	A	B
=		
1	p	
2	4	1
3	q	
4	-2	5
5		
6		
7		
8		
9		
10		
11		

B5

P(4,1) **Q = (⁻2,5)**

change in x from P to Q $= {}^{-}2 - 4 = {}^{-}6$

change in y from P to Q $= 5 - 1 = 4$

1) $PQ = \begin{bmatrix} {}^{-}6 \\ 4 \end{bmatrix}$

$= \langle {}^{-}6, 4 \rangle$

$= {}^{-}6\mathbf{i} + 4\mathbf{j}$

2) $|PQ| = \sqrt{[({}^{-}6)^2 + (4)^2]} = \sqrt{52}$

$= 2 \cdot \sqrt{13}$

P(4,1) **Q = (⁻2,5)** |PQ| = $=2\cdot\sqrt{13}$

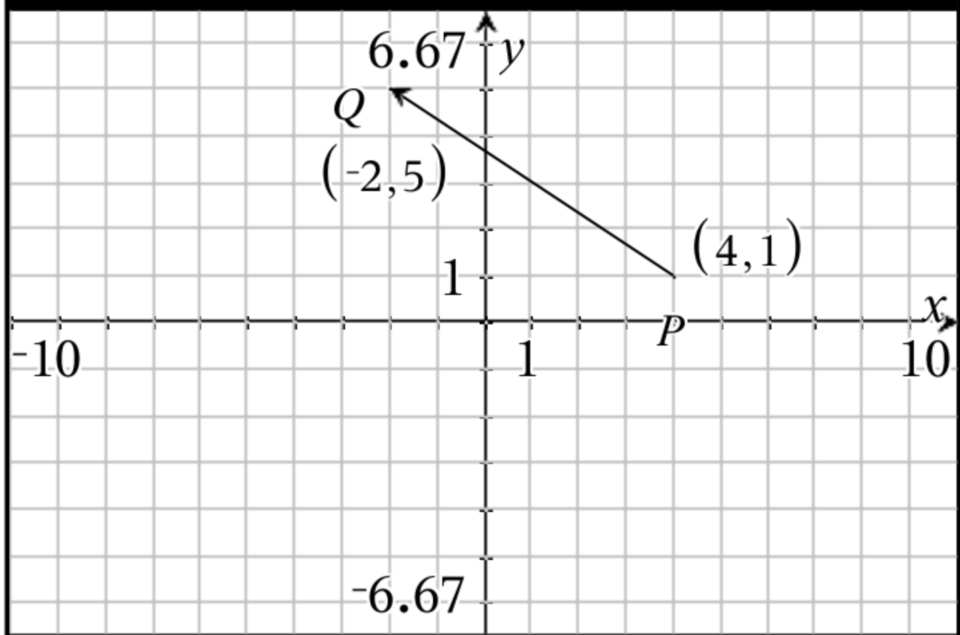
change in x from P to Q = ⁻6 **change in y from P to Q =** 4

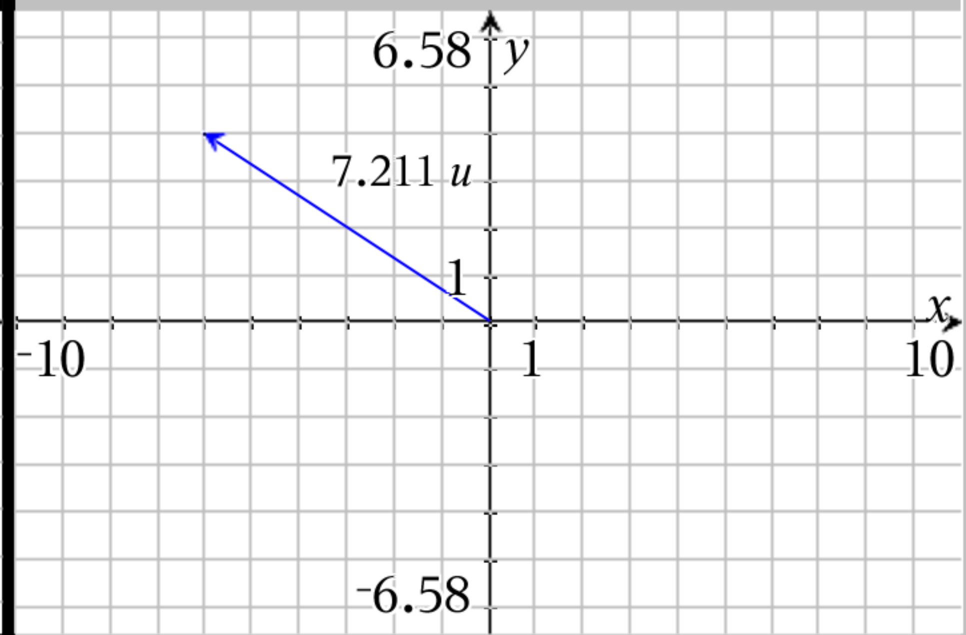
$$PQ = \begin{bmatrix} ^{-}6 \\ 4 \end{bmatrix} = \langle ^{-}6, 4 \rangle = {^{-}6}\mathbf{i} + 4\mathbf{j}$$

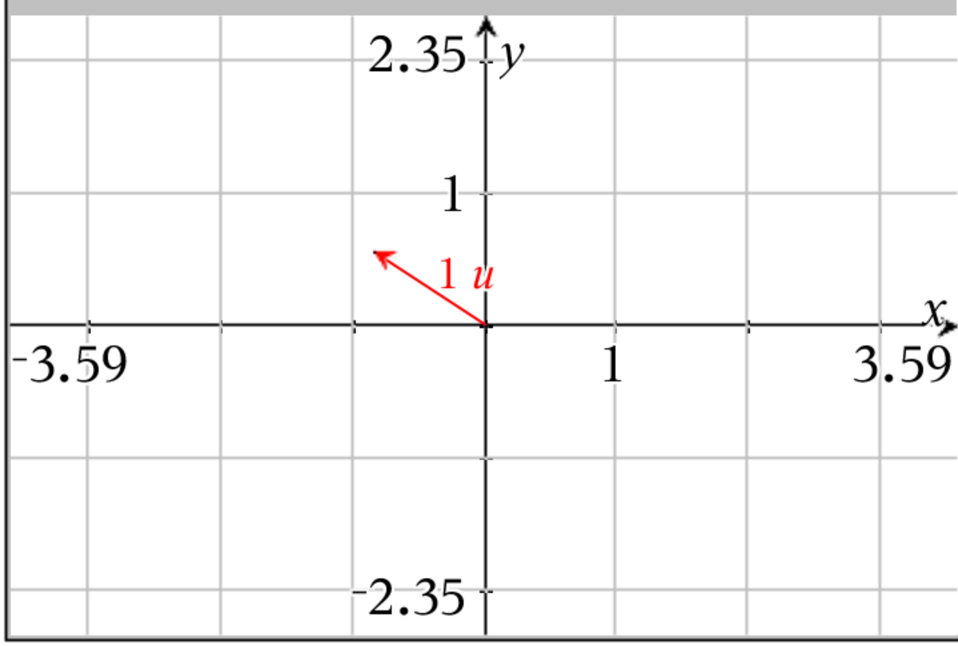
$$\text{3) Unit Vector Related to PQ} = \begin{bmatrix} \frac{^{-}3\cdot\sqrt{13}}{13} \\ \frac{2\cdot\sqrt{13}}{13} \end{bmatrix}$$

$= \langle ^{-}6/\sqrt{52}, 4/\sqrt{52} \rangle$ **unsimplified**

$= {^{-}6}/\sqrt{52}\,\mathbf{i} + 4/\sqrt{52}\,\mathbf{j}$ **unsimplified**



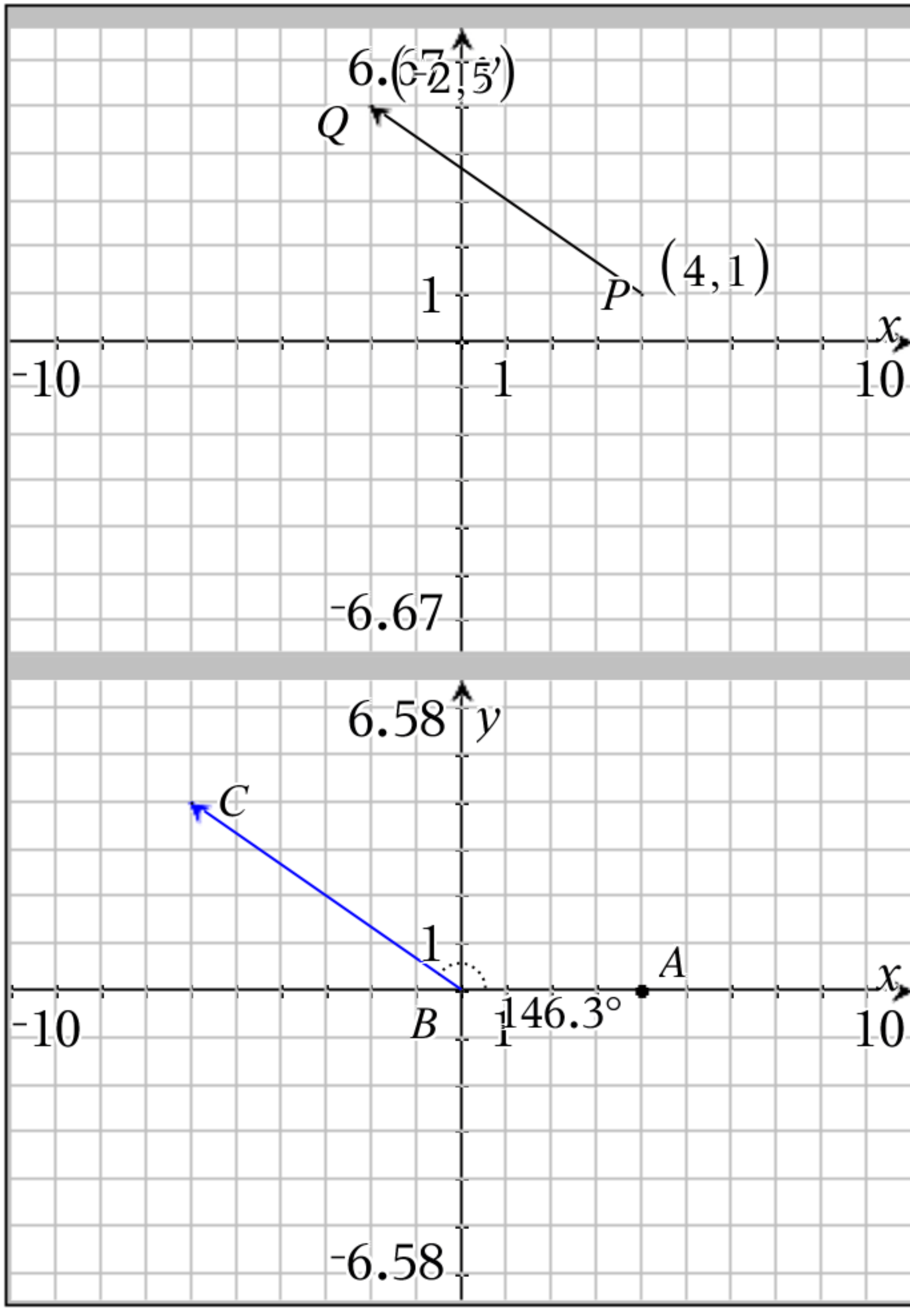




Vector PQ is drawn in the upper left hand corner

the vector that is parallel to vector PQ with same direction that starts at the origin is in the upper right hand corner

the related unit vector is drawn in the lower left hand corner (note scale change)



Vector PQ is drawn in the upper left hand corner

To find the directional angle formed with the x axis look to the lower left

To find the angle (not the direction angle, but the angle itself) use tangent function

$m\angle ABC = \tan^{-1}(4/6) = 33.6901\ °$

Since this is in quadrant 2 we reference from 180°

directional angle $= 180° - 33.6901\ °$

$= 146.31\ °$

IF we keep signs in tangent, then

$m\angle ABC = \tan^{-1}(4/{}^{-}6) = {}^{-}33.6901\ °$

(this is in Quadrant 4 not Quadrant 2)

directional angle $= 180° + {}^{-}33.6901\ °$

$= 146.31\ °$

	_list8	y_list8
=		
1	0	0
2	-2	-6
3	1	3
4		
5		
6		
7		
8		
9		
10		
11		

V3 $= ^{-}\mathbf{scalar9} \cdot v2$

vector a = $\begin{bmatrix} 8 \\ -2 \end{bmatrix}$ = <8, -2> = 8i − 2j

vector b = $\begin{bmatrix} 10 \\ 4 \end{bmatrix}$ = <10, 4> = 10i + 4j

5) $2\mathbf{a} + 5\mathbf{b} = 2\begin{bmatrix} 8 \\ -2 \end{bmatrix} + 5\begin{bmatrix} 10 \\ 4 \end{bmatrix}$

$$= \begin{bmatrix} 16 \\ -4 \end{bmatrix} + \begin{bmatrix} 50 \\ 20 \end{bmatrix}$$

$$= \begin{bmatrix} 66 \\ 16 \end{bmatrix}$$

vector a $= \begin{bmatrix} 8 \\ -2 \end{bmatrix} = \langle 8, -2 \rangle = 8\,i - 2\,j$

vector b $= \begin{bmatrix} 10 \\ 4 \end{bmatrix} = \langle 10, 4 \rangle = 10\,i + 4\,j$

6) $\frac{-3}{4}\mathbf{a} + \frac{3}{10}\mathbf{b} = \frac{-3}{4}\begin{bmatrix} 8 \\ -2 \end{bmatrix} + \frac{3}{10}\begin{bmatrix} 10 \\ 4 \end{bmatrix}$

$$= \begin{bmatrix} -6 \\ \frac{3}{2} \end{bmatrix} + \begin{bmatrix} 3 \\ \frac{6}{5} \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ \frac{27}{10} \end{bmatrix} = \begin{bmatrix} -3. \\ 2.7 \end{bmatrix}$$

vector a = $\begin{bmatrix} 8 \\ -2 \end{bmatrix}$ = <8, -2> = 8i − 2j

vector b = $\begin{bmatrix} 10 \\ 4 \end{bmatrix}$ = <10, 4> = 10i + 4j

7) |5a − 4b| =

First Step Find 5a − 4b

$$5\begin{bmatrix} 8 \\ -2 \end{bmatrix} - 4\begin{bmatrix} 10 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 40 \\ -10 \end{bmatrix} + \begin{bmatrix} -40 \\ -16 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -26 \end{bmatrix}$$

Now find the magnitude $\sqrt{[(0)^2 + (-26)^2]}$

$= \sqrt{[676]}$

$= 26$

vector a = $\begin{bmatrix} 8 \\ -2 \end{bmatrix}$ =<8,-2> = 8i − 2j

vector b = $\begin{bmatrix} 10 \\ 4 \end{bmatrix}$ =<10,4> = 10i + 4j

8) Write a vector that is parallel (this means any multiple of (a−b))

Write a vector that travels in the opposite direction as (a−b)

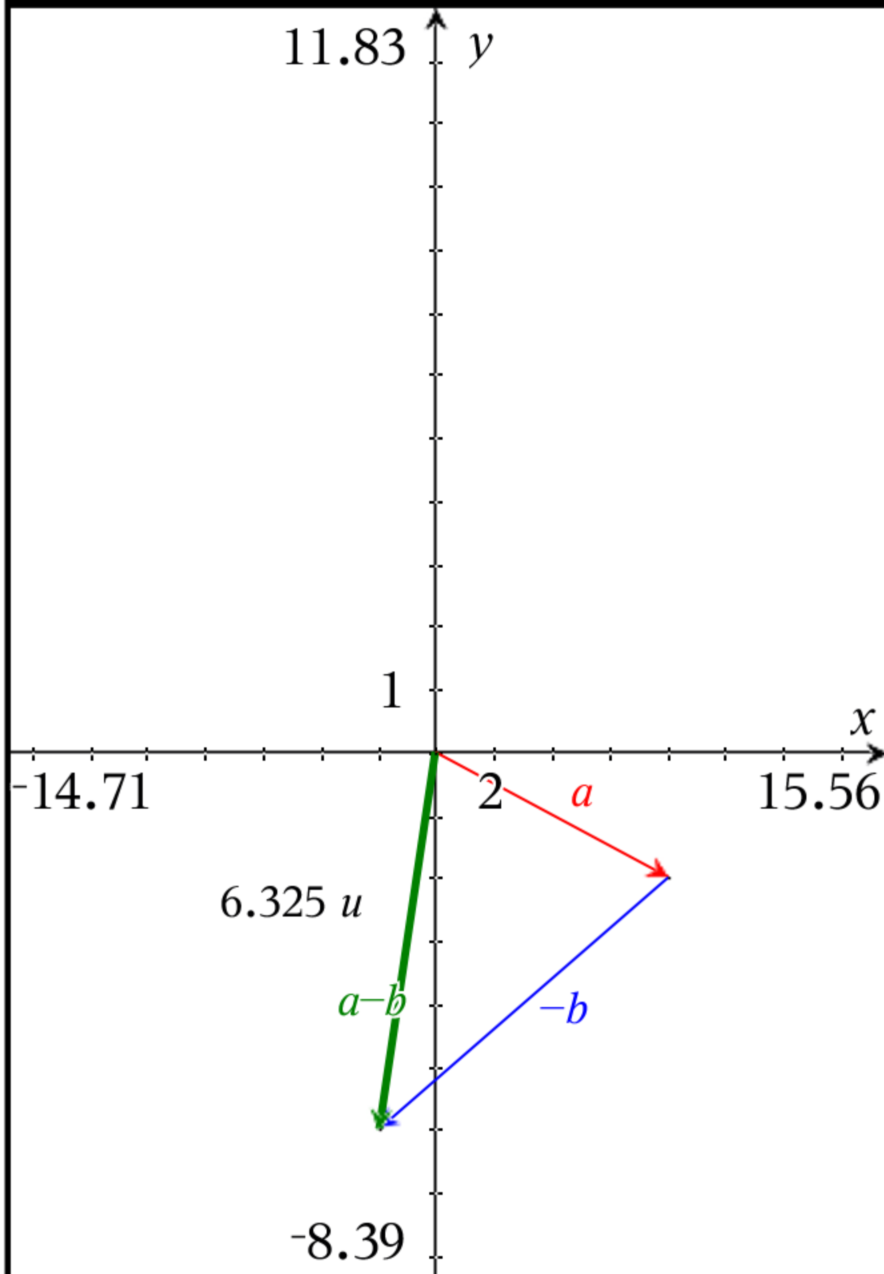
(this means that once you find a−b you must change the signs of change in x and change in y)

Write a vector that is a quarter as long (this means scalar is $\frac{1}{2}$ or $\frac{-1}{2}$)

Step 1) find a−b $\begin{bmatrix} 8 \\ -2 \end{bmatrix} - \begin{bmatrix} 10 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ -6 \end{bmatrix}$

Step 2) change its direction and size by multiplying by $\frac{-1}{2}$

$$\frac{-1}{2}(a-b) = \frac{-1}{2}\begin{bmatrix} -2 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$



Now, this vector "triangle" represents a−b

We know that a−b has direction $\begin{bmatrix} -2 \\ -6 \end{bmatrix}$

This means that it has length $2\cdot\sqrt{10}$

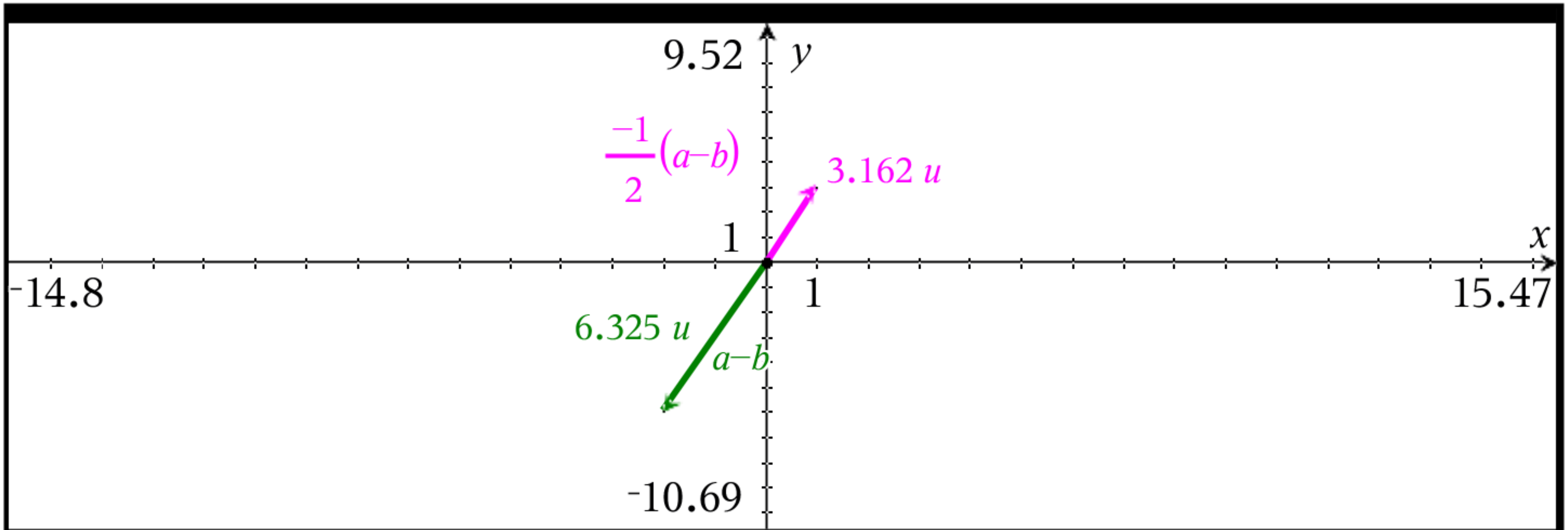
If we want this vector to travel in the opposite direction we would need to multiply it by −1

If we want this vector to be $\frac{1}{2}$ as long,

we need to multiply it by $\frac{1}{2}$

If we want it to be both $\frac{1}{2}$ as long and travel in the

opposite direction, we need to multiply by $\frac{-1}{2}$



So the direction of the vector that is

1) in the opposite direction of a−b

2) $\frac{1}{2}$ as long as a−b

Can be found by performing the following $\frac{-1}{2}\begin{bmatrix}-2\\-6\end{bmatrix}=\begin{bmatrix}1\\3\end{bmatrix}$

vector a = $\begin{bmatrix} 8 \\ -2 \end{bmatrix}$ = $\langle 8, -2 \rangle$ = $8i - 2j$

vector b = $\begin{bmatrix} 10 \\ 4 \end{bmatrix}$ = $\langle 10, 4 \rangle$ = $10i + 4j$

9) write the unit vector related to $2a + 5b$

Step 1 Find $2\mathbf{a} + 5\mathbf{b} = 2\begin{bmatrix} 8 \\ -2 \end{bmatrix} + 5\begin{bmatrix} 10 \\ 4 \end{bmatrix}$

$$= \begin{bmatrix} 16 \\ -4 \end{bmatrix} + \begin{bmatrix} 50 \\ 20 \end{bmatrix} = \begin{bmatrix} 66 \\ 16 \end{bmatrix}$$

Step 2 find magnitude of $2\mathbf{a} + 5\mathbf{b}$ $\sqrt{[(66)^2+(16)^2]}$

$$=\sqrt{[4612]} = 2\cdot\sqrt{1153}$$

Step 3 divide $2\mathbf{a} + 5\mathbf{b}$ **by** $\sqrt{[4612]}$

$$\langle 66/\sqrt{[4612]}, 16/\sqrt{[4612]} \rangle = \langle 66/2\cdot\sqrt{1153}, 16/2\cdot\sqrt{1153} \rangle = \begin{bmatrix} \frac{33\cdot\sqrt{1153}}{1153} \\ \frac{8\cdot\sqrt{1153}}{1153} \end{bmatrix}$$

vector a = $\begin{bmatrix} 8 \\ ^{-}2 \end{bmatrix}$ =<8 ,⁻2 > = 8 i − 2 j

vector b = $\begin{bmatrix} 10 \\ 4 \end{bmatrix}$ =<10 ,4 > = 10 i + 4 j

6/10) $\frac{^{-}3}{4}\mathbf{a} + \frac{3}{10}\mathbf{b} = \frac{^{-}3}{4}\begin{bmatrix} 8 \\ ^{-}2 \end{bmatrix} + \frac{3}{10}\begin{bmatrix} 10 \\ 4 \end{bmatrix} = \begin{bmatrix} ^{-}6 \\ \frac{3}{2} \end{bmatrix} + \begin{bmatrix} 3 \\ \frac{6}{5} \end{bmatrix} = \begin{bmatrix} ^{-}3 \\ \frac{27}{10} \end{bmatrix} = \begin{bmatrix} ^{-}3. \\ 2.7 \end{bmatrix}$

Vertical component form $\begin{bmatrix} ^{-}3 \\ \frac{27}{10} \end{bmatrix} = \begin{bmatrix} ^{-}3. \\ 2.7 \end{bmatrix}$

Horizontal component form $< ^{-}3 , \frac{27}{10} > = < ^{-}3. , 2.7 >$

UNIT VECTOR FORM $^{-}3\ \mathbf{i} + \frac{27}{10}\mathbf{j} = {}^{-}3.\ \mathbf{i} + 2.7\ \mathbf{j}$

	A	B
=		
1		
2	2	-2
3		
4	-4	-5
5		
6		
7		
8		
9		
10		
11		

B3

$T(2, {}^-2)$ $\mathbf{W} = ({}^-4, {}^-5)$

change in x from T to W $= {}^-4 - 2 = {}^-6$

change in y from T to W $= {}^-5 - {}^-2 = {}^-3$

1) $TW = \begin{bmatrix} {}^-6 \\ {}^-3 \end{bmatrix}$

$= \langle {}^-6, {}^-3 \rangle$

$= {}^-6\mathbf{i} - 3\mathbf{j}$

2) $|TW| = \sqrt{[({}^-6)^2 + ({}^-3)^2]} = \sqrt{45}$

$= 3 \cdot \sqrt{5}$

T(2,⁻2)   **W = (⁻4,⁻5)** |TW| = $=3\cdot\sqrt{5}$

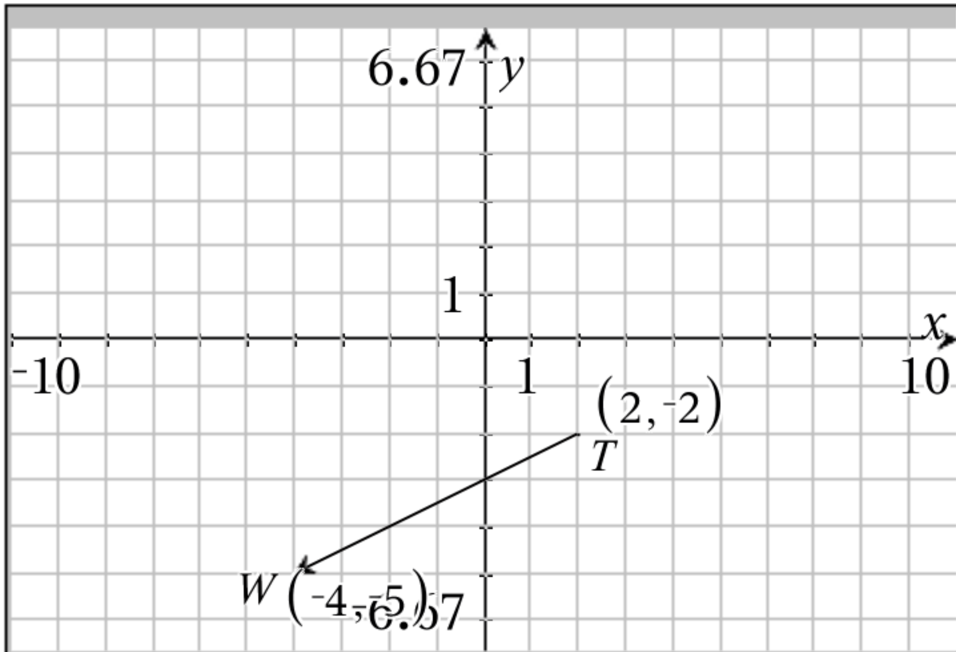
change in x from T to W =⁻6   **change in y from T to W =**⁻3

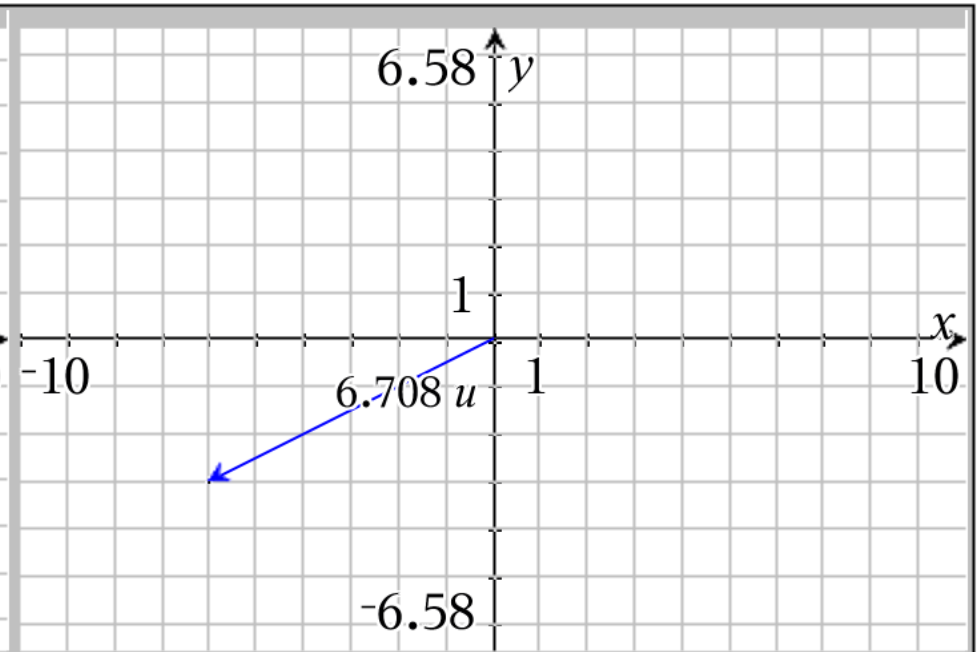
TW = $\begin{bmatrix} ^{-}6 \\ ^{-}3 \end{bmatrix}$ = <⁻6,⁻3>=⁻6**i** − 3**j**

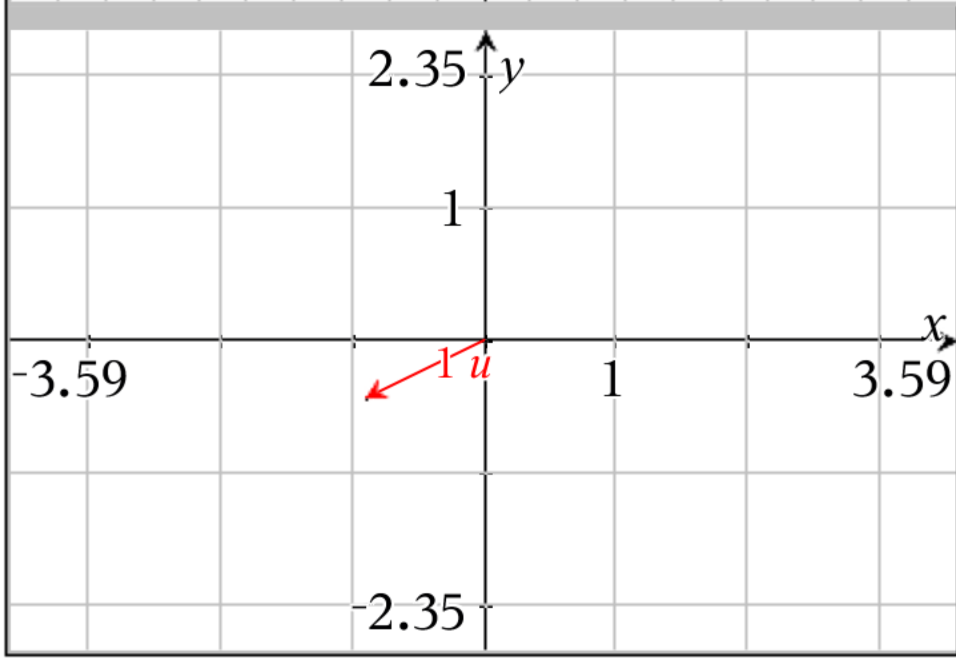
3) Unit Vector Related to TW = $\begin{bmatrix} \frac{^{-}2\cdot\sqrt{5}}{5} \\ \frac{^{-}\sqrt{5}}{5} \end{bmatrix}$

= <⁻6/√45,⁻3/√45> **unsimplified**

=⁻6/√45**i** − 3/√45**j** **unsimplified**



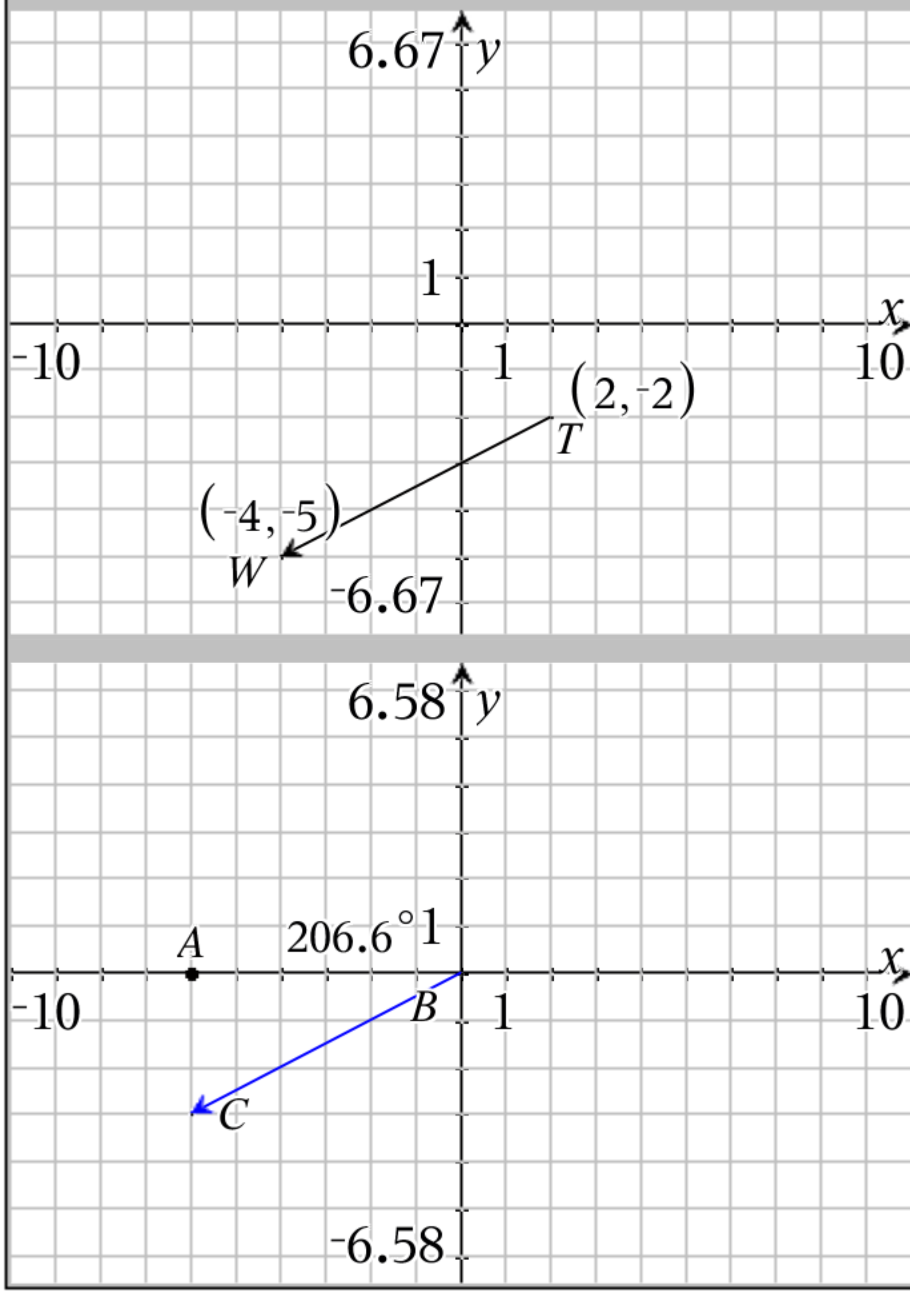




Vector TW is drawn in the upper left hand corner

the vector that is parallel to vector TW with same direction that starts at the origin is in the upper right hand corner

the related unit vector is drawn in the lower left hand corner (note scale change)



Vector PTW is drawn in the upper left hand corner

To find the directional angle formed with the x axis look to the lower left

To find the angle (not the direction angle, but the angle itself) use tangent function

$m\angle ABC = \tan^{-1}(3/6) = 26.5651°$

Since this is in quadrant 3 we reference from 180°

$\text{directional angle} = 180° + 26.5651°$

$= 206.565°$

IF we keep signs in tangent, then

$m\angle ABC = \tan^{-1}(^-3/^-6) = 26.5651°$

(this is in Q1 not Q3)

$\text{directional angle} = 180° + 26.5651°$

$= 206.565°$

Problem 4

	x_list	y_list
1	0	0
2	1	2
3	2	4
4	3	6
5	6	5
6	9	4
7	12	3
8	15	2
9	17	7
10	19	12
11		

X11