Solutions to Entry Slip 2−3−17

There were three ferris wheel problems that we look at on 2−3−17

problem 1) Started at the minimum height (this is probably best modeled by cosine model)

problem 2) Started at the maximum height (this is probably best modeled by cosine model)

problem 3) Started at midline (this is probably best modeled by sine model)

Recall: a < 0 if a reflection is present (problem 1)

$y=a\cdot \cos\left(\frac{2\text{pi}}{period}(x-\text{shift})\right)+midline$ or $y=a\cdot \sin\left(\frac{2\text{pi}}{period}(x-\text{shift})\right)+midline$

I will also write a shifted model for each of the models to show those that attempted models how to do this as well

A carnival Ferris wheel with a radius of 20 m. makes one complete revolution every 248 seconds. The bottom of the wheel is 1.8 m above the ground. If a person is at the maximum height when a stopwatch is started, then determine how high above the ground that person will be after 9 minutes and 4 seconds.

4. What is the equation of the midline that this Ferris wheel model is expecting?

5. Since we are at a maximum at time 0, write a model that predicts the height above the ground in terms of seconds. Let y=height above the ground and x = time in seconds since start of motion.

6. After 9 minutes and 4 second the person is approximately _________ meters off the ground

Question

A carnival Ferris wheel with a radius of 20 m. makes one complete revolution every 248 seconds. The bottom of the wheel is 1.8 m above the ground. If a person is at the maximum height when a stopwatch is started, then determine how high above the ground that person will be after 9 minutes and 4 seconds.

NOTE: MANY REALLY SMART PEOPLE READ TOO QUICKLY AND ASSUME THINGS ARE THE SAME IN EACH PROBLEM

4. What is the equation of the midline that this Ferris wheel model is expecting?

Answer

We find the midline by adding any platform height or height above ground to the radius

so d = radius + platform

d = $20+1.8 = 21.8$

We know the midline of the equation regardless of sine or cosine is y=21.8

Question

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What is the "b" of this model?

Answer

Since we are using radians b $= \frac{2\cdot\pi}{period} = \frac{2\cdot\pi}{248} \blacktriangleright \frac{\pi}{124}$

So we know regardless of which model we build b $= \frac{2\cdot\pi}{248} = \frac{\pi}{124}$

Question

A carnival Ferris wheel with a radius of 20 m. makes one complete revolution every 248 seconds. The bottom of the wheel is 1.8 m above the ground. If a person is at the maximum height when a stopwatch is started, then determine how high above the ground that person will be after 9 minutes and 4 seconds.

When does cool stuff occur?

Answer

cool stuff happens every 1/4 of period, so $\frac{1}{4} \cdot 248 = 62$ (every 62 seconds)

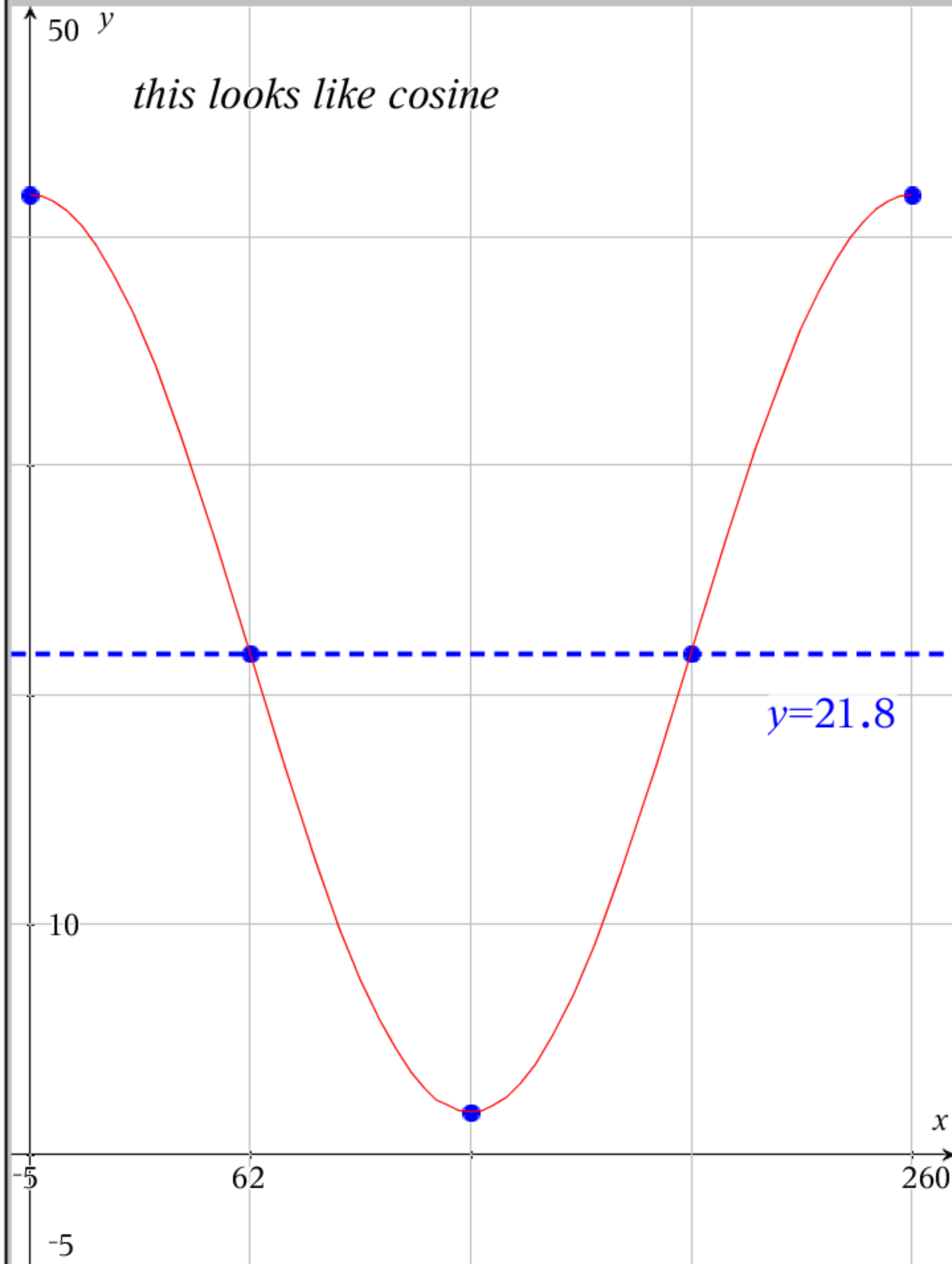
at time 0 we are at $1.8+40 = 41.8$ m above ground (maximum height)

at time 62 we are at $1.8+20 = 21.8$ m above ground (midline on way down)

at time 124 we are at 1.8 m above the ground (minimum height)

at time 186 we are at $1.8+20 = 21.8$ m above ground (midline on way up)

at time 248 we are at $1.8+40 = 41.8$ m above ground (maximum)



So as of right now, we know that the midline is

y = 21.8

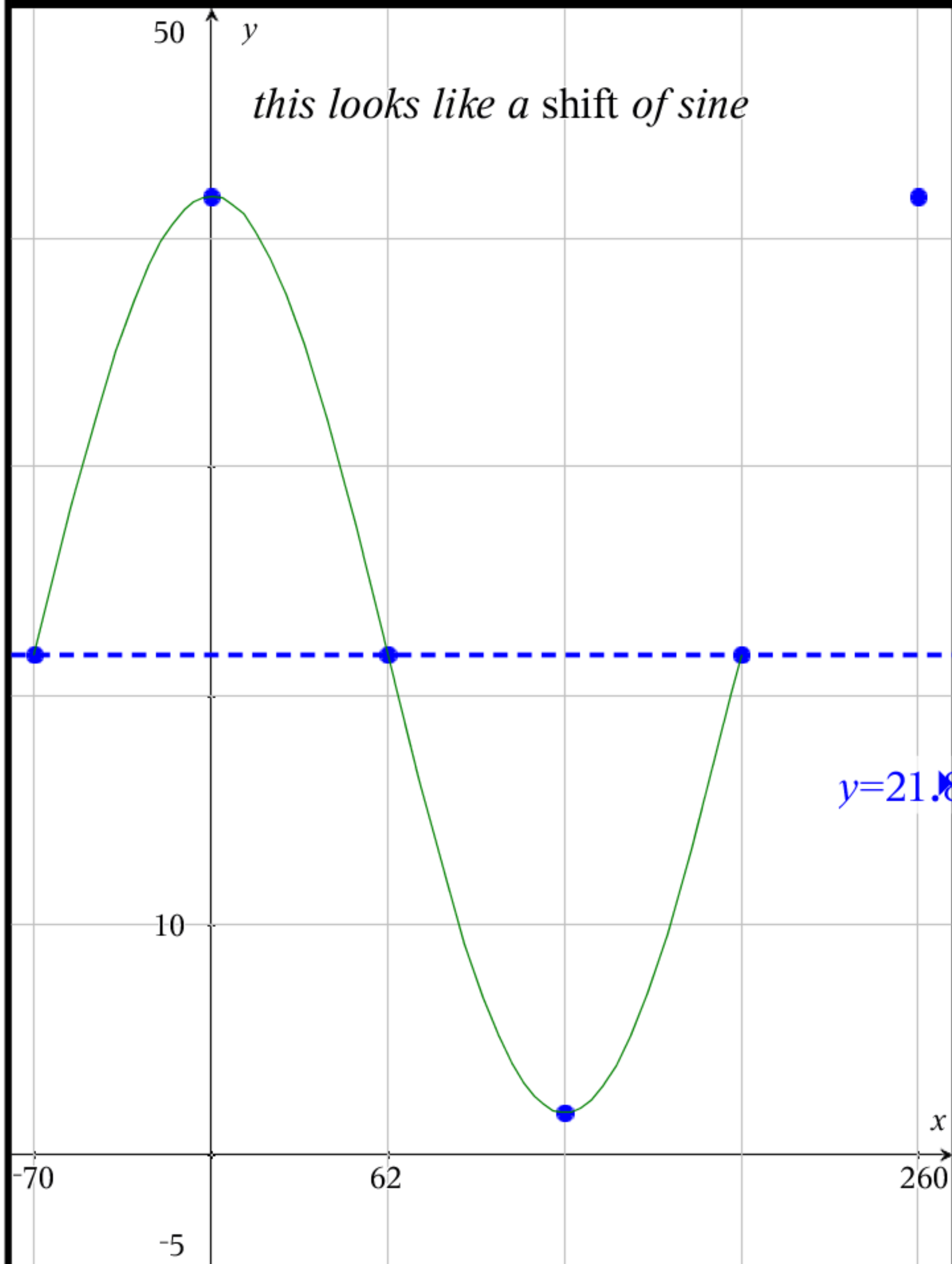
we also know that there are five easily found points to put on our graph

(0,41.8) (62,21.8) (124, 1.8) (186, 21.8) and(248,41.8)

and b = $\frac{2\cdot\pi}{248} = \frac{\pi}{124}$

	A time	B height	C	D	E	F
=						
1	0	41.8				
2	62	21.8				
3	124	1.8				
4	186	21.8				
5	248	41.8				
6	-62	21.8				
7						
8						
9						
10						

C1



So as of right now, we know that the midline is

y = 21.8

(0,41.8) (62,21.8) (124, 1.8) (186, 21.8), (248,41.8), and (−62, 21.8)

and b = $\frac{2\cdot\pi}{248} = \frac{\pi}{124}$

	A time	B height	C	D	E	F
=						
1	0	41.8				
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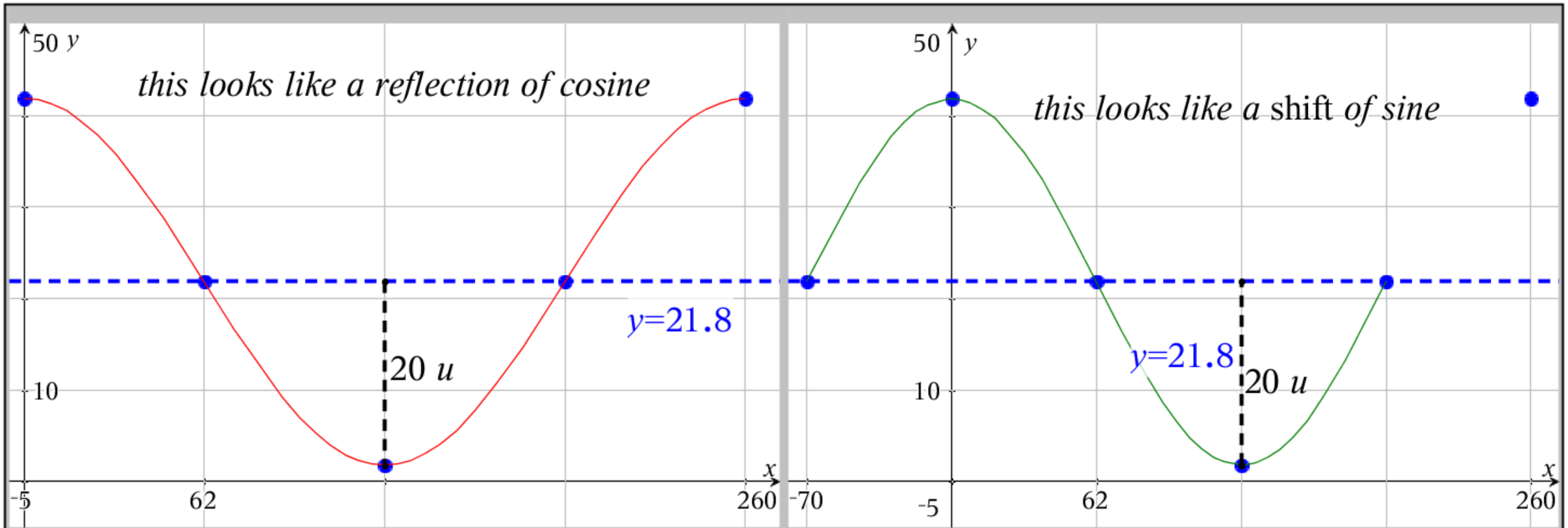
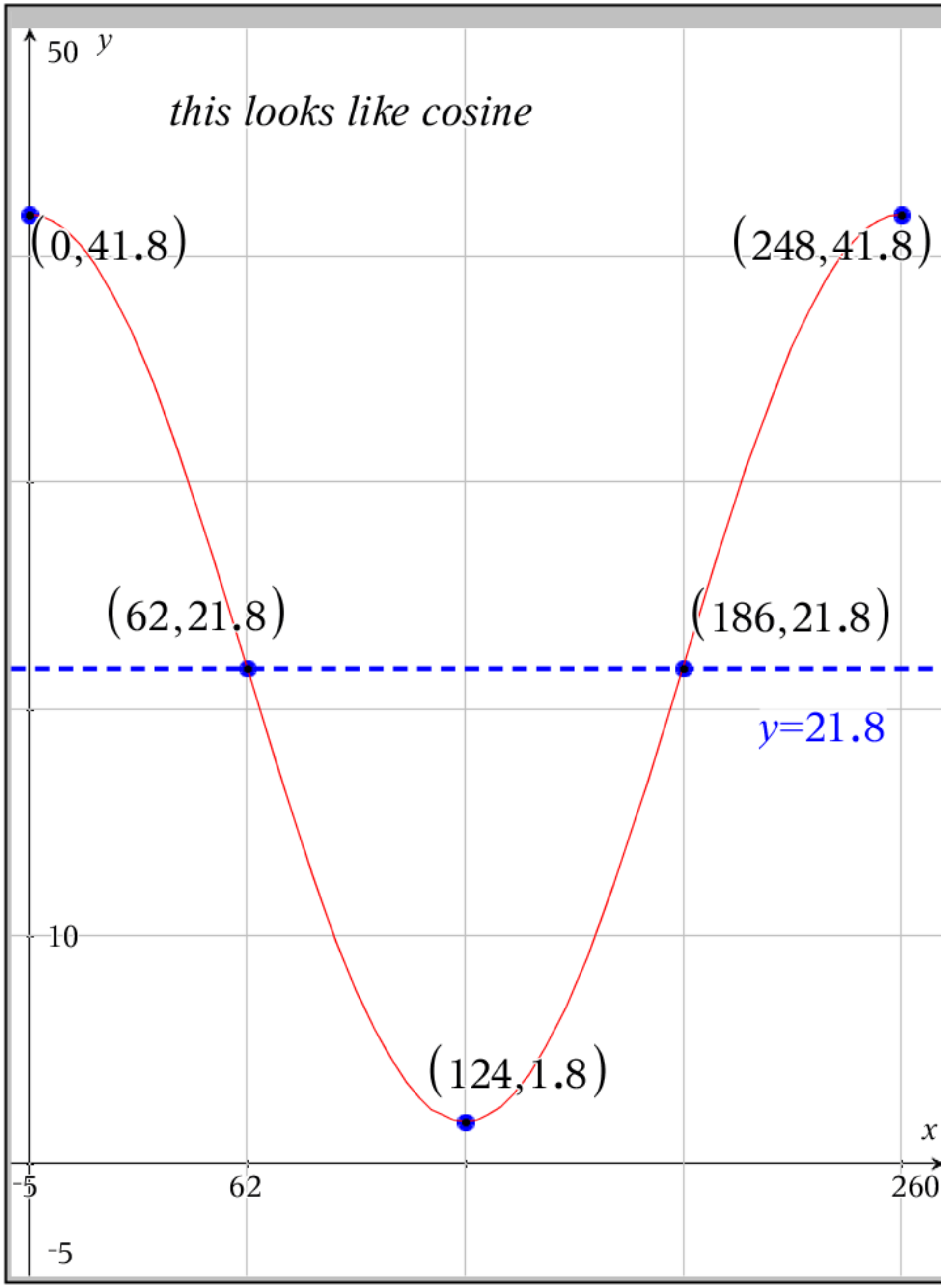
7						

C1



Either way we look at it the amplitude we have is $|20|$ either 20 or 20

both models use a = 20



So as of right now, we know that the midline is

y = 21.8 & b = $\frac{2\cdot\pi}{248}$ & "a" = 20

So since we started at maximum we can write a NO SHIFT model

$$y=20\cdot\cos\left(\frac{2\cdot\pi}{248}\cdot x\right)+21.8$$

test of model

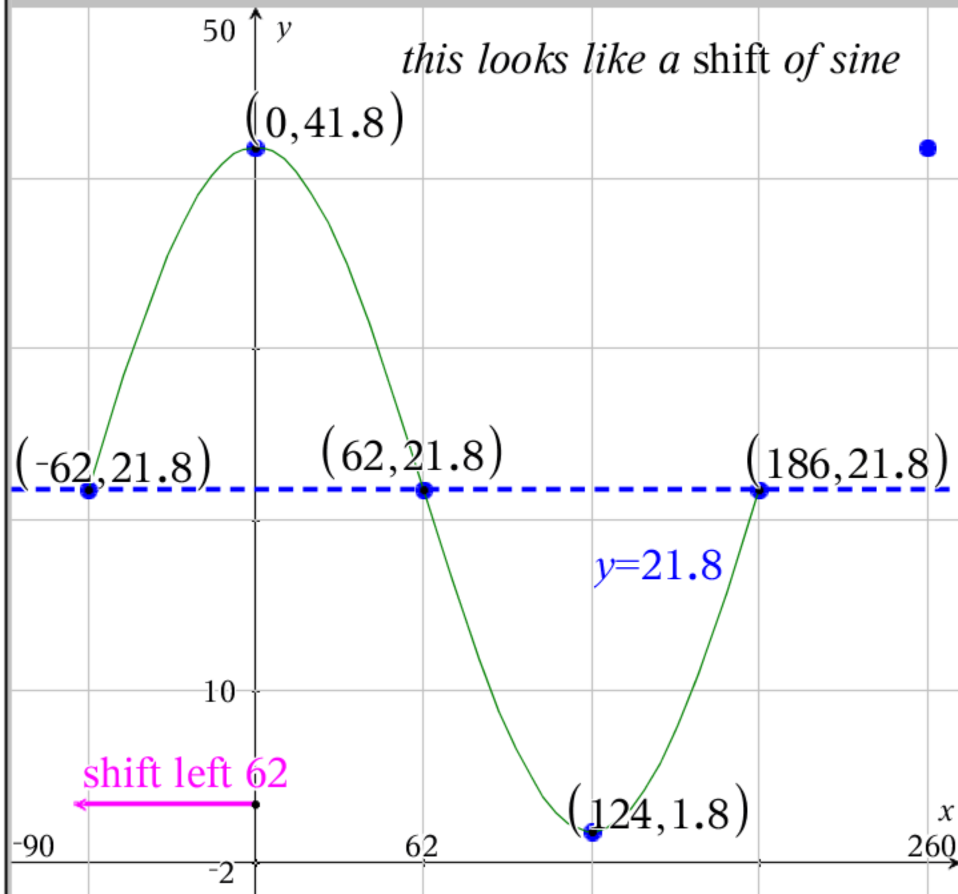
$$y=20\cdot\cos\left(\frac{2\cdot\pi}{248}\cdot 0\right)+21.8 = 41.8$$

$$y=20\cdot\cos\left(\frac{2\cdot\pi}{248}\cdot 62\right)+21.8 = 21.8$$

$$y=20\cdot\cos\left(\frac{2\cdot\pi}{248}\cdot 124\right)+21.8 = 1.8$$

$$y=20\cdot\cos\left(\frac{2\cdot\pi}{248}\cdot 186\right)+21.8 = 21.8$$

$$y=20\cdot\cos\left(\frac{2\cdot\pi}{248}\cdot 248\right)+21.8 = 41.8$$



we can build the model without solving because we know the shift (62 left)

(I'll show the algebra after this, but not necessary when you know about transformations and the shift)

So as of right now, we know that the midline is

y = 21.8 & b = $\frac{2\cdot\pi}{248}$ & "a" = 20

We also know that we want to shift the graph to the left of 0 by 62

$$y=20\cdot\sin\left(\frac{2\cdot\pi}{248}\cdot(x+62)\right)+21.8$$

test of model

$$y=20\cdot\sin\left(\frac{2\cdot\pi}{248}\cdot(-62+62)\right)+21.8 = 21.8$$

$$y=20\cdot\sin\left(\frac{2\cdot\pi}{248}\cdot(0+62)\right)+21.8 = 41.8$$

$$y=20\cdot\sin\left(\frac{2\cdot\pi}{248}\cdot(62+62)\right)+21.8 = 21.8$$

$$y=20\cdot\sin\left(\frac{2\cdot\pi}{248}\cdot(124+62)\right)+21.8 = 1.8$$

$$y=20\cdot\sin\left(\frac{2\cdot\pi}{248}\cdot(186+62)\right)+21.8 = 21.8$$

Algebra of this problem

we know (0,41.8) lies on this sine model

we know $y=20\cdot\sin\left(\frac{2\cdot\pi}{248}\cdot(x+\text{shift})\right)+21.8$

So replace and solve for shift!

$$41.8=20\cdot\sin\left(\frac{2\cdot\pi}{248}\cdot(0+\text{shift})\right)+21.8$$

$$41.8=20\cdot\sin\left(\frac{2\cdot\pi}{248}\cdot(\text{shift})\right)+21.8$$

$$41.8-21.8=20\cdot\sin\left(\frac{2\cdot\pi}{248}\cdot(\text{shift})\right)+21.8-21.8$$

$$20=20\cdot\sin\left(\frac{2\cdot\pi}{248}\cdot(\text{shift})\right)$$

$$\frac{20}{20}=\frac{20\cdot\sin\left(\frac{2\cdot\pi}{248}\cdot(\text{shift})\right)}{20}$$

$$1=\sin\left(\frac{2\cdot\pi}{248}\cdot(\text{shift})\right)$$

Now we know the equation to solve using inverse trigonometry

$$\sin^{-1}(1)=\sin^{-1}\left(\sin\left(\frac{2\cdot\pi}{248}\cdot(\text{shift})\right)\right)$$

$$\frac{\pi}{2}=\frac{2\cdot\pi}{248}\cdot(\text{shift})$$

$$\frac{\pi}{2}\cdot\frac{248}{2\cdot\pi}=\frac{248}{2\cdot\pi}\cdot\frac{2\cdot\pi}{248}\cdot\text{shift}$$

62 = shift

This was the process we use if we DON'T know the shift amount

This is tedious and error prone so be careful!

Question

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5. Since we are at a maximum at time 0, write a model that predicts the height above the ground in terms of seconds. Let y=height above the ground and x = time in seconds since start of motion.

Answer

cosine model $y=20\cdot \cos\left(\frac{2\cdot \pi}{248}\cdot x\right)+21.8$ sine model $y=20\cdot \sin\left(\frac{2\cdot \pi}{248}\cdot (x+62)\right)+21.8$

Question

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6. After 9 minutes and 4 seconds the person is approximately _________ meters off the ground

Answer

Since this model has time in seconds we need to convert 6minutes and 41 seconds to seconds $9\cdot 60+4=544$ so we can plug $x=544$ into either model

$y=20\cdot \cos\left(\frac{2\cdot \pi}{248}\cdot 544\right)+21.8=28.7461$ $\quad$ $y=20\cdot \sin\left(\frac{2\cdot \pi}{248}\cdot (544+62)\right)+21.8 ▸ 28.7461$

So the height after 9 minutes and 4 seconds is approximately 28.746 meters above ground