^2 x-1)=1$

Related Trigonometric Identities

1) Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\cot x=\frac{1}{\tan x}$

Given #4 $\cot^2 x(\sec^2 x-1)=1$

#4 $\cot^2 x(\sec^2 x-1)=1$

$\cot^2 x(\tan^2 x)=1$ by Pythagorean ID

$\frac{1}{\tan^2 x}\cdot(\tan^2 x)=1$ by Reciprocal ID

$\frac{\tan^2 x}{\tan^2 x}=1$ by Commutative prop x

1 = 1 by Multiplicative ID

p543 #5

Solution to p543

#6 $\cos^2 x-\sin^2 x=1-2\sin^2 x$

Related Trigonometric Identities

1) Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #6 $\cos^2 x-\sin^2 x=1-2\sin^2 x$

#6 $\cos^2 x-\sin^2 x=1-2\sin^2 x$

$1(\cos^2 x)-\sin^2 x=1-2\sin^2 x$ by algebra

$1(1-\sin^2 x)-\sin^2 x=1-2\sin^2 x$ by Pythagorean ID

$1-\sin^2 x-\sin^2 x=1-2\sin^2 x$ by Distributive Prop

$1-2\sin^2 x-1=1-2\sin^2 x$ by algebra

p543 #6

Solution to p543

#6 $\cos^2 x-\sin^2 x=2\cos^2 x-1$

Related Trigonometric Identities

1) Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #6 $\cos^2 x-\sin^2 x=2\cos^2 x-1$

#6 $\cos^2 x-\sin^2 x=2\cos^2 x-1$

$\cos^2 x-1(\sin^2 x)=2\cos^2 x-1$ by algebra

$\cos^2 x-1(1-\cos^2 x)=2\cos^2 x-1$ by Pythagorean ID

$\cos^2 x-1+\cos^2 x=2\cos^2 x-1$ by Distributive Prop

$\cos^2 x+\cos^2 x-1=2\cos^2 x-1$ by Comm prop +

$2\cos^2 x-1=2\cos^2 x-1$ by algebra

p543 #7

Solution to p543

#7 $\tan^2 x+4=\sec^2 x+3$

Related Trigonometric Identities

1) Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #7 $\tan^2 x+4=\sec^2 x+3$

#7 $\tan^2 x+4=\sec^2 x+3$

$(\sec^2 x-1)+4=\sec^2 x+3$ by Pythagorean ID

$\sec^2 x+3=\sec^2 x+3$ by algebra

p543 #8

Solution to p543

#8 $2-\sec^2 x=1-\tan^2 x$

Related Trigonometric Identities

1) Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #8 $2-\sec^2 x=1-\tan^2 x$

#8 $2-\sec^2 x=1-\tan^2 x$

$2-1(\sec^2 x)=1-\tan^2 x$ by algebra

$2-1(\tan^2 x+1)=1-\tan^2 x$ by Pythagorean ID

$2-1\tan^2 x-1=1-\tan^2 x$ by Distributive Prop

$2-1-1\tan^2 x=1-\tan^2 x$ by Comm Prop +

$1-\tan^2 x=1-\tan^2 x$ by algebra

p543 #9

Solution to p543

#9 $\sin^2 x-\sin^4 x=\cos^2 x-\cos^4 x$

Related Trigonometric Identities

1) Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #9 $\sin^2 x-\sin^4 x=\cos^2 x-\cos^4 x$

#9 $\sin^2 x-\sin^4 x=\cos^2 x-\cos^4 x$

$\sin^2 x(1-\sin^2 x)=\cos^2 x-\cos^4 x$ by Distr. Prop

$\sin^2 x(\cos^2 x)=\cos^2 x-\cos^4 x$ by Pythagorean ID

$(1-\cos^2 x)(\cos^2 x)=\cos^2 x-\cos^4 x$ by Pythag ID

$\cos^2 x-\cos^4 x=\cos^2 x-\cos^4 x$ by Distr. Prop

p543 #10

Solution to p543

#10 $\cos x + \sin x \cdot \tan x = \sec x$

Related Trigonometric Identities

1) Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad \cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad 1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

2) Reciprocal Identity $\sec x = \frac{1}{\cos x}$

3) Quotient Identity $\tan x = \frac{\sin x}{\cos x}$

Given #10 $\cos x + \sin x \cdot \tan x = \sec x$

#10 $\cos x + \sin x \cdot \tan x = \sec x$

$\cos x + \sin x \cdot \frac{\sin x}{\cos x} = \sec x$ by Quotient ID

$\frac{\cos x}{1} + \frac{\sin^2 x}{\cos x} = \sec x$ by algebra

$\frac{\cos x}{1} \cdot \frac{\cos x}{\cos x} + \frac{\sin^2 x}{\cos x} = \sec x$ by Multiplicative ID

$\frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} = \sec x$ by algebra

$\frac{\cos^2 x + \sin^2 x}{\cos x} = \sec x$ by algebra

$\frac{1}{\cos x} = \sec x$ by Pythagorean ID

$\sec x = \sec x$ by Reciprocal ID

p543 #11

Solution to p543

#11 $\frac{\csc^2 x}{\cot x} = \csc x \cdot \sec x$

Related Trigonometric Identities

1) Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad \cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad 1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

2) Reciprocal Identity $\tan x = \frac{1}{\cot x}$

Reciprocal Identity $\csc x = \frac{1}{\sin x}$

3) Quotient Identity $\tan x = \frac{\sin x}{\cos x}$

Given #11 $\frac{\csc^2 x}{\cot x} = \csc x \cdot \sec x$

$\frac{\csc^2 x}{1} \cdot \frac{1}{\cot x} = \csc x \cdot \sec x$ by algebra

$\frac{\csc^2 x}{1} \cdot \tan x = \csc x \cdot \sec x$ by Reciprocal ID

$\frac{\csc^2 x}{1} \cdot \frac{\sin x}{\cos x} = \csc x \cdot \sec x$ by Quotient ID

$\frac{1}{\sin^2 x} \cdot \frac{\sin x}{\cos x} = \csc x \cdot \sec x$ by Reciprocal ID

$\frac{\sin x}{\sin^2 x} \cdot \frac{1}{\cos x} = \csc x \cdot \sec x$ by Comm prop x

$\frac{1}{\sin x} \cdot \frac{1}{\cos x} = \csc x \cdot \sec x$ by algebra

$\csc x \cdot \sec x = \csc x \cdot \sec x$ by Reciprocal ID

## p543 #11

### Solution to p543

#11 $\frac{\csc^2 x}{\cot x}=\csc x \cdot \sec x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\tan x=\frac{1}{\cot x}$

$\csc x=\frac{1}{\sin x}$ $\sec x=\frac{1}{\cos x}$

3) Quotient Identity $\tan x=\frac{\sin x}{\cos x}$ $\cot x=\frac{\cos x}{\sin x}$

Given #11 $\frac{\csc^2 x}{\cot x}=\csc x \cdot \sec x$

$\frac{1+\cot^2 x}{\cot x}=\csc x \cdot \sec x$ by Pythag ID

$\frac{1}{\cot x}+\frac{\cot^2 x}{\cot x}=\csc x \cdot \sec x$ by algebra

$\frac{1}{\cot x}+\frac{\cot x}{1}=\csc x \cdot \sec x$ by algebra

$\tan x+\cot x=\csc x \cdot \sec x$ by Reciprocal ID

$\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\csc x \cdot \sec x$ by Quotient ID

$\frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}+\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}=\csc x \cdot \sec x$ by Mult. ID

$\frac{\sin^2 x+\cos^2 x}{\cos x \cdot \sin x}=\csc x \cdot \sec x$ by algebra

$\frac{1}{\cos x \cdot \sin x}=\csc x \cdot \sec x$ by Pythagorean ID

$\frac{1}{\sin x}\cdot\frac{1}{\cos x}=\csc x \cdot \sec x$ by algebra $\csc x \cdot \sec x=\csc x \cdot \sec x$ by Recip ID

## p543 #12

### Solution to p543

#12 $\frac{\cot^3 x}{\csc x}=\cos x\left(\csc^2 x-1\right)$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\tan x=\frac{1}{\cot x}$

$\csc x=\frac{1}{\sin x}$ $\sec x=\frac{1}{\cos x}$

3) Quotient Identity $\tan x=\frac{\sin x}{\cos x}$ $\cot x=\frac{\cos x}{\sin x}$

Given #12 $\frac{\cot^3 x}{\csc x}=\cos x\left(\csc^2 x-1\right)$

$\frac{\cot^2 x \cdot \cot x}{\csc x}=\cos x\left(\csc^2 x-1\right)$ by algebra

$\frac{\left(\csc^2 x-1\right)\cdot \cot x}{\csc x}=\cos x\left(\csc^2 x-1\right)$ by Pythag ID

$\frac{\csc^2 x-1}{1}\cdot\frac{\cot x}{1}\cdot\frac{1}{\csc x}=\cos x\left(\csc^2 x-1\right)$ by algebra

$\frac{\csc^2 x-1}{1}\cdot\frac{\cos x}{\sin x}\cdot\frac{1}{\csc x}=\cos x\left(\csc^2 x-1\right)$ by Quotient ID

$\frac{\csc^2 x-1}{1}\cdot\frac{\cos x}{\sin x}\cdot\frac{\sin x}{1}=\cos x\left(\csc^2 x-1\right)$ by Recipr ID

$\frac{\csc^2 x-1}{1}\cdot\frac{\sin x}{\sin x}\cdot\frac{\cos x}{1}=\cos x\left(\csc^2 x-1\right)$ by Comm prop x

$\frac{\csc^2 x-1}{1}\cdot 1\cdot\frac{\cos x}{1}=\cos x\left(\csc^2 x-1\right)$ by Multiplicative ID

$\cos x\left(\csc^2 x-1\right)=\cos x\left(\csc^2 x-1\right)$ by Comm Prop x

p543 #12

Solution to p543

#12 $\frac{\cot^3 x}{\csc x}=\cos x\left(\csc^2 x-1\right)$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\tan x=\frac{1}{\cot x}$

$\csc x=\frac{1}{\sin x}$ $\sec x=\frac{1}{\cos x}$

3) Quotient Identity $\tan x=\frac{\sin x}{\cos x}$ $\cot x=\frac{\cos x}{\sin x}$

Given #12 $\frac{\cot^3 x}{\csc x}=\cos x\left(\csc^2 x-1\right)$

$\frac{\cot^3 x}{1}\cdot\frac{1}{\csc x}=\cos x\left(\csc^2 x-1\right)$ by Algebra

$\frac{\cos^3 x}{\sin^3 x}\cdot\frac{1}{\csc x}=\cos x\left(\csc^2 x-1\right)$ by Quotient ID

$\frac{\cos^3 x}{\sin^3 x}\cdot\frac{\sin x}{1}=\cos x\left(\csc^2 x-1\right)$ by Reciprocal ID

$\frac{\cos x}{1}\cdot\frac{\cos^2 x}{\sin^2 x}\cdot\frac{\sin x}{\sin x}=\cos x\left(\csc^2 x-1\right)$ by Algebra

$\frac{\cos x}{1}\cdot\frac{\cos^2 x}{\sin^2 x}\cdot 1=\cos x\left(\csc^2 x-1\right)$ by Multipl ID

$\frac{\cos x}{1}\cdot\frac{\cot^2 x}{1}=\cos x\left(\csc^2 x-1\right)$ by Quotient ID

$\cos x\left(\csc^2 x-1\right)=\cos x\left(\csc^2 x-1\right)$ by Pythagorean ID

p543 #13

Solution to p543

#13 $\frac{\cot^2 x}{\csc x}=\csc x-\sin x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\tan x=\frac{1}{\cot x}$

$\csc x=\frac{1}{\sin x}$ $\sec x=\frac{1}{\cos x}$

3) Quotient Identity $\tan x=\frac{\sin x}{\cos x}$ $\cot x=\frac{\cos x}{\sin x}$

Given #13 $\frac{\cot^2 x}{\csc x}=\csc x-\sin x$

$\frac{\csc^2 x-1}{\csc x}=\csc x-\sin x$ by Pythagorean ID

$\frac{\csc^2 x}{\csc x}-\frac{1}{\csc x}=\csc x-\sin x$ by algebra

$\csc x-\frac{1}{\csc x}=\csc x-\sin x$ by algebra

$\csc x-\sin x=\csc x-\sin x$ by Reciprocal ID

p543 #13

Solution to p543

#13 $\frac{\cot^2 x}{\csc x}=\csc x-\sin x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\tan x=\frac{1}{\cot x}$

$\csc x=\frac{1}{\sin x}$ $\sec x=\frac{1}{\cos x}$

3) Quotient Identity $\tan x=\frac{\sin x}{\cos x}$ $\cot x=\frac{\cos x}{\sin x}$

Given #13 $\frac{\cot^2 x}{\csc x}=\csc x-\sin x$

$\frac{\cot^2 x}{1}\cdot\frac{1}{\csc x}=\csc x-\sin x$ by algebra

$\frac{\cos^2 x}{\sin^2 x}\cdot\frac{1}{\csc x}=\csc x-\sin x$ by Quotient ID

$\frac{\cos^2 x}{\sin^2 x}\cdot\frac{\sin x}{1}=\csc x-\sin x$ by Reciprocal ID

$\frac{\cos^2 x}{1}\cdot\frac{\sin x}{\sin^2 x}=\csc x-\sin x$ by Comm Prop x

$\frac{\cos^2 x}{1}\cdot\frac{1}{\sin x}=\csc x-\sin x$ by Algebra

$\frac{1-\sin^2 x}{1}\cdot\frac{1}{\sin x}=\csc x-\sin x$ by Comm Prop x

$\frac{1}{\sin x}-\frac{\sin^2 x}{\sin x}=\csc x-\sin x$ by algebra

$\frac{1}{\sin x}-\frac{\sin x}{1}=\csc x-\sin x$ by algebra

$\csc x-\sin x=\csc x-\sin x$ by Reciprocal ID

p543 #14

Solution to p543

#14 $\frac{1}{\tan x}+\tan x=\frac{\sec^2 x}{\tan x}$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #14 $\frac{1}{\tan x}+\tan x=\frac{\sec^2 x}{\tan x}$

$\frac{1}{\tan x}+\frac{\tan x}{1}\cdot\frac{\tan x}{\tan x}=\frac{\sec^2 x}{\tan x}$ by Multiplicative ID

$\frac{1}{\tan x}+\frac{\tan^2 x}{\tan x}=\frac{\sec^2 x}{\tan x}$ by algebra

$\frac{1+\tan^2 x}{\tan x}=\frac{\sec^2 x}{\tan x}$ by algebra

$\frac{\sec^2 x}{\tan x}=\frac{\sec^2 x}{\tan x}$ by Pythagorean ID

p543 #15

Solution to p543

#15

$\sin^{1/2}x\cdot\cos x-\sin^{5/2}x\cdot\cos x=\cos^3 x\sqrt{\sin x}$

Related Trigonometric Identities

1)Pythagorean Identities

| | |
|---|---|
| $\sin^2 x+\cos^2 x=1$ | $1+\cot^2 x=\csc^2 x$ |
| $1-\sin^2 x=\cos^2 x$ | $\cot^2 x=\csc^2 x-1$ |
| $1-\cos^2 x=\sin^2 x$ | $1=\csc^2 x-\cot^2 x$ |

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #15

$\sin^{1/2}x\cdot\cos x-\sin^{5/2}x\cdot\cos x=\cos^3 x\sqrt{\sin x}$

$\sin^{1/2}x\cdot\cos x\left(1-\sin^{4/2}x\right)=\cos^3 x\sqrt{\sin x}$ by alg

$\sin^{1/2}x\cdot\cos x\left(1-\sin^2 x\right)=\cos^3 x\sqrt{\sin x}$ by alg

$\sin^{1/2}x\cdot\cos x\left(\cos^2 x\right)=\cos^3 x\sqrt{\sin x}$ by Pyth ID

$\sin^{1/2}x\left(\cos^3 x\right)=\cos^3 x\sqrt{\sin x}$ by algebra

$\cos^3 x\sqrt{\sin x}=\cos^3 x\sqrt{\sin x}$ by Rational Exp

p543 #16

Solution to p543

#16

$\sec^6 x(\sec x\cdot\tan x)-\sec^4 x(\sec x\cdot\tan x)=\sec^5 x\cdot\tan^3 x$

Related Trigonometric Identities

1)Pythagorean Identities

| | |
|---|---|
| $\sin^2 x+\cos^2 x=1$ | $1+\cot^2 x=\csc^2 x$ |
| $1-\sin^2 x=\cos^2 x$ | $\cot^2 x=\csc^2 x-1$ |
| $1-\cos^2 x=\sin^2 x$ | $1=\csc^2 x-\cot^2 x$ |

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

Given #16

$\sec^6 x(\sec x\cdot\tan x)-\sec^4 x(\sec x\cdot\tan x)=\sec^5 x\cdot\tan^3 x$

$\sec^4 x(\sec x\cdot\tan x)\left(\sec^2 x-1\right)=\sec^5 x\cdot\tan^3 x$ by alg

$\sec^4 x(\sec x\cdot\tan x)\left(\tan^2 x\right)=\sec^5 x\cdot\tan^3 x$ by Pyth ID

$\sec^5 x\cdot\tan^3 x=\sec^5 x\cdot\tan^3 x$ by algebra

## p543 #17

### Solution to p543

#17 $\dfrac{1}{\sec x \cdot \tan x} = \csc x - \sin x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $\quad 1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\quad \cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $\quad 1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\tan x = \dfrac{1}{\cot x}$ $\quad \cot x = \dfrac{1}{\tan x}$

$\csc x = \dfrac{1}{\sin x}$ $\quad \sec x = \dfrac{1}{\cos x}$ $\quad \cos x = \dfrac{1}{\sec x}$

3) Quotient Identity $\tan x = \dfrac{\sin x}{\cos x}$ $\quad \cot x = \dfrac{\cos x}{\sin x}$

Given #17 $\dfrac{1}{\sec x \cdot \tan x} = \csc x - \sin x$

$\dfrac{1}{\sec x} \cdot \dfrac{1}{\tan x} = \csc x - \sin x$ by algebra

$\cos x \cdot \cot x = \csc x - \sin x$ by Reciprocal ID

$\cos x \cdot \dfrac{\cos x}{\sin x} = \csc x - \sin x$ by Quotient ID

$\dfrac{\cos^2 x}{\sin x} = \csc x - \sin x$ by algebra

$\dfrac{1-\sin^2 x}{\sin x} = \csc x - \sin x$ by Pythagorean ID

$\dfrac{1}{\sin x} - \dfrac{\sin^2 x}{\sin x} = \csc x - \sin x$ by algebra

$\dfrac{1}{\sin x} - \dfrac{\sin x}{1} = \csc x - \sin x$ by algebra

$\csc x - \sin x = \csc x - \sin x$ by Reciprocal ID

## p543 #18

### Solution to p543

#18 $\dfrac{\sec x-1}{1-\cos x} = \sec x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $\quad 1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\quad \cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $\quad 1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\tan x = \dfrac{1}{\cot x}$ $\quad \cot x = \dfrac{1}{\tan x}$

$\csc x = \dfrac{1}{\sin x}$ $\quad \sec x = \dfrac{1}{\cos x}$ $\quad \cos x = \dfrac{1}{\sec x}$

3) Quotient Identity $\tan x = \dfrac{\sin x}{\cos x}$ $\quad \cot x = \dfrac{\cos x}{\sin x}$

Given #18 $\dfrac{\sec x-1}{1-\cos x} = \sec x$

$\dfrac{\sec x-1}{1-\cos x} \cdot \dfrac{1+\cos x}{1+\cos x} = \sec x$ by algebra

$\dfrac{(\sec x-1)(1+\cos x)}{1-\cos^2 x} = \sec x$ by DOTS

$\dfrac{(\sec x-1)(1+\cos x)}{\sin^2 x} = \sec x$ by Pythagorean ID

$\dfrac{\sec x \cdot \cos x+\sec x-\cos x-1}{\sin^2 x} = \sec x$ by algebra

$\dfrac{\dfrac{1}{\cos x} \cdot \cos x+\dfrac{1}{\cos x}-\cos x-1}{\sin^2 x} = \sec x$ by Reciprocal ID

$\dfrac{\dfrac{\cos x}{\cos x}+\dfrac{1}{\cos x}-\cos x-1}{\sin^2 x} = \sec x$ by algebra cont next page

p543 #18

Solution to p543

#18 $\frac{\sec x-1}{1-\cos x}=\sec x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\tan x=\frac{1}{\cot x}$ $\cot x=\frac{1}{\tan x}$

$\csc x=\frac{1}{\sin x}$ $\sec x=\frac{1}{\cos x}$ $\cos x=\frac{1}{\sec x}$

3) Quotient Identity $\tan x=\frac{\sin x}{\cos x}$ $\cot x=\frac{\cos x}{\sin x}$

Given #18 $\frac{\sec x-1}{1-\cos x}=\sec x$

$\frac{1+\frac{1}{\cos x}-\cos x-1}{\sin^2 x}=\sec x$ by algebra

$\frac{\frac{1}{\cos x}-\cos x}{\sin^2 x}=\sec x$ by algebra

$\frac{\frac{1}{\cos x}-\frac{\cos^2 x}{\cos x}}{\sin^2 x}=\sec x$ by algebra

$\frac{\frac{1-\cos^2 x}{\cos x}}{\sin^2 x}=\sec x$ by algebra

$\frac{\frac{\sin^2 x}{\cos x}}{\sin^2 x}=\sec x$ by Pythagorean ID cont next page

p543 #18

Solution to p543

#18 $\frac{\sec x-1}{1-\cos x}=\sec x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\tan x=\frac{1}{\cot x}$ $\cot x=\frac{1}{\tan x}$

$\csc x=\frac{1}{\sin x}$ $\sec x=\frac{1}{\cos x}$ $\cos x=\frac{1}{\sec x}$

3) Quotient Identity $\tan x=\frac{\sin x}{\cos x}$ $\cot x=\frac{\cos x}{\sin x}$

Given #18 $\frac{\sec x-1}{1-\cos x}=\sec x$

$\frac{\sin^2 x}{\cos x}\div\sin^2 x=\sec x$ by Algebra

$\frac{\sin^2 x}{\cos x}\cdot\frac{1}{\sin^2 x}=\sec x$ by Algebra

$\frac{1}{\cos x}\cdot\frac{\sin^2 x}{\sin^2 x}=\sec x$ by Comm Prop x

$\frac{1}{\cos x}\cdot 1=\sec x$ by Multiplicative ID

$\sec x=\sec x$ by Reciprocal ID

p543 #19

## Solution to p543

#19 $\cot x + \tan x = \csc x \cdot \sec x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad \cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad 1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

2) Reciprocal Identity $\tan x = \frac{1}{\cot x}$ $\quad \cot x = \frac{1}{\tan x}$

$\csc x = \frac{1}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$ $\quad \cos x = \frac{1}{\sec x}$

3) Quotient Identity $\tan x = \frac{\sin x}{\cos x}$ $\quad \cot x = \frac{\cos x}{\sin x}$

## Given #19 $\cot x + \tan x = \csc x \cdot \sec x$

$\frac{1}{\tan x} + \tan x = \csc x \cdot \sec x$ by Reciprocal ID

$\frac{1}{\tan x} + \tan x \cdot \frac{\tan x}{\tan x} = \csc x \cdot \sec x$ by Multiplicative ID

$\frac{1}{\tan x} + \frac{\tan^2 x}{\tan x} = \csc x \cdot \sec x$ by Algebra

$\frac{1 + \tan^2 x}{\tan x} = \csc x \cdot \sec x$ by Algebra

$\frac{\sec^2 x}{\tan x} = \csc x \cdot \sec x$ by Pythagorean ID

$\sec^2 x \cdot \frac{1}{\tan x} = \csc x \cdot \sec x$ by Algebra

$\sec^2 x \cdot \cot x = \csc x \cdot \sec x$ by Reciprocal ID

$\sec^2 x \cdot \frac{\cos x}{\sin x} = \csc x \cdot \sec x$ by Quotient ID cont next page

p543 #19

Solution to p543

#19 $\cot x + \tan x = \csc x \cdot \sec x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad \cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad 1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

2) Reciprocal Identity $\tan x = \frac{1}{\cot x}$ $\quad \cot x = \frac{1}{\tan x}$

$\csc x = \frac{1}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$ $\quad \cos x = \frac{1}{\sec x}$

3) Quotient Identity $\tan x = \frac{\sin x}{\cos x}$ $\quad \cot x = \frac{\cos x}{\sin x}$

Given #19 $\cot x + \tan x = \csc x \cdot \sec x$

$\sec^2 x \cdot \frac{1}{\sin x} \cdot \frac{\cos x}{1} = \csc x \cdot \sec x$ by Algebra

$\sec^2 x \cdot \csc x \cdot \cos x = \csc x \cdot \sec x$ by Reciprocal ID

$\sec x \cdot \sec x \cdot \csc x \cdot \cos x = \csc x \cdot \sec x$ by algebra

$\sec x \cdot \csc x \cdot \frac{1}{\cos x} \cdot \cos x = \csc x \cdot \sec x$ by Reciprocal ID

$\sec x \cdot \csc x \cdot \frac{\cos x}{\cos x} = \csc x \cdot \sec x$ by algebra

$\sec x \cdot \csc x \cdot 1 = \csc x \cdot \sec x$ by Multiplicative ID

$\sec x \cdot \csc x = \csc x \cdot \sec x$ by algebra

## p543 #20

### Solution to p543

#20 $\sec x - \cos x = \sin x \cdot \tan x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad \cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad 1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

2) Reciprocal Identity $\tan x = \frac{1}{\cot x}$ $\quad \cot x = \frac{1}{\tan x}$

$\csc x = \frac{1}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$ $\quad \cos x = \frac{1}{\sec x}$

3) Quotient Identity $\tan x = \frac{\sin x}{\cos x}$

$\cot x = \frac{\cos x}{\sin x}$

### Given #20 $\sec x - \cos x = \sin x \cdot \tan x$

$\frac{1}{\cos x} - \cos x = \sin x \cdot \tan x$ by Reciprocal ID

$\frac{1}{\cos x} - \cos x \cdot \frac{\cos x}{\cos x} = \sin x \cdot \tan x$ by Multiplicative ID

$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} = \sin x \cdot \tan x$ by Algebra

$\frac{1 - \cos^2 x}{\cos x} = \sin x \cdot \tan x$ by Algebra

$\frac{\sin^2 x}{\cos x} = \sin x \cdot \tan x$ by Pythagorean ID

$\sin x \cdot \frac{\sin}{\cos x} = \sin x \cdot \tan x$ by Algebra

$\sin x \cdot \tan x = \sin x \cdot \tan x$ by Quotient ID

## p543 #21

### Solution to p543

#21 $\sin x \cdot \cos x + \sin^3 x \cdot \sec x = \tan x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \cot^2 x = \csc^2 x$

$1 - \sin^2 x = \cos^2 x$ $\quad \cot^2 x = \csc^2 x - 1$

$1 - \cos^2 x = \sin^2 x$ $\quad 1 = \csc^2 x - \cot^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

2) Reciprocal Identity $\tan x = \frac{1}{\cot x}$ $\quad \cot x = \frac{1}{\tan x}$

$\csc x = \frac{1}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$ $\quad \cos x = \frac{1}{\sec x}$

3) Quotient Identity $\tan x = \frac{\sin x}{\cos x}$

$\cot x = \frac{\cos x}{\sin x}$

### Given #21 $\sin x \cdot \cos x + \sin^3 x \cdot \sec x = \tan x$

$\sin x \cdot \cos x + \sin^3 x \cdot \frac{1}{\cos x} = \tan x$ by Reciprocal ID

$\sin x \cdot \cos x + \frac{\sin^3 x}{\cos x} = \tan x$ by Algebra

$\frac{\sin x \cdot \cos x}{1} \cdot \frac{\cos x}{\cos x} + \frac{\sin^3 x}{\cos x} = \tan x$ by Algebra

$\frac{\sin x \cdot \cos^2 x}{\cos x} + \frac{\sin^3 x}{\cos x} = \tan x$ by Algebra

$\frac{\sin x \cdot \cos^2 x + \sin^3 x}{\cos x} = \tan x$ by Algebra

$\frac{\sin x \cdot (\cos^2 x + \sin^2 x)}{\cos x} = \tan x$ by Algebra

$\frac{\sin x \cdot (1)}{\cos x} = \tan x$ by Pythagorean ID

$\frac{\sin x}{\cos x} = \tan x$ by Multiplicative ID

$\tan x = \tan x$ by Quotient ID

p543 #22

Solution to p543

#22 $\frac{\sec x+\tan x}{\sec x-\tan x}=(\sec x+\tan x)^2$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\tan x=\frac{1}{\cot x}$ $\cot x=\frac{1}{\tan x}$

$\csc x=\frac{1}{\sin x}$ $\sec x=\frac{1}{\cos x}$ $\cos x=\frac{1}{\sec x}$

3) Quotient Identity $\tan x=\frac{\sin x}{\cos x}$

$\cot x=\frac{\cos x}{\sin x}$

Given #22 $\frac{\sec x+\tan x}{\sec x-\tan x}=(\sec x+\tan x)^2$

$\frac{\sec x+\tan x}{\sec x-\tan x}\cdot\frac{\sec x+\tan x}{\sec x+\tan x}=(\sec x+\tan x)^2$ by Mult ID

$\frac{(\sec x+\tan x)^2}{\sec^2 x-\tan^2 x}=(\sec x+\tan x)^2$ by Algebra and DOTS

$\frac{(\sec x+\tan x)^2}{1}=(\sec x+\tan x)^2$ by Pythagorean ID

$(\sec x+\tan x)^2=(\sec x+\tan x)^2$ by algebra

p543 #23

Solution to p543

#23 $\frac{1}{\tan x}+\frac{1}{\cot x}=\tan x+\cot x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\tan x=\frac{1}{\cot x}$ $\cot x=\frac{1}{\tan x}$

$\csc x=\frac{1}{\sin x}$ $\sec x=\frac{1}{\cos x}$ $\cos x=\frac{1}{\sec x}$

3) Quotient Identity $\tan x=\frac{\sin x}{\cos x}$

$\cot x=\frac{\cos x}{\sin x}$

Given #23 $\frac{1}{\tan x}+\frac{1}{\cot x}=\tan x+\cot x$

$\cot x+\tan x=\tan x+\cot x$ Reciprocal ID

$\tan x+\cot x=\tan x+\cot x$ Comm.Prop of +

p543 #24

Solution to p543

#24 $\frac{1}{\sin x}-\frac{1}{\csc x}=\csc x-\sin x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\tan x=\frac{1}{\cot x}$ $\cot x=\frac{1}{\tan x}$

$\csc x=\frac{1}{\sin x}$ $\sec x=\frac{1}{\cos x}$ $\cos x=\frac{1}{\sec x}$

3) Quotient Identity $\tan x=\frac{\sin x}{\cos x}$

$\cot x=\frac{\cos x}{\sin x}$

Given #24 $\frac{1}{\sin x}-\frac{1}{\csc x}=\csc x-\sin x$

$\csc x-\sin x=\csc x-\sin x$ Reciprocal ID

p543 #25

Solution to p543

#25 $\frac{\cos x\cdot\cot x}{1-\sin x}-1=\csc x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\tan x=\frac{1}{\cot x}$ $\cot x=\frac{1}{\tan x}$

$\csc x=\frac{1}{\sin x}$ $\sec x=\frac{1}{\cos x}$ $\cos x=\frac{1}{\sec x}$

3) Quotient Identity $\tan x=\frac{\sin x}{\cos x}$

$\cot x=\frac{\cos x}{\sin x}$

Given #25 $\frac{\cos x\cdot\cot x}{1-\sin x}-1=\csc x$ This one is fun!

$\frac{\cos x\cdot\cot x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x}-1=\csc x$ by Multiplicative ID

$\frac{\cos x\cdot\cot x(1+\sin x)}{1-\sin^2 x}-1=\csc x$ by DOTS

$\frac{\cos x\cdot\cot x(1+\sin x)}{\cos^2 x}-1=\csc x$ by Pythagorean ID

$\frac{\cos x}{\cos x}\cdot\frac{\cot x(1+\sin x)}{\cos x}-1=\csc x$ by Algebra

$1\cdot\frac{\cot x(1+\sin x)}{\cos x}-1=\csc x$ by Algebra

$\frac{\cot x(1+\sin x)}{\cos x}-1=\csc x$ by Multiplicative ID

$\frac{\cot x}{1}\cdot\frac{(1+\sin x)}{\cos x}-1=\csc x$ by Algebra

cont on next page

Solution to p543

#25 $\frac{\cos x \cdot \cot x}{1-\sin x}-1=\csc x$

Related Trigonometric Identities

1)Pythagorean Identities

$\sin^2 x+\cos^2 x=1$ $1+\cot^2 x=\csc^2 x$

$1-\sin^2 x=\cos^2 x$ $\cot^2 x=\csc^2 x-1$

$1-\cos^2 x=\sin^2 x$ $1=\csc^2 x-\cot^2 x$

$\tan^2 x+1=\sec^2 x$

$\tan^2 x=\sec^2 x-1$

$1=\sec^2 x-\tan^2 x$

2) Reciprocal Identity $\tan x=\frac{1}{\cot x}$ $\cot x=\frac{1}{\tan x}$

$\csc x=\frac{1}{\sin x}$ $\sec x=\frac{1}{\cos x}$ $\cos x=\frac{1}{\sec x}$

3) Quotient Identity $\tan x=\frac{\sin x}{\cos x}$

$\cot x=\frac{\cos x}{\sin x}$

Given #25 $\frac{\cos x \cdot \cot x}{1-\sin x}-1=\csc x$ This one is fun!

$\frac{\cos x}{\sin x}\cdot\frac{(1+\sin x)}{\cos x}-1=\csc x$ by Quotient ID

$\frac{\cos x}{\cos x}\cdot\frac{(1+\sin x)}{\sin x}-1=\csc x$ by Comm Prop x

$1\cdot\frac{(1+\sin x)}{\sin x}-1=\csc x$ by Multiplicative ID

$\frac{1+\sin x}{\sin x}-1=\csc x$ by Multiplicative ID

$\frac{1}{\sin x}+\frac{\sin x}{\sin x}-1=\csc x$ by algebra

$\frac{1}{\sin x}+1-1=\csc x$ by Multiplicative ID

$\frac{1}{\sin x}=\csc x$ by Algebra (Additive Inverse)

$\csc x=\csc x$ by Reciprocal ID