Solutions to Entry Slip 2−3−17

There were three ferris wheel problems that we look at on 2−3−17

problem 1) Started at the minimum height  (this is probably best modeled by cosine model)

problem 2) Started at the maximum height (this is probably best modeled by cosine model)

problem 3) Started at midline (this is probably best modeled by sine model)

Recall: a < 0 if a reflection is present (problem 1)

$y=a\cdot \cos\left(\frac{2\pi}{period}\left(x+\text{shift}\right)\right)+midline$ or $y=a\cdot \sin\left(\frac{2\pi}{period}\left(x+\text{shift}\right)\right)+midline$

I will also write a shifted model for each of the models to show those that attempted models how to do this as well

NOTE: some books use −shift, but then you have to deal with more negatives in solution process

A carnival Ferris wheel with a diameter of 9 m. makes one complete revolution every 44 seconds.  The bottom of the wheel is 1.2 m above the ground.  If a person is at the minimum height when a stopwatch is started, then determine how high above the ground that person will be after 6 minutes and 41 seconds.

1. What is the equation of the midline that this Ferris wheel model is expecting?

2. Since we are at a minimum at time 0, write a model that predicts the height above the ground in terms of seconds.  Let y=height above the ground and x = time in seconds since start of motion.

3. After 6 minutes and 41 second the person is approximately _________ meters off the ground

## Question

A carnival Ferris wheel with a diameter of 9 m. makes one complete revolution every 44 seconds. The bottom of the wheel is 1.2 m above the ground. If a person is at the minimum height when a stopwatch is started, then determine how high above the ground that person will be after 6 minutes and 41 seconds.

1. What is the equation of the midline that this Ferris wheel model is expecting?

## Answer

We find the midline by adding any platform height or height above ground to the radius

so d = radius + platform

$$d = \frac{9}{2} + 1.2$$

We know the midline of the equation regardless of sine or cosine is y=5.7

**Question**

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What is the "b" of this model?

**Answer**

Since we are using radians b $= \frac{2\cdot\pi}{period} = \frac{2\cdot\pi}{44}$

So we know regardless of which model we build b $= \frac{2\cdot\pi}{44}$

**Question**

A carnival Ferris wheel with a diameter of 9 m. makes one complete revolution every 44 seconds. The bottom of the wheel is 1.2 m above the ground. If a person is at the minimum height when a stopwatch is started, then determine how high above the ground that person will be after 6 minutes and 41 seconds.

When does cool stuff occur?

**Answer**

cool stuff happens every 14 of period, so $\frac{1}{4} \cdot 44$ (every 11 seconds)

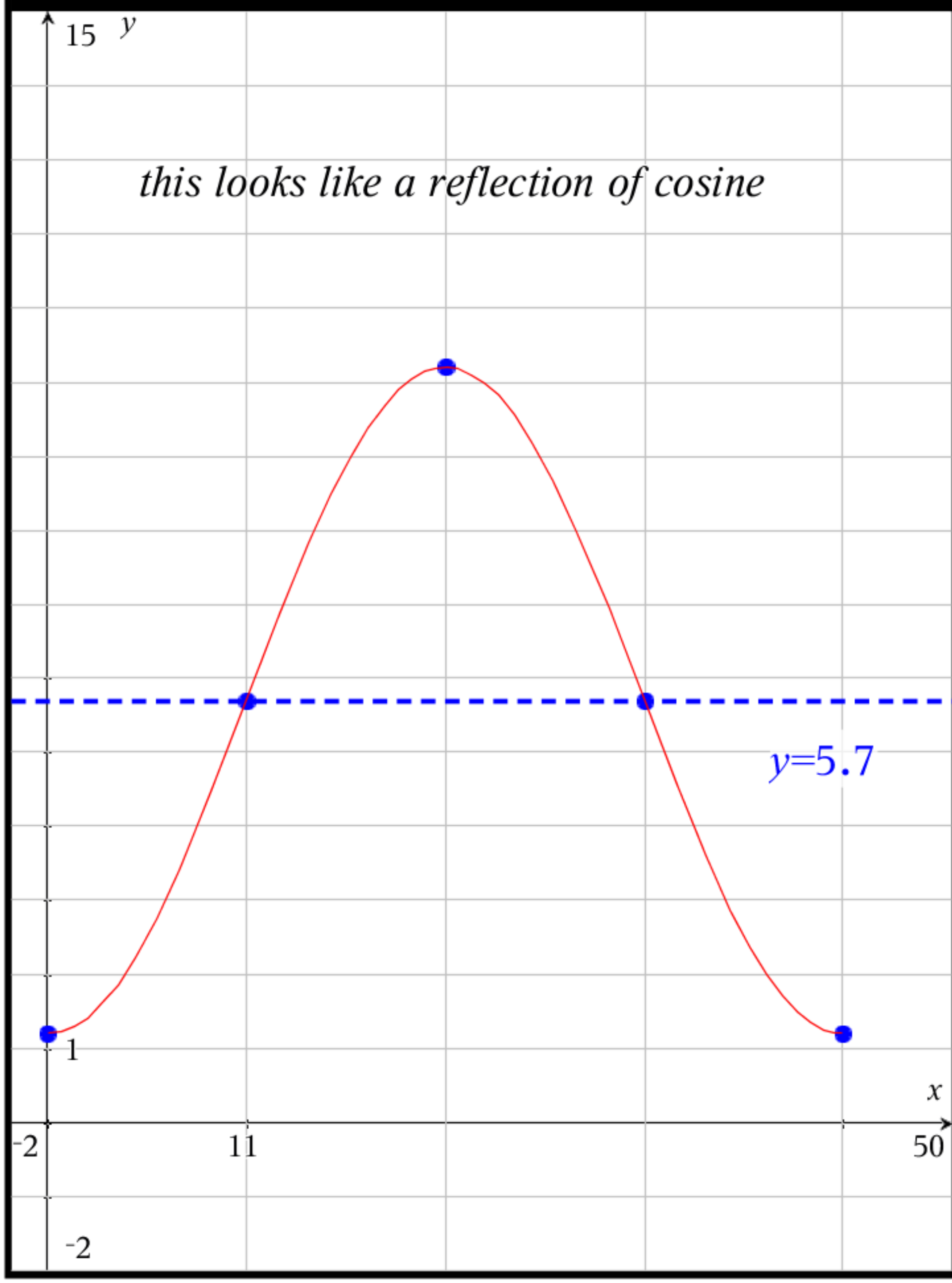
at time 0 we are at $1.2$ m above ground (minimum height )

at time 11 we are at $1.2+4.5$ m above ground (midline)

at time 22 we are at $1.2+9$ m above the ground (maximum height)

at time 33 we are at $1.2+4.5$ m above ground (midline)

at time 44 we are at $1.2$ m above ground (minimum)




So as of right now, we know that the midline is

y = 5.7

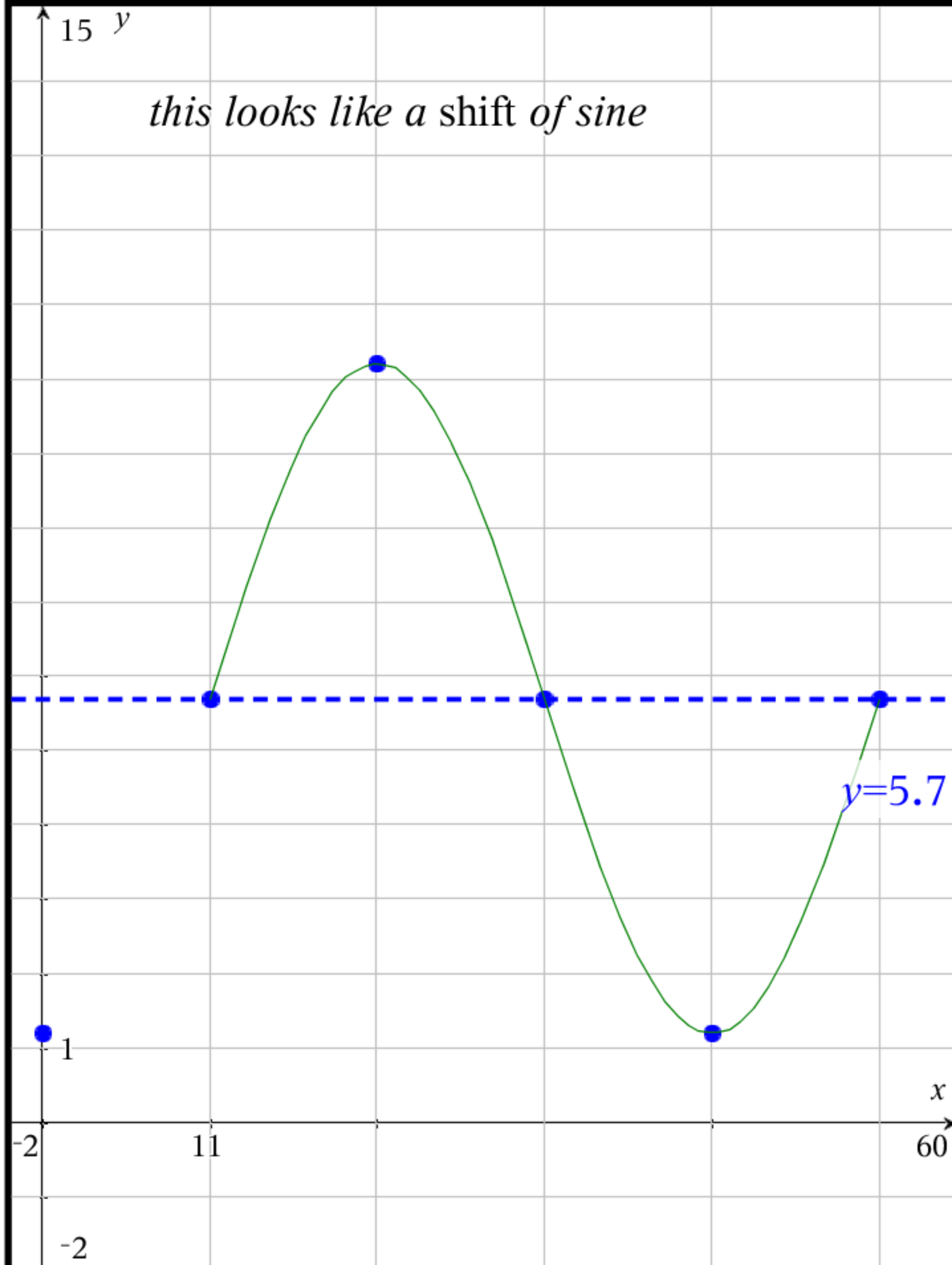
we also know that there are five easily found points to put on our graph

(0,1,2) (11,5.7) (22, 10.2) (33, 5.7) and(44,1.2)

and b = $\frac{2\cdot\pi}{44}$

| | A time | B height | C | D | E | F |
|---|---|---|---|---|---|---|
| = | | | | | | |
| 1 | 0 | 1.2 | | | | |
| 2 | 11 | 5.7 | | | | |
| 3 | 22 | 10.2 | | | | |
| 4 | 33 | 5.7 | | | | |
| 5 | 44 | 1.2 | | | | |
| 6 | 55 | 5.7 | | | | |
| 7 | | | | | | |
| 8 | | | | | | |
| 9 | | | | | | |
| 10 | | | | | | |

A1 =0




So as of right now, we know that the midline is

y = 5.7

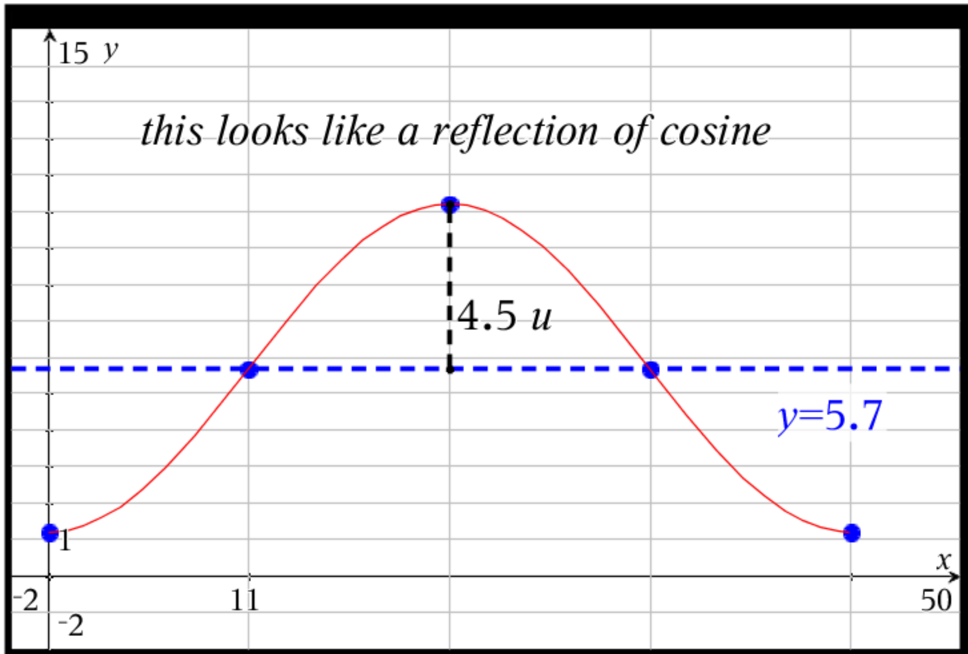
we also know that there are six easily found points to put on our graph

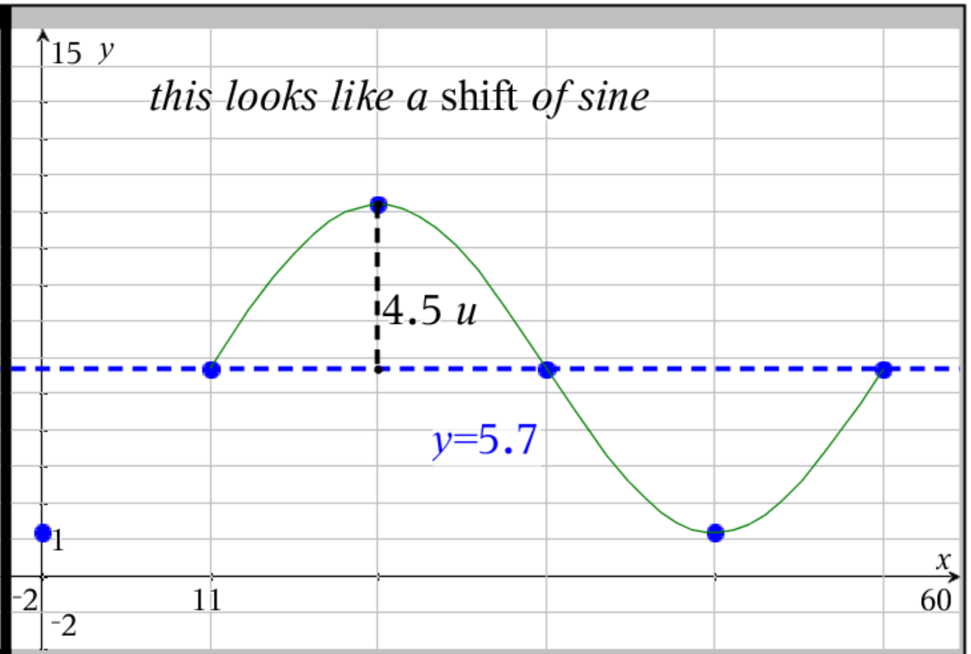
(0,1,2) (11,5.7) (22, 10.2) (33, 5.7), (44,1.2) and (55, 5.7)

and b = $\frac{2\cdot\pi}{44}$

| | A time | B height | C | D | E | F |
|---|---|---|---|---|---|---|
| = | | | | | | |
| 1 | 0 | 1.2 | | | | |
| 2 | 11 | 5.7 | | | | |
| 3 | 22 | 10.2 | | | | |
| 4 | 33 | 5.7 | | | | |
| 5 | 44 | 1.2 | | | | |
| 6 | 55 | 5.7 | | | | |
| 7 | | | | | | |

A1 0

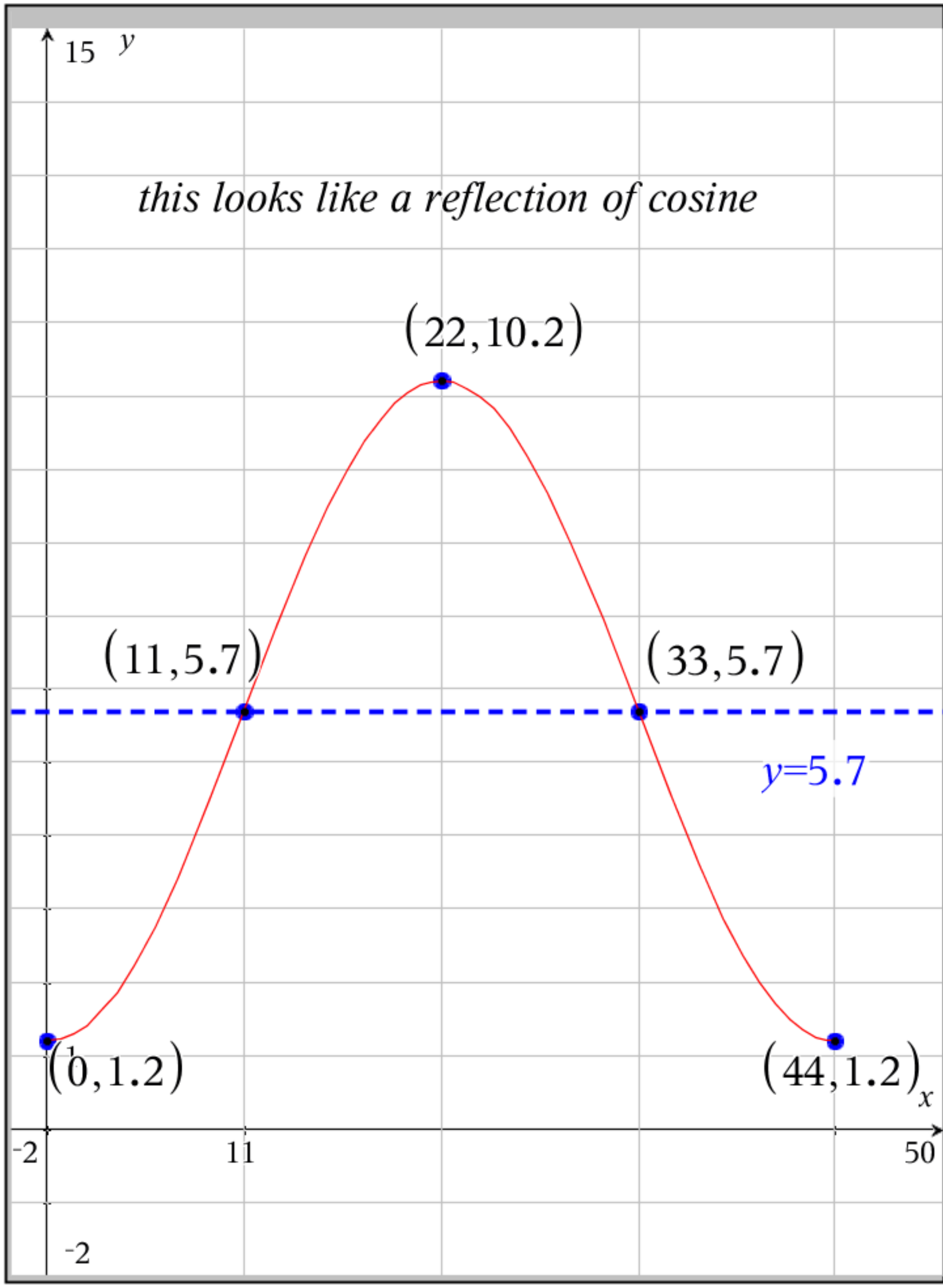



Either way we look at it the amplitude we have is $|4.5|$ either −4.5 or 4.5

In the case of minimum models, we use −4.5 as "a"

In the case of midline models,  we use  4.5 as "a"




So as of right now, we know that the midline is

y = 5.7 & b = $\frac{2\cdot\pi}{44}$ & "a" = $^-4.5$

So since we started at minimum we can write a NO SHIFT model

$$y=^-4.5\cdot\cos\left(\frac{2\cdot\pi}{44}\cdot x\right)+5.7$$

test of model

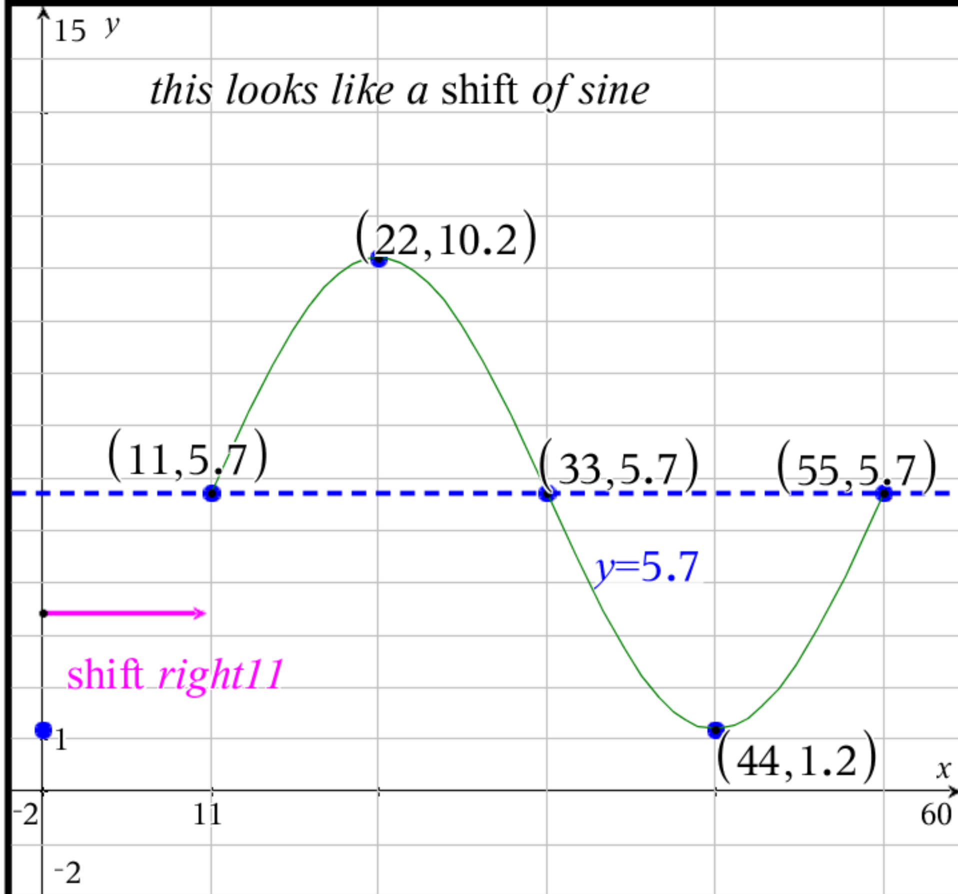
$$y=^-4.5\cdot\cos\left(\frac{2\cdot\pi}{44}\cdot 0\right)+5.7$$

$$y=^-4.5\cdot\cos\left(\frac{2\cdot\pi}{44}\cdot 11\right)+5.7$$

$$y=^-4.5\cdot\cos\left(\frac{2\cdot\pi}{44}\cdot 22\right)+5.7$$

$$y=^-4.5\cdot\cos\left(\frac{2\cdot\pi}{44}\cdot 33\right)+5.7$$

$$y=^-4.5\cdot\cos\left(\frac{2\cdot\pi}{44}\cdot 44\right)+5.7$$




we can build the model without solving because we know the shift (11 right)

(I'll show the algebra after this, but not necessary when you know about transformations and the shift)

So as of right now, we know that the midline is

y = 5.7 b = $\frac{2\cdot\pi}{44}$ & "a" = 4.5

We also know that we want to shift the graph to the right of 0 by 11

$$y=4.5\cdot\sin\left(\frac{2\cdot\pi}{44}\cdot(x-11)\right)+5.7$$

test of model

$$y=4.5\cdot\sin\left(\frac{2\cdot\pi}{44}\cdot(0-11)\right)+5.7$$

$$y=4.5\cdot\sin\left(\frac{2\cdot\pi}{44}\cdot(11-11)\right)+5.7$$

$$y=4.5\cdot\sin\left(\frac{2\cdot\pi}{44}\cdot(22-11)\right)+5.7$$

$$y=4.5\cdot\sin\left(\frac{2\cdot\pi}{44}\cdot(33-11)\right)+5.7$$

$$y=4.5\cdot\sin\left(\frac{2\cdot\pi}{44}\cdot(44-11)\right)+5.7$$

Algebra of this problem

we know (0,1.2) lies on this sine model

we know $y=4.5\cdot\sin\left(\frac{2\cdot\pi}{44}\cdot(x+\text{shift})\right)+5.7$

So replace and solve for shift!

$$1.2=4.5\cdot\sin\left(\frac{2\cdot\pi}{44}\cdot(0+\text{shift})\right)+5.7$$

$$1.2=4.5\cdot\sin\left(\frac{2\cdot\pi}{44}\cdot(\text{shift})\right)+5.7$$

$$1.2-5.7=4.5\cdot\sin\left(\frac{2\cdot\pi}{44}\cdot(\text{shift})\right)+5.7-5.7$$

$$-4.5=4.5\cdot\sin\left(\frac{2\cdot\pi}{44}\cdot(\text{shift})\right)$$

$$\frac{-4.5}{4.5}=\frac{4.5\cdot\sin\left(\frac{2\cdot\pi}{44}\cdot(\text{shift})\right)}{4.5}$$

$$-1=\sin\left(\frac{2\cdot\pi}{44}\cdot(\text{shift})\right)$$

Now we know the equation to solve using inverse trigonometry

$$\sin^{-1}(-1)=\sin^{-1}\left(\sin\left(\frac{2\cdot\pi}{44}\cdot(\text{shift})\right)\right)$$

$$\frac{-\pi}{2}=\frac{2\cdot\pi}{44}\cdot(\text{shift})$$

$$\frac{-\pi}{2}\cdot\frac{44}{2\cdot\pi}=\frac{44}{2\cdot\pi}\cdot\frac{-2\cdot\pi}{44}\cdot\text{shift}$$

$-11$ = shift

This was the process we use if we DON'T know the shift amount

This is tedious and error prone so be careful!

## Question

A carnival Ferris wheel with a diameter of 9 m. makes one complete revolution every 44 seconds. The bottom of the wheel is 1.2 m above the ground. If a person is at the minimum height when a stopwatch is started, then determine how high above the ground that person will be after 6 minutes and 41 seconds.

2. Since we are at a minimum at time 0, write a model that predicts the height above the ground in terms of seconds. Let y=height above the ground and x = time in seconds since start of motion.

## Answer

cosine model $y=^{-}4.5\cdot \cos\left(\frac{2\cdot \pi}{44}\cdot x\right)+5.7$ sine model $y=4.5\cdot \sin\left(\frac{2\cdot \pi}{44}\cdot (x-11)\right)+5.7$

**Question**

A carnival Ferris wheel with a diameter of 9 m. makes one complete revolution every 44 seconds. The bottom of the wheel is 1.2 m above the ground. If a person is at the minimum height when a stopwatch is started, then determine how high above the ground that person will be after 6 minutes and 41 seconds.

3. After 6 minutes and 41 second the person is approximately _________ meters off the ground

**Answer**

Since this model has time in seconds we need to convert 6minutes and 41 seconds to seconds $6\cdot 60+41 = 401$ so we can plug x = 401 into either model

$y=-4.5\cdot \cos\left(\frac{2\cdot\pi}{44}\cdot 401\right)+5.7 = 2.29913$ $\quad y=4.5\cdot \sin\left(\frac{2\cdot\pi}{44}\cdot (401-11)\right)+5.7 = 2.29913$

So the height after 6 minutes and 41 seconds is approximately 2.299 meters above ground