Solutions to Entry Slip 2−3−17

There were three ferris wheel problems that we look at on 2−3−17

problem 1) Started at the minimum height (this is probably best modeled by cosine model)

problem 2) Started at the maximum height (this is probably best modeled by cosine model)

problem 3) Started at midline (this is probably best modeled by sine model)

Recall: a < 0 if a reflection is present (problem 1)

$y=a\cdot\cos\left(\frac{2\text{pi}}{period}(x-\text{shift})\right)+midline$ or $y=a\cdot\sin\left(\frac{2\text{pi}}{period}(x-\text{shift})\right)+midline$

I will also write a shifted model for each of the models to show those that attempted models how to do this as well

A carnival Ferris wheel with a radius of 18 m. makes one complete revolution every 308 seconds. The bottom of the wheel is 0.9 m above the ground. If a person is at the height of 18.9 m and is on the way up when a stopwatch is started, then determine how high above the ground that person will be after 12 minutes and 46 seconds.

7. What is the equation of the midline that this Ferris wheel model is expecting?

8. Since we are at a height of 18.9 m and on the way up when the stopwatch starts, write a model that predicts the height above the ground in terms of seconds. Let y=height above the ground and x = time in seconds since start of motion.

9. After 12 minutes and 46 second the person is approximately _________ meters off the ground

Question

A carnival Ferris wheel with a radius of 18 m. makes one complete revolution every 308 seconds. The bottom of the wheel is 0.9 m above the ground. If a person is at the height of 18.9 m and is on the way up when a stopwatch is started, then determine how high above the ground that person will be after 12 minutes and 46 seconds.

NOTE: MANY REALLY SMART PEOPLE READ TOO QUICKLY AND ASSUME THINGS ARE THE SAME IN EACH PROBLEM

7. What is the equation of the midline that this Ferris wheel model is expecting?

Answer

We find the midline by adding any platform height or height above ground to the radius

so d = radius + platform

d = $18+0.9=18.9$ (hey that is the same as the height we were given!)

We know the midline of the equation regardless of sine or cosine is y=18.9

Question

A carnival Ferris wheel with a radius of 18 m. makes one complete revolution every 308 seconds. The bottom of the wheel is 0.9 m above the ground. If a person is at the height of 18.9 m and is on the way up when a stopwatch is started, then determine how high above the ground that person will be after 12 minutes and 46 seconds.

What is the "b" of this model?

Answer

Since we are using radians b $= \frac{2\cdot\pi}{period} = \frac{2\cdot\pi}{308} \blacktriangleright \frac{\pi}{154}$

So we know regardless of which model we build b $= \frac{2\cdot\pi}{308} = \frac{\pi}{154}$

Question

A carnival Ferris wheel with a radius of 18 m. makes one complete revolution every 308 seconds. The bottom of the wheel is 0.9 m above the ground. If a person is at the height of 18.9 m and is on the way up when a stopwatch is started, then determine how high above the ground that person will be after 12 minutes and 46 seconds.

When does cool stuff occur?

Answer

cool stuff happens every 1/4 of period, so $\frac{1}{4} \cdot 308 = 77$ (every 77 seconds)

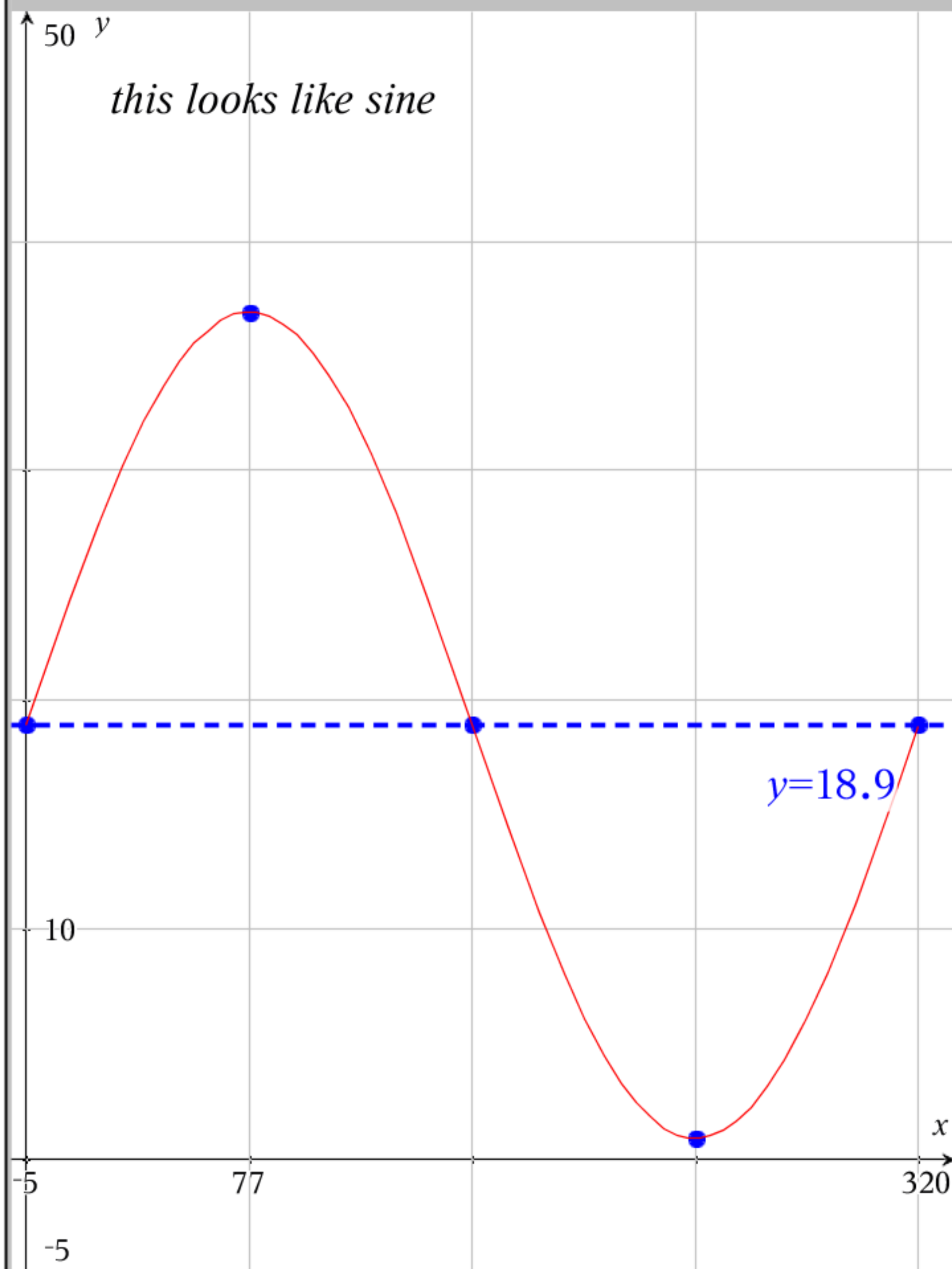
at time 0 we are at 18.9 m above ground (midline on way up)

at time 77 we are at $0.9+36 = 36.9$ m above ground (maximum height)

at time 154 we are at $0.9+18 = 18.9$ m above the ground (midline on way down)

at time 231 we are at 0.9 m above ground (minimum height)

at time 308 we are at $0.9+18 = 18.9$ m above ground (midline on way up)



So as of right now, we know that the midline is

y = 18.9

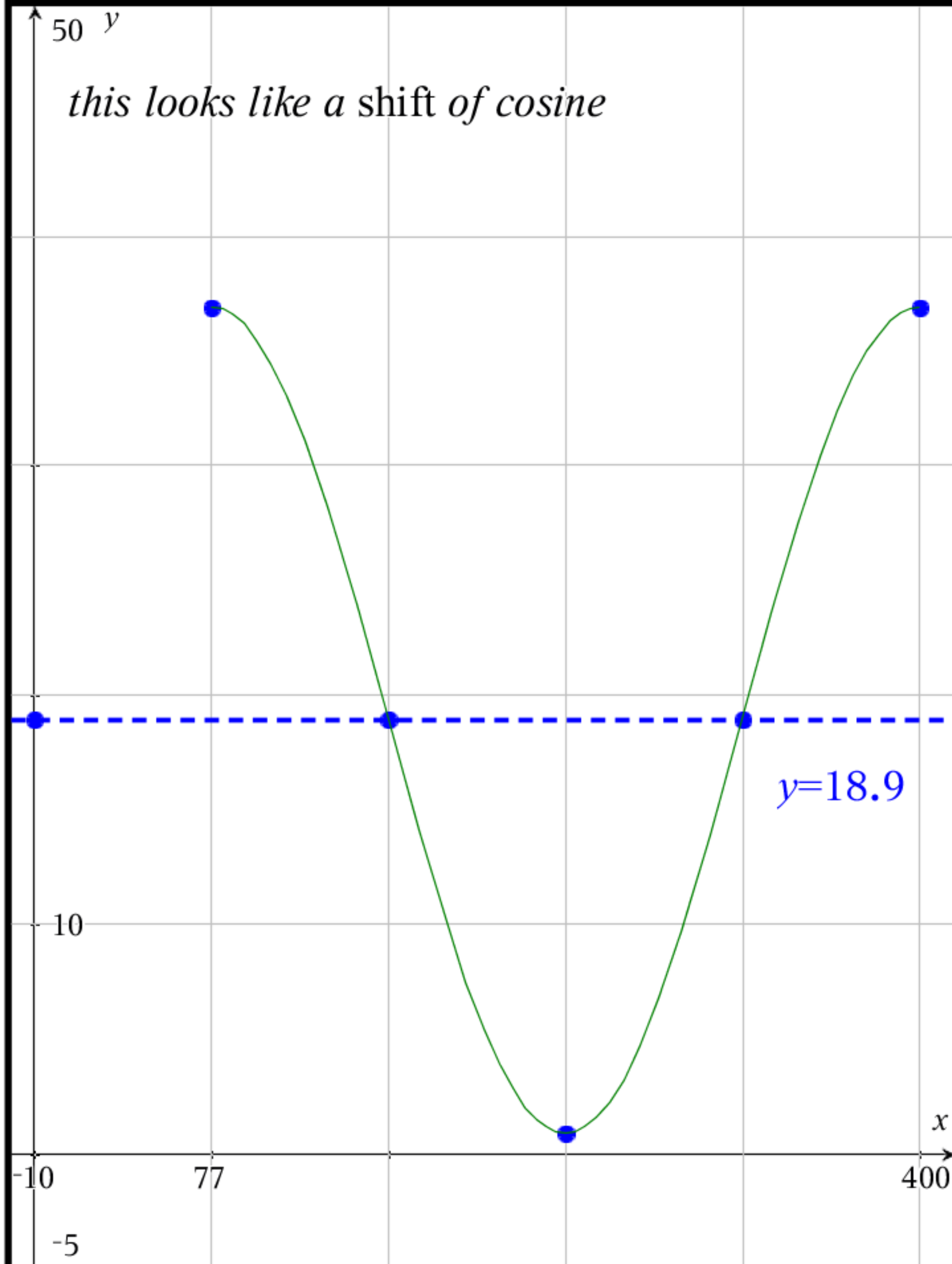
we also know that there are five easily found points to put on our graph

(0,18.9) (77,36.9) (154, 18.9) (231, 0.9) and(308,18.9)

and b = $\frac{2\cdot\pi}{308} = \frac{\pi}{154}$

	A time	B height	C	D	E	F
=						
1	0	18.9				
2	77	36.9				
3	154	18.9				
4	231	0.9				
5	308	18.9				
6	385	36.9				
7						
8						
9						
10						

B3 18.9



So as of right now, we know that the midline is

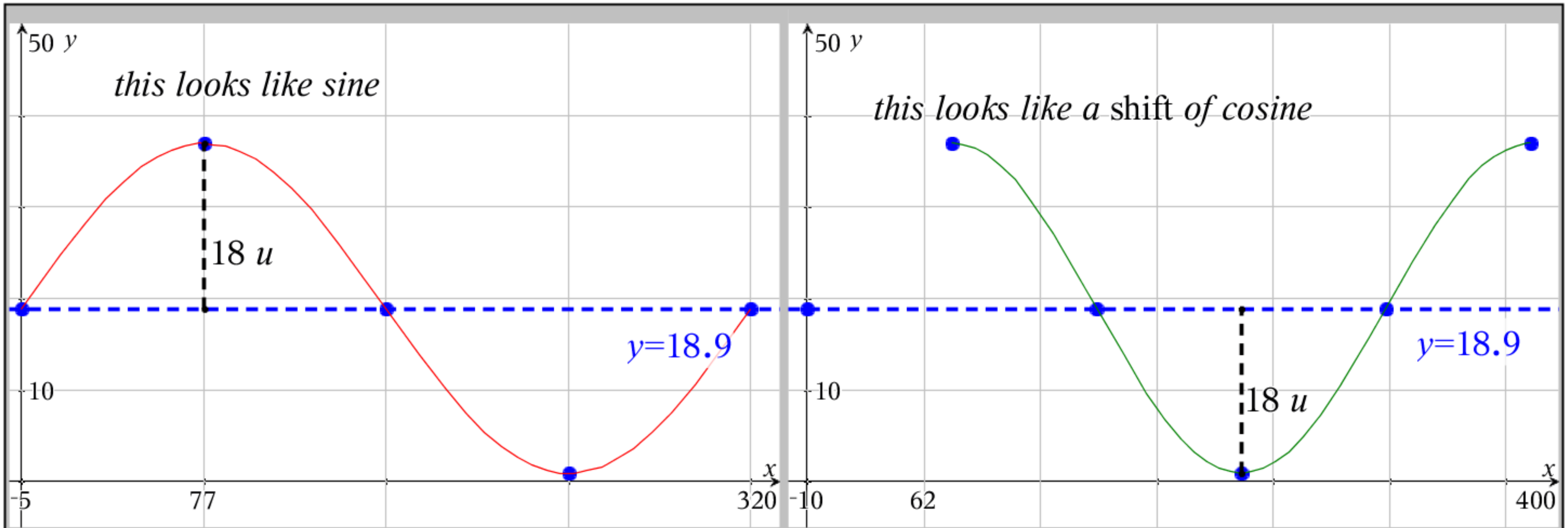
y = 18.9

(0,18.9) (77,36.9) (154, 18.9) (231, 0.9), (308,18.9), and (385, 36.9)

and b $= \frac{2 \cdot \pi}{308} = \frac{\pi}{154}$

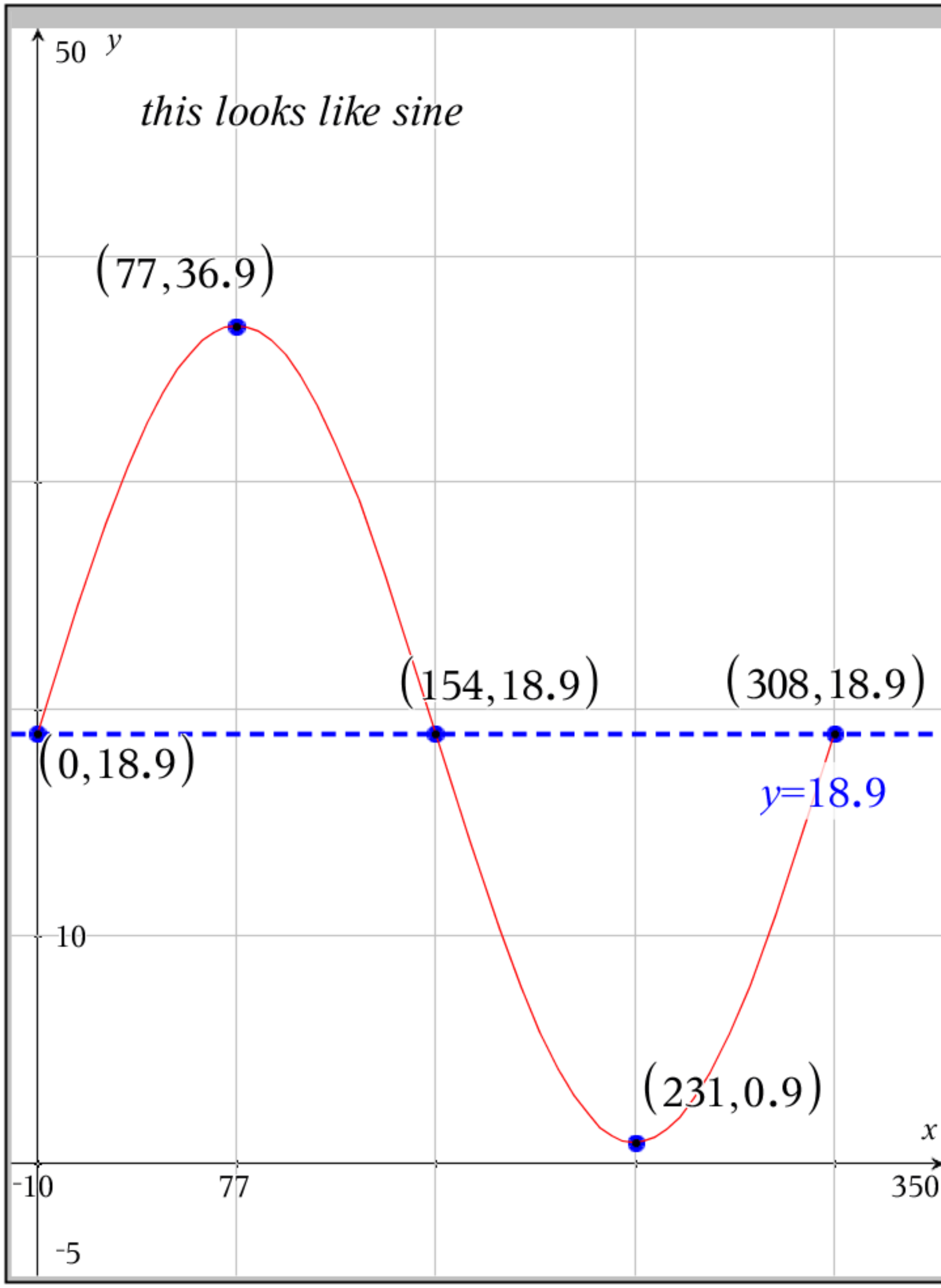
	A time	B height	C	D	E	F
=						
1	0	18.9				
2	77	36.9				
3	154	18.9				
4	231	0.9				
5	308	18.9				
6	385	36.9				
7						

A1 =0



Either way we look at it the amplitude we have is $|18|$ either 18 or −18

both models use a = 18



So as of right now, we know that the midline is

y = 18.9 & b = $\frac{2\cdot\pi}{308}$ & "a" = 18

So since we started at midline we can write a NO SHIFT model

$$y=18\cdot\sin\left(\frac{2\cdot\pi}{308}\cdot x\right)+18.9$$

test of model

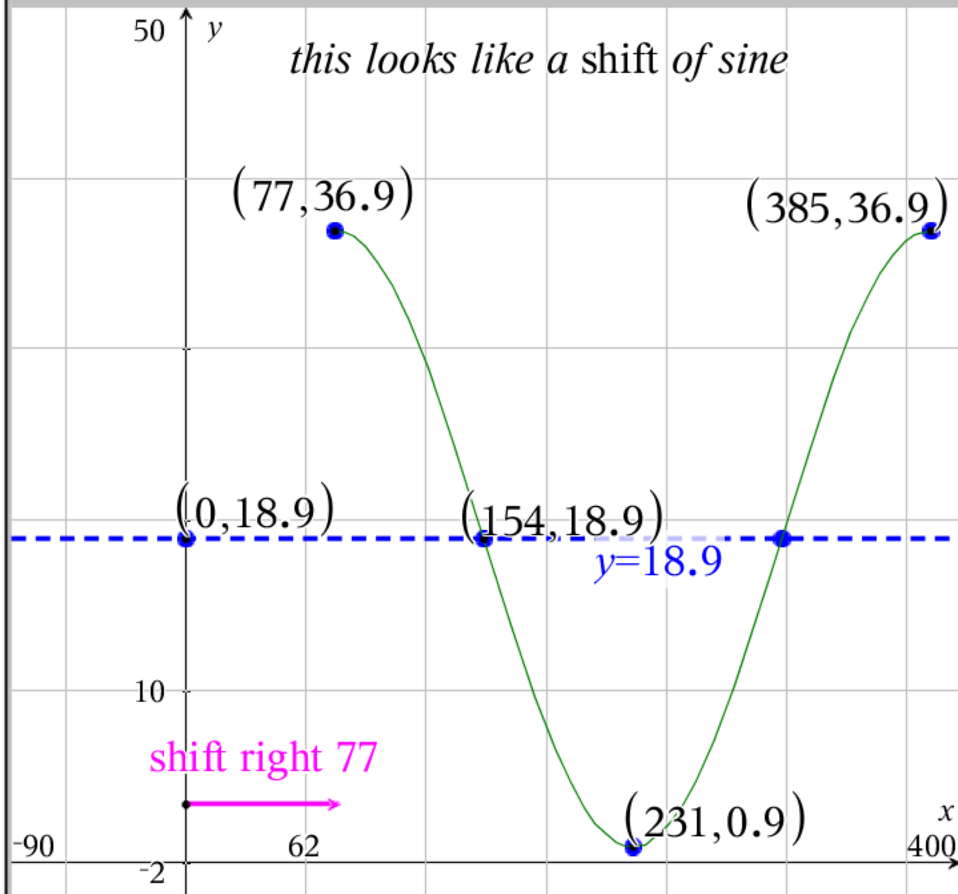
$$y=18\cdot\sin\left(\frac{2\cdot\pi}{308}\cdot 0\right)+18.9 = 18.9$$

$$y=18\cdot\sin\left(\frac{2\cdot\pi}{308}\cdot 77\right)+18.9 = 36.9$$

$$y=18\cdot\sin\left(\frac{2\cdot\pi}{308}\cdot 154\right)+18.9 = 18.9$$

$$y=18\cdot\sin\left(\frac{2\cdot\pi}{308}\cdot 231\right)+18.9 = 0.9$$

$$y=18\cdot\sin\left(\frac{2\cdot\pi}{308}\cdot 308\right)+18.9 = 18.9$$



we can build the model without solving because we know the shift (77 right)

(I'll show the algebra after this, but not necessary when you know about transformations and the shift)

So as of right now, we know that the midline is

y = 18.9 & b = $\frac{2\cdot\pi}{308}$ & "a" = 18

We also know that we want to shift the graph to the right of 0 by 77

$$y=18\cdot\cos\left(\frac{2\cdot\pi}{308}\cdot(x-77)\right)+18.9$$

test of model

$$y=18\cdot\cos\left(\frac{2\cdot\pi}{308}\cdot(0-77)\right)+18.9 = 18.9$$

$$y=18\cdot\cos\left(\frac{2\cdot\pi}{308}\cdot(77-77)\right)+18.9 \blacktriangleright 36.9$$

$$y=18\cdot\cos\left(\frac{2\cdot\pi}{308}\cdot(154-77)\right)+18.9 = 18.9$$

$$y=18\cdot\cos\left(\frac{2\cdot\pi}{308}\cdot(231-77)\right)+18.9 = 0.9$$

$$y=18\cdot\cos\left(\frac{2\cdot\pi}{308}\cdot(308-77)\right)+18.9 = 18.9$$

Algebra of this problem

we know (0,18.9) lies on this cosine model

we know $y=18\cdot\cos\left(\frac{2\cdot\pi}{308}\cdot(x+\text{shift})\right)+18.9$

So replace and solve for shift!

$$18.9=18\cdot\cos\left(\frac{2\cdot\pi}{308}\cdot(0+\text{shift})\right)+18.9$$

$$18.9=18\cdot\cos\left(\frac{2\cdot\pi}{308}\cdot(\text{shift})\right)+18.9$$

$$18.9-18.9=18\cdot\cos\left(\frac{2\cdot\pi}{308}\cdot(\text{shift})\right)+18.9-18.9$$

$$0=18\cdot\cos\left(\frac{2\cdot\pi}{308}\cdot(\text{shift})\right)$$

$$\frac{0}{18}=\frac{18\cdot\cos\left(\frac{2\cdot\pi}{308}\cdot(\text{shift})\right)}{18}$$

$$0=\cos\left(\frac{2\cdot\pi}{248}\cdot(\text{shift})\right)$$

Now we know the equation to solve using inverse trigonometry

$$\cos^{-1}(0)=\cos^{-1}\left(\cos\left(\frac{2\cdot\pi}{308}\cdot(\text{shift})\right)\right)$$

$$\frac{\pi}{2}=\frac{2\cdot\pi}{308}\cdot(\text{shift})$$

$$\frac{\pi}{2}\cdot\frac{308}{2\cdot\pi}=\frac{308}{2\cdot\pi}\cdot\frac{2\cdot\pi}{308}\cdot\text{shift}$$

77 = shift

This was the process we use if we DON'T know the shift amount

This is tedious and error prone so be careful!

Question

A carnival Ferris wheel with a radius of 18 m. makes one complete revolution every 308 seconds. The bottom of the wheel is 0.9 m above the ground. If a person is at the height of 18.9 m and is on the way up when a stopwatch is started, then determine how high above the ground that person will be after 12 minutes and 46 seconds.

5. Since we are at a midline at time 0, write a model that predicts the height above the ground in terms of seconds. Let y=height above the ground and x = time in seconds since start of motion.

Answer

cosine model $y=18\cdot \cos\left(\frac{2\cdot \pi}{308}\cdot (x-77)\right)+18.9$ sine model $y=18\cdot \sin\left(\frac{2\cdot \pi}{308}\cdot x\right)+18.9$

Question

A carnival Ferris wheel with a radius of 18 m. makes one complete revolution every 308 seconds. The bottom of the wheel is 0.9 m above the ground. If a person is at the height of 18.9 m and is on the way up when a stopwatch is started, then determine how high above the ground that person will be after 12 minutes and 46 seconds.

9. After 12 minutes and 46 seconds the person is approximately _________ meters off the ground

Answer

Since this model has time in seconds we need to convert 6minutes and 41 seconds to seconds $12\cdot 60+46 = 766$ so we can plug $x = 766$ into either model

$y=18\cdot \sin\left(\frac{2\cdot \pi}{308}\cdot 766\right)+18.9 = 20.3672$ $\quad$ $y=18\cdot \cos\left(\frac{2\cdot \pi}{308}\cdot (766-77)\right)+18.9 \blacktriangleright 20.3672$

So the height after 12 minutes and 46 seconds is approximately 30.367 meters above ground