

Problem 1

	A	B
1	p	
2		2
3	q	
4		5
5		-3
6		
7		
8		
9		
10		
11		

A1 p

$$P(2,5) \quad Q = (5,-3)$$

$$\text{change in } x \text{ from } P \text{ to } Q = 5 - 2 = 3$$

$$\text{change in } y \text{ from } P \text{ to } Q = -3 - 5 = -8$$

$$1) \text{ PQ} = \begin{bmatrix} 3 \\ -8 \end{bmatrix}$$

$$= \langle 3, -8 \rangle$$

$$= 3\mathbf{i} - 8\mathbf{j}$$

$$2) |\text{PQ}| = \sqrt{[(3)^2 + (-8)^2]} = \sqrt{73}$$

$$= \sqrt{73}$$

$$P(2,5) \quad Q = (5,-3) \quad |PQ| = \sqrt{73}$$

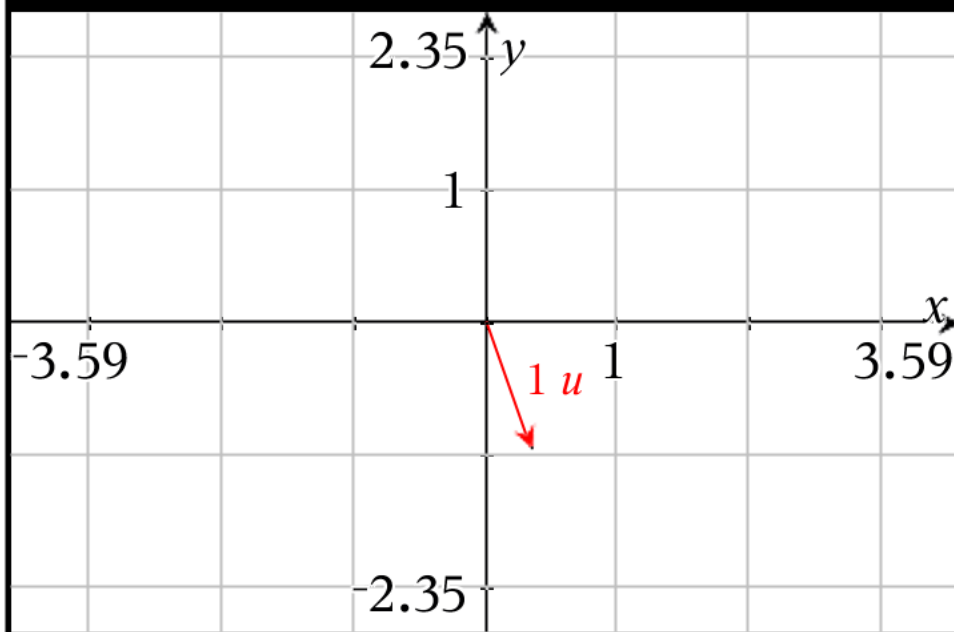
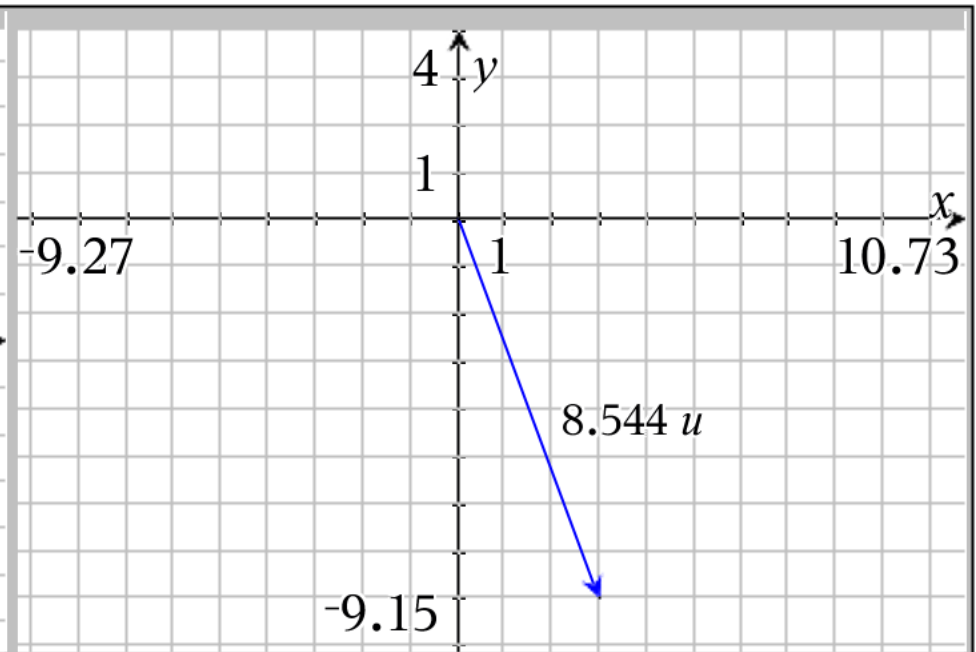
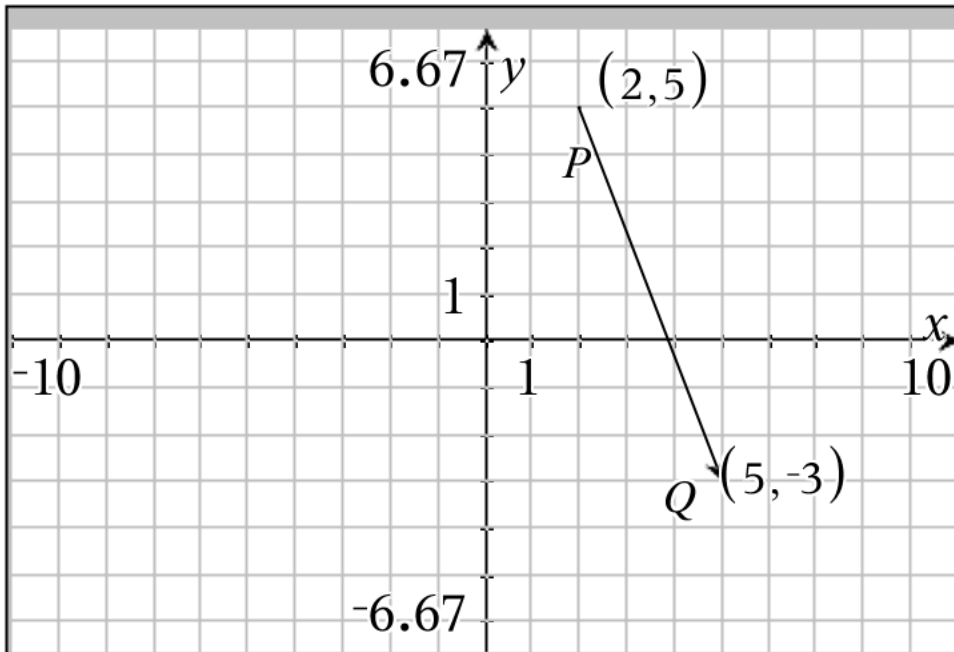
change in x from P to Q = 3 change in y from P to Q = -8

$$PQ = \begin{bmatrix} 3 \\ -8 \end{bmatrix} = \langle 3, -8 \rangle = 3\mathbf{i} - 8\mathbf{j}$$

$$3) \text{ Unit Vector Related to } PQ = \begin{bmatrix} \frac{3 \cdot \sqrt{73}}{73} \\ \frac{-8 \cdot \sqrt{73}}{73} \end{bmatrix}$$

$$= \langle 3/\sqrt{73}, -8/\sqrt{73} \rangle \text{ unsimplified}$$

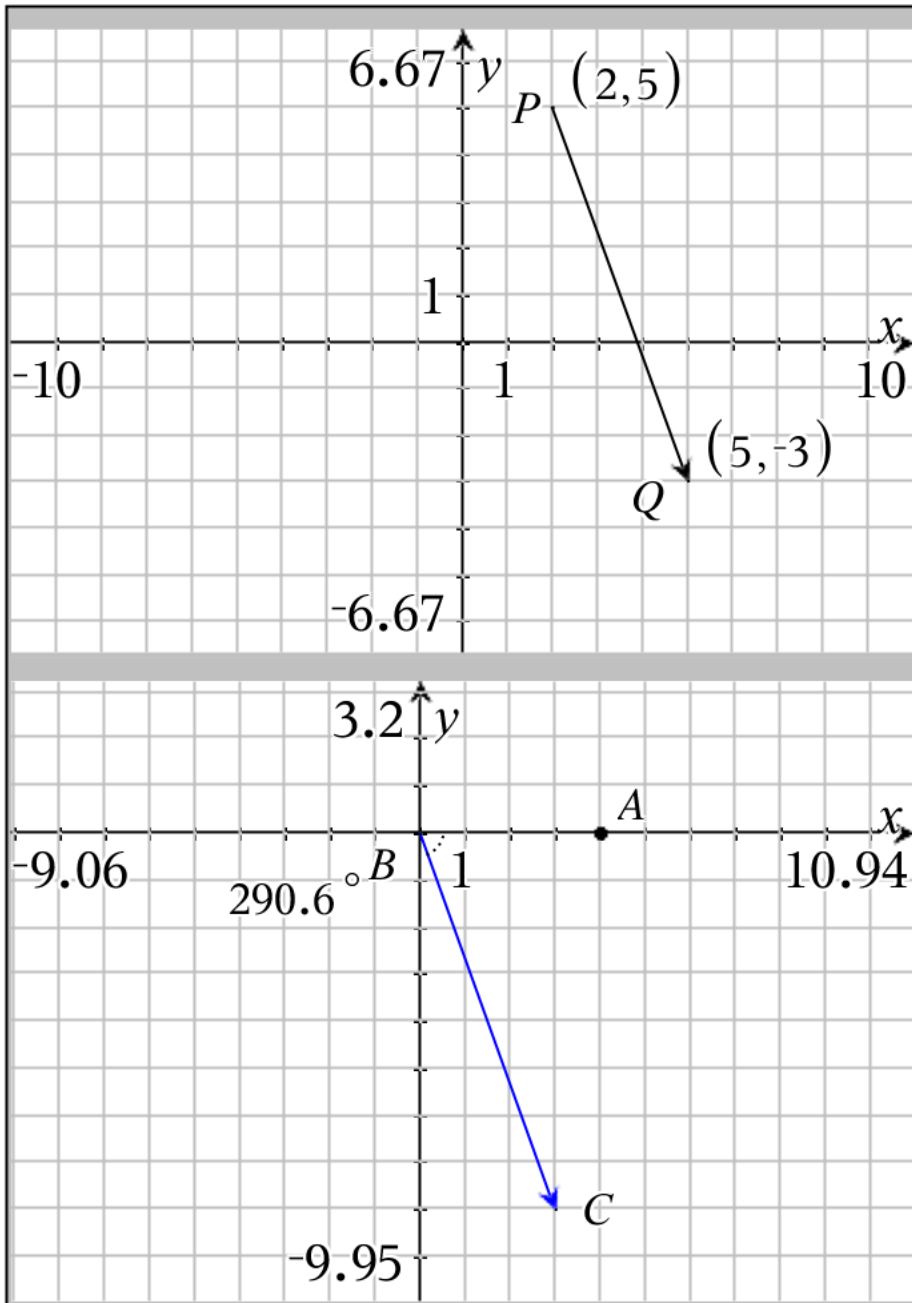
$$= 3/\sqrt{73}\mathbf{i} - 8/\sqrt{73}\mathbf{j} \text{ unsimplified}$$



Vector PQ is drawn in the upper left hand corner

the vector that is parallel to vector PQ with same direction that starts at the origin is in the upper right hand corner

the related unit vector is drawn in the lower left hand corner (note scale change)



Vector PQ is drawn in the upper left hand corner
 To find the directional angle formed with the x axis
 look to the lower left

To find the angle (not the direction angle, but the
 angle itself) use tangent function

$$m\angle ABC = \tan^{-1}(8/3) = 69.444^\circ$$

Since this is in quadrant 4 we reference from 360°

$$\begin{aligned} \text{directional angle} &= 360^\circ - 69.444^\circ \\ &= 290.556^\circ \end{aligned}$$

IF we keep signs in tangent, then

$$m\angle ABC = \tan^{-1}(-8/3) = -69.444^\circ$$

(this is a directional angle itself)

$$\begin{aligned} \text{directional angle} &= 360^\circ + -69.444^\circ \\ &= 290.556^\circ \end{aligned}$$

Problem 2

	_list8...	T y_list8...
=		
1	0	0
2	3	-3
3	-3	3
4		
5		
6		
7		
8		
9		
10		
11		

vector $\mathbf{a} = \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \langle -2, 3 \rangle = -2\mathbf{i} + 3\mathbf{j}$

vector $\mathbf{b} = \begin{bmatrix} 3 \\ -3 \end{bmatrix} = \langle 3, -3 \rangle = 3\mathbf{i} - 3\mathbf{j}$

5) $6\mathbf{a} + 5\mathbf{b} = 6 \begin{bmatrix} -2 \\ 3 \end{bmatrix} + 5 \begin{bmatrix} 3 \\ -3 \end{bmatrix}$

$= \begin{bmatrix} -12 \\ 18 \end{bmatrix} + \begin{bmatrix} 15 \\ -15 \end{bmatrix}$

$= \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

$$\text{vector a} = \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \langle -2, 3 \rangle = -2\mathbf{i} + 3\mathbf{j}$$

$$\text{vector b} = \begin{bmatrix} 3 \\ -3 \end{bmatrix} = \langle 3, -3 \rangle = 3\mathbf{i} - 3\mathbf{j}$$

$$\begin{aligned} 6) \frac{1}{2}\mathbf{a} - \frac{2}{3}\mathbf{b} &= \frac{1}{2} \begin{bmatrix} -2 \\ 3 \end{bmatrix} - \frac{2}{3} \begin{bmatrix} 3 \\ -3 \end{bmatrix} \\ &= \begin{bmatrix} -1 \\ \frac{3}{2} \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} -3 \\ \frac{7}{2} \end{bmatrix} = \begin{bmatrix} -3. \\ 3.5 \end{bmatrix} \end{aligned}$$

$$\text{vector a} = \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \langle -2, 3 \rangle = -2i + 3j$$

$$\text{vector b} = \begin{bmatrix} 3 \\ -3 \end{bmatrix} = \langle 3, -3 \rangle = 3i - 3j$$

$$7) |5\mathbf{a}| =$$

First Step Find 5 a

$$5 \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -10 \\ 15 \end{bmatrix}$$

Now find the magnitude $\sqrt{(-10)^2 + (15)^2}$

$$= \sqrt{325}$$

$$= 5 \cdot \sqrt{13}$$

$$\text{vector a} = \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \langle -2, 3 \rangle = -2i + 3j$$

$$\text{vector b} = \begin{bmatrix} 3 \\ -3 \end{bmatrix} = \langle 3, -3 \rangle = 3i - 3j$$

8) Write a vector that is parallel (this means any multiple of b)

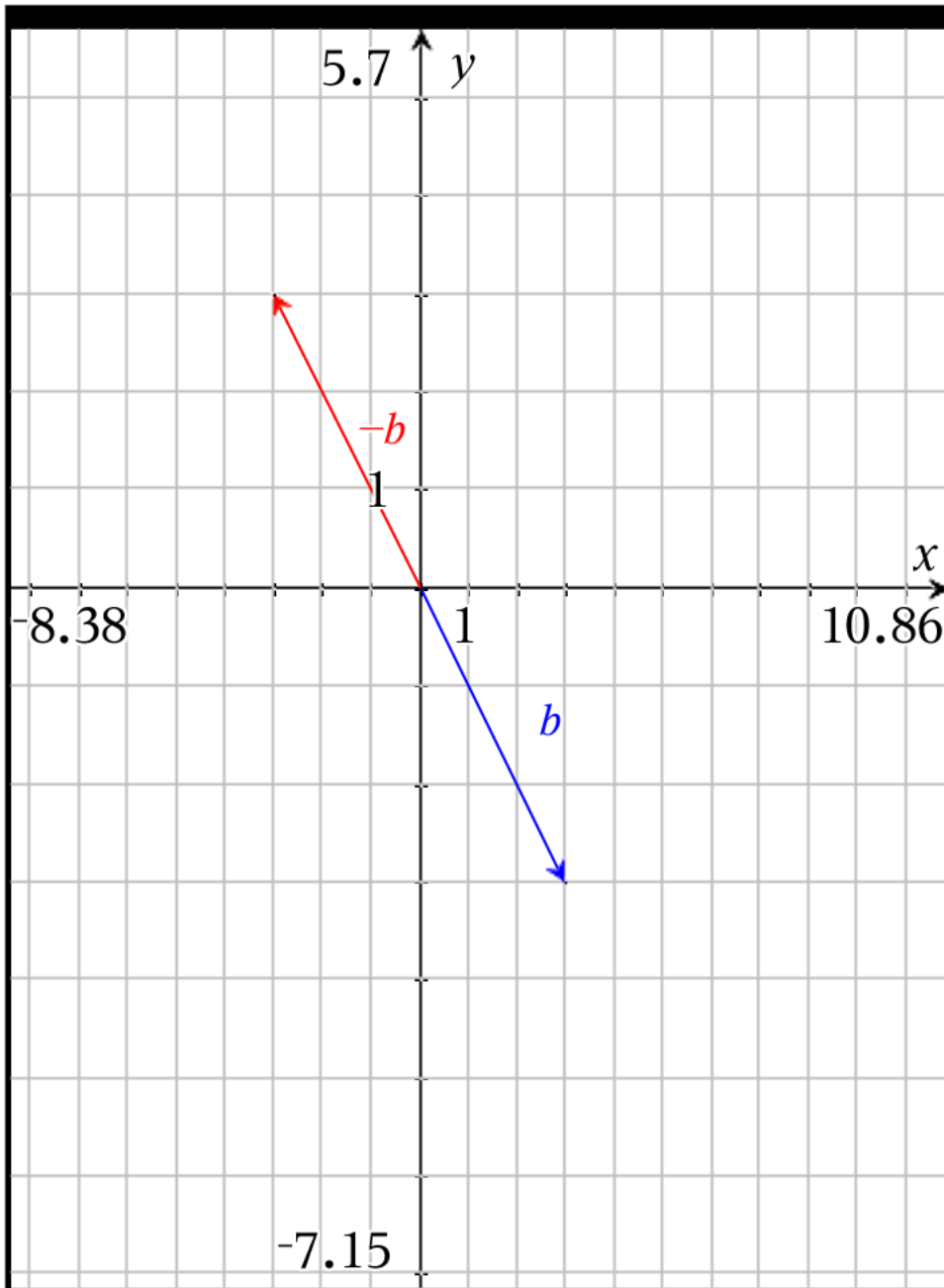
Write a vector that travels in the opposite direction as b)

(you must change the signs of change in x and change in y)

$$\text{Step 1) find } -b = -\begin{bmatrix} 3 \\ -3 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$$

Technically, this is all that is required to satisfy the given conditions

A more complete answer would be $\begin{bmatrix} -3 \cdot n \\ 3 \cdot n \end{bmatrix}$ for all $n > 0$



Now, this vector $-b$

We know that $-b$ has direction $\begin{bmatrix} -3 \\ 3 \end{bmatrix}$

For a vector to be parallel,
it simply must be a multiple of direction
vector that is nonzero

For a vector to travel in the opposite
direction it must have a negative scalar
applied to original direction vector

$$\text{vector a} = \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \langle -2, 3 \rangle = -2i + 3j$$

$$\text{vector b} = \begin{bmatrix} 3 \\ -3 \end{bmatrix} = \langle 3, -3 \rangle = 3i - 3j$$

9) write the unit vector related to $6a + 5b$

$$\begin{aligned} \text{Step 1 Find } 6a + 5b &= 6 \begin{bmatrix} -2 \\ 3 \end{bmatrix} + 5 \begin{bmatrix} 3 \\ -3 \end{bmatrix} \\ &= \begin{bmatrix} -12 \\ 18 \end{bmatrix} + \begin{bmatrix} 15 \\ -15 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{Step 2 find magnitude of } 6a + 5b &= \sqrt{(3)^2 + (3)^2} \\ &= \sqrt{18} = 3 \cdot \sqrt{2} \end{aligned}$$

Step 3 divide $6a + 5b$ by $\sqrt{18}$

$$\langle 3/\sqrt{18}, 3/\sqrt{18} \rangle = \langle 3/3 \cdot \sqrt{2}, 3/3 \cdot \sqrt{2} \rangle = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} \text{ these are coordinates on } 45^\circ$$

$$\text{vector a} = \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \langle -2, 3 \rangle = -2\mathbf{i} + 3\mathbf{j}$$

$$\text{vector b} = \begin{bmatrix} 3 \\ -3 \end{bmatrix} = \langle 3, -3 \rangle = 3\mathbf{i} - 3\mathbf{j}$$

$$6/10) \frac{1}{2}\mathbf{a} - \frac{2}{3}\mathbf{b} = \frac{1}{2} \begin{bmatrix} -2 \\ 3 \end{bmatrix} - \frac{2}{3} \begin{bmatrix} 3 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \\ \frac{3}{2} \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ \frac{7}{2} \end{bmatrix} = \begin{bmatrix} -3. \\ 3.5 \end{bmatrix}$$

$$\text{Vertical component form} \begin{bmatrix} -3 \\ \frac{7}{2} \end{bmatrix} = \begin{bmatrix} -3. \\ 3.5 \end{bmatrix}$$

$$\text{Horizontal component form} \langle -3, \frac{7}{2} \rangle = \langle -3., 3.5 \rangle$$

$$\text{UNIT VECTOR FORM} -3\mathbf{i} + \frac{7}{2}\mathbf{j} = -3.\mathbf{i} + 3.5\mathbf{j}$$