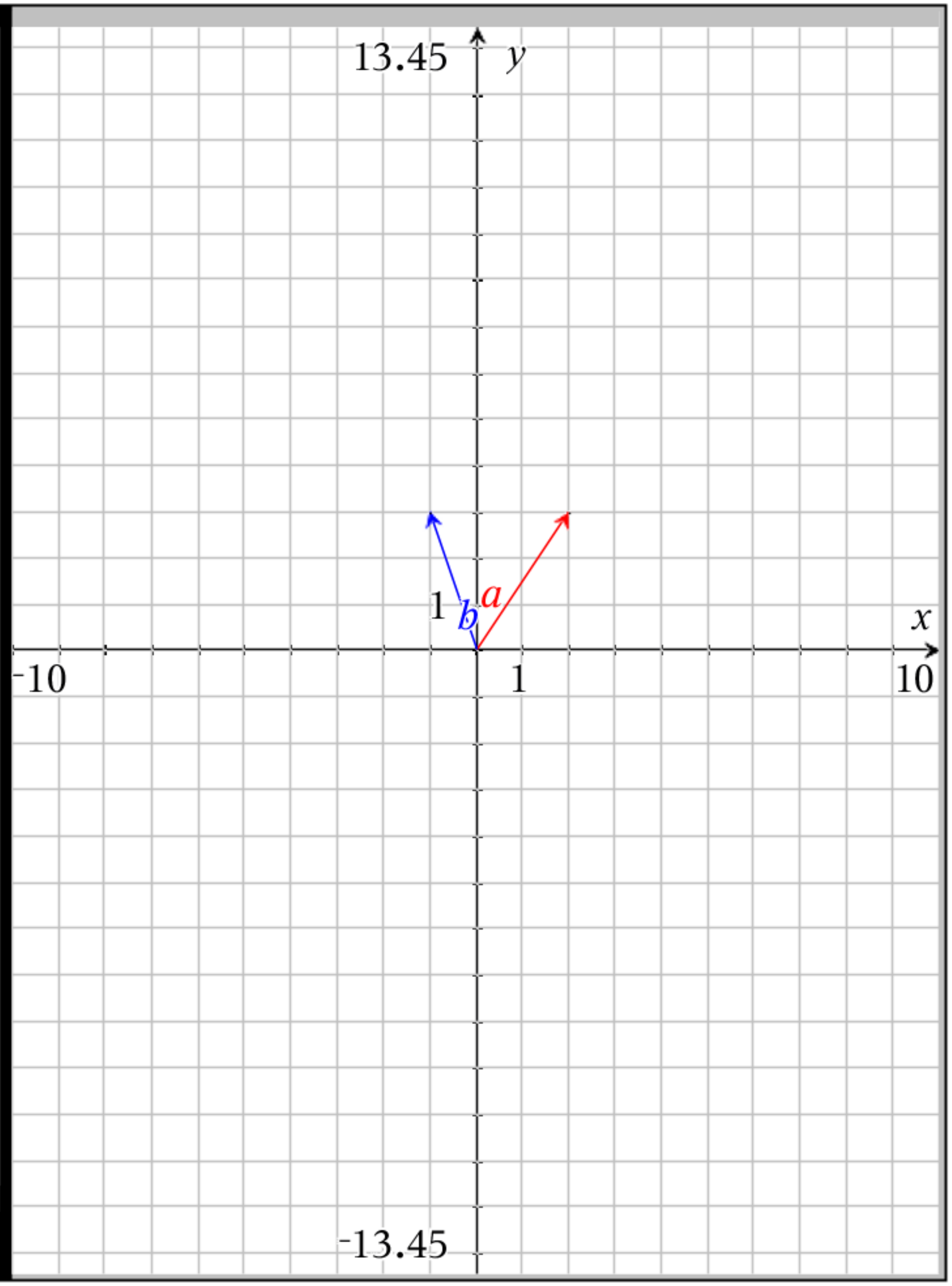
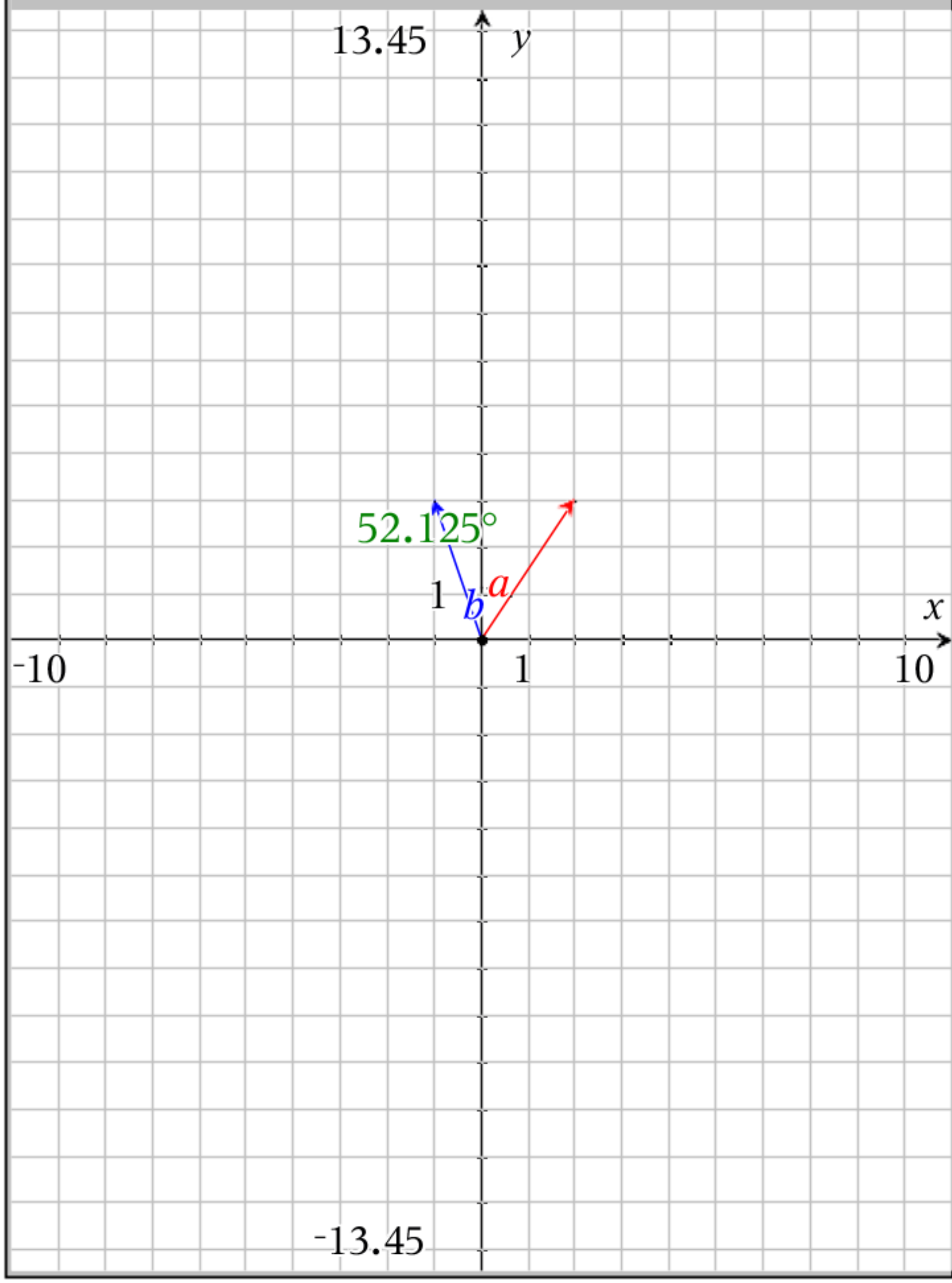
Questions 15 and 18

	A	B	C	D	E
1	vector_a	dx_1	2		
2		dy_1	3		
3					
4	vector_b	dx_2	⁻1		
5		dy_2	3		
6					
7		scalar_1	⁻4		
8		scalar_2	⁻1		
9					
10					
11					
12					
13					
14					
15					

A1 *vector_a*





To find angle between vectors

1) Find magnitudes of vectors

$a = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $|a| = \sqrt{(13)} = \sqrt{13}$

$b = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ $|b| = \sqrt{(10)} = \sqrt{10}$

2) Find dot product of vectors

$a \cdot b = (2)(-1) + (3)(3) = 7$

3) Apply $\cos(\theta) = \dfrac{a \cdot b}{|a| \cdot |b|}$

so cos θ has ratios

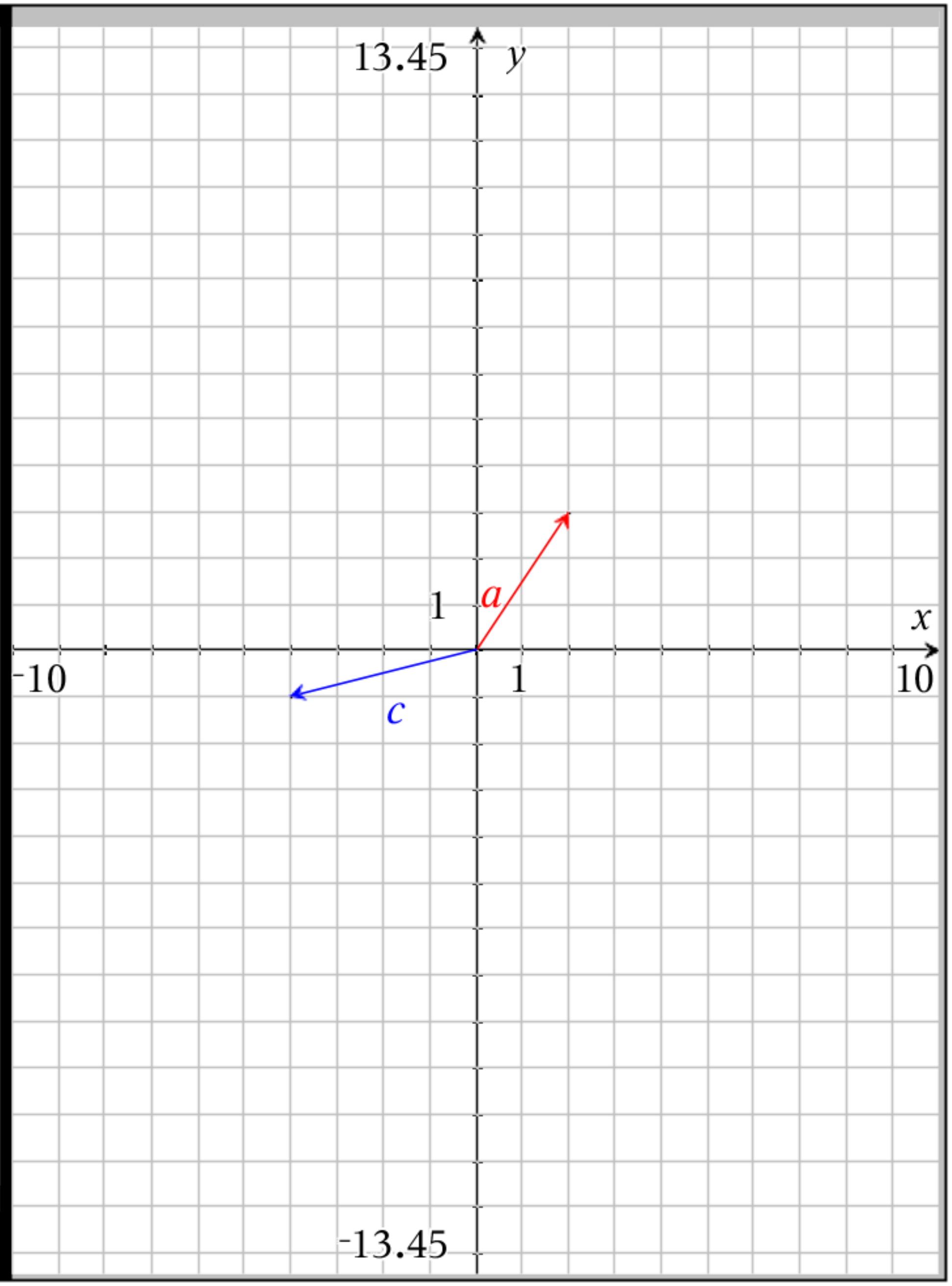
$= 7/(\sqrt{13}\sqrt{10}) = 7/(\sqrt{130}) = \dfrac{7 \cdot \sqrt{130}}{130}$

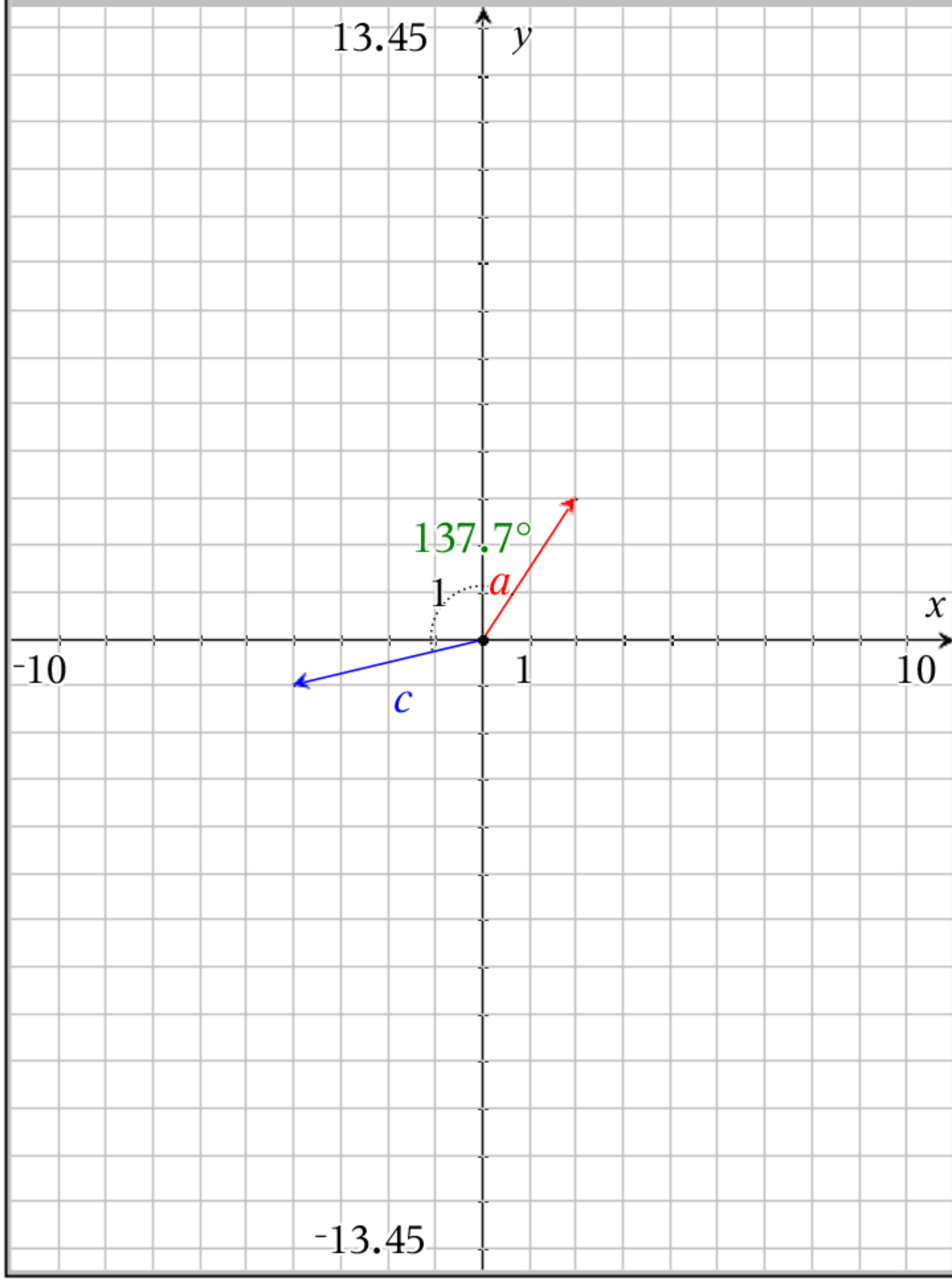
4) Use inverse cos to find θ

$\theta = \cos^{-1}(7/(\sqrt{130})) = 52.125°$

	A	B	C	D	E
=					
1	vector_a	dx_1	2		
2		dy_1	3		
3					
4	vector_c	dx_2	-4		
5		dy_2	-1		
6					
7		scalar_1	-4		
8		scalar_2	-1		
9					
10					
11					
12					
13					
14					
15					

C6





To find angle between vectors

1) Find magnitudes of vectors

$a = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $|a| = \sqrt{(13)} = \sqrt{13}$

$c = \begin{bmatrix} -4 \\ -1 \end{bmatrix}$ $|c| = \sqrt{(17)} = \sqrt{17}$

2) Find dot product of vectors

$a \cdot c = (2)(-4) + (3)(-1) = -11$

3) Apply $\cos(\theta) = \dfrac{a \cdot b}{|a| \cdot |b|}$

so cos θ has ratios

$= -11 / (\sqrt{13}\sqrt{17}) = -11 / (\sqrt{221}) = \dfrac{-11 \cdot \sqrt{221}}{221}$

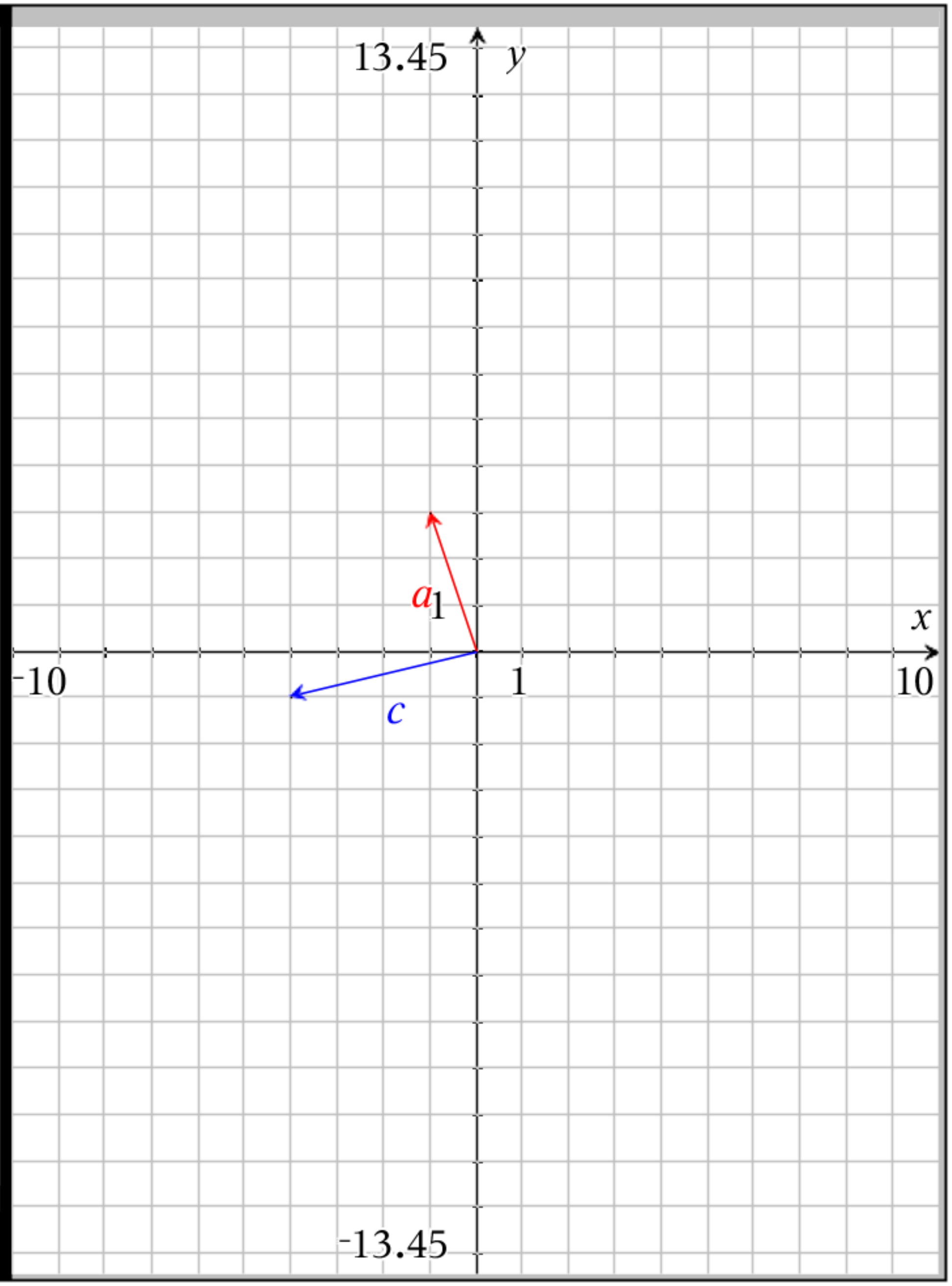
4) Use inverse cos to find θ

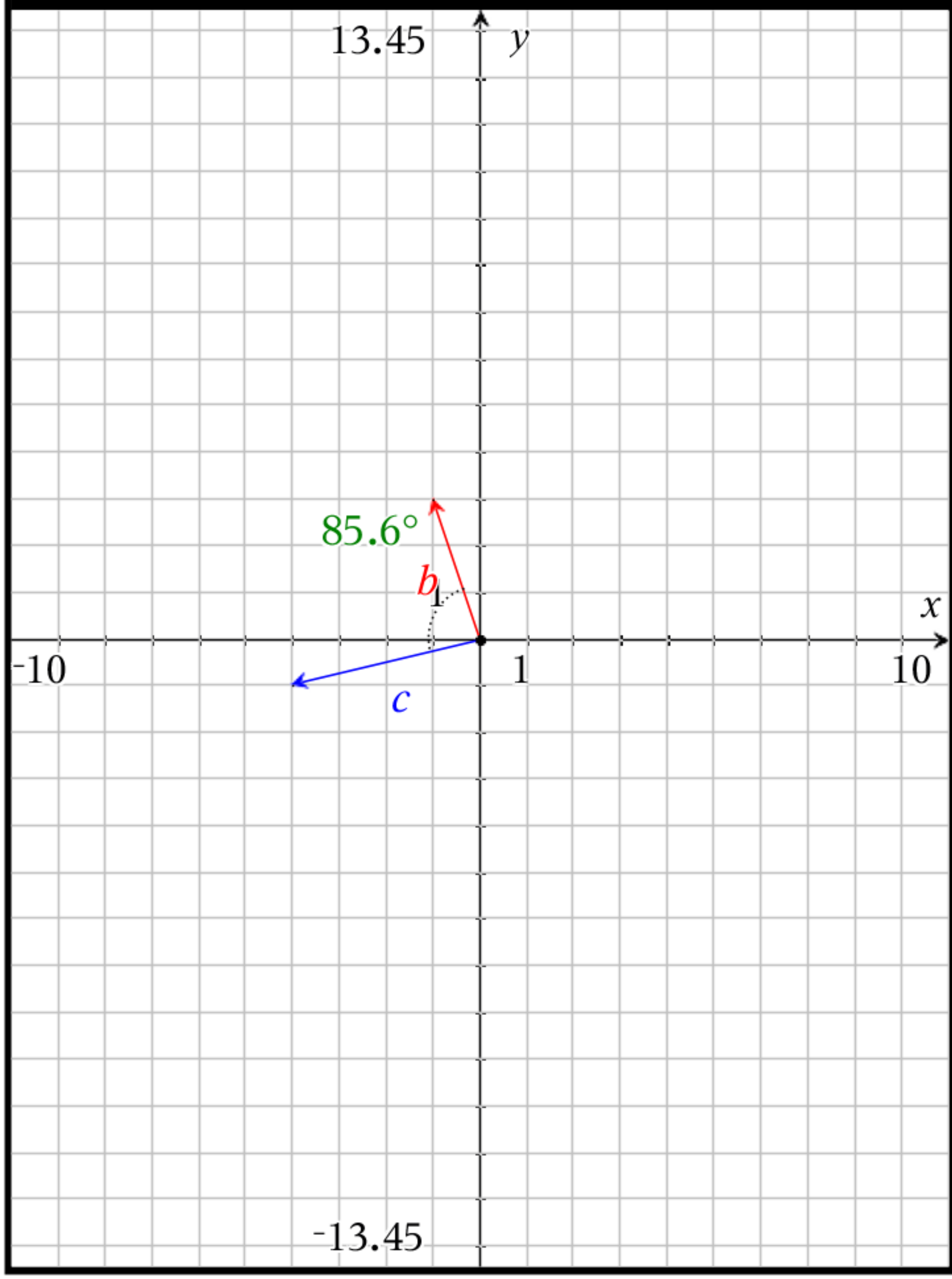
$\theta = \cos^{-1}(-11 / (\sqrt{221})) = 137.726°$

Questions 17 and 20

	A	B	C	D	E
=					
1	vector_b	dx_1	-1		
2		dy_1	3		
3					
4	vector_c	dx_2	-4		
5		dy_2	-1		
6					
7		scalar_1	1		
8		scalar_2	-2		
9					
10					
11					
12					
13					
14					
15					

C6





To find angle between vectors

1) Find magnitudes of vectors

$b = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ $|b| = \sqrt{(10)} = \sqrt{10}$

$c = \begin{bmatrix} -4 \\ -1 \end{bmatrix}$ $|c| = \sqrt{(17)} = \sqrt{17}$

2) Find dot product of vectors

$a \cdot b = (-1)(-4) + (3)(-1) = 1$

3) Apply $\cos(\theta) = \dfrac{a \cdot b}{|a| \cdot |b|}$

so cos θ has ratios

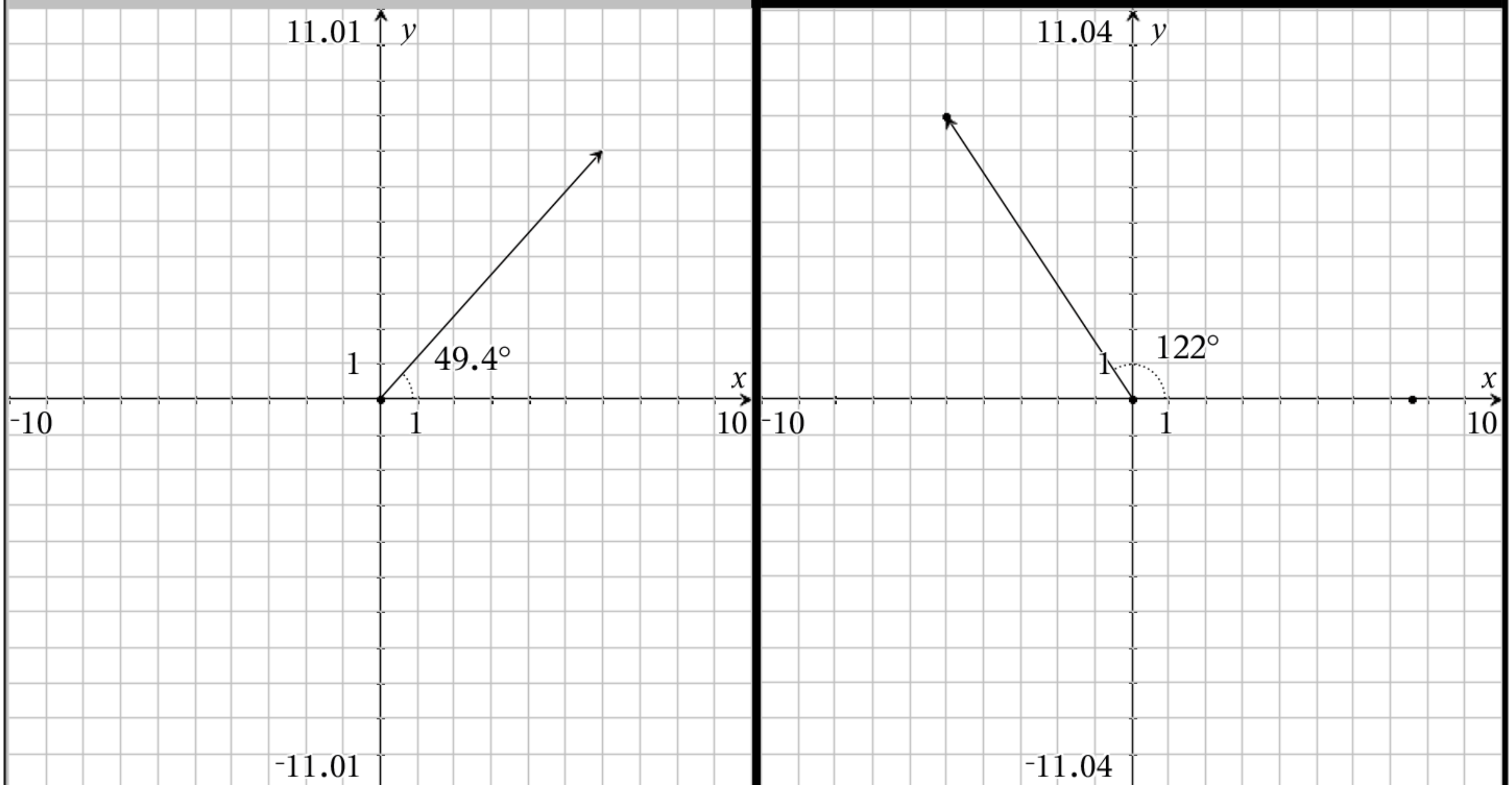
$= \mathbf{dot} / (\sqrt{10}\sqrt{17}) = 1 / (\sqrt{170}) = \dfrac{\sqrt{170}}{170}$

4) Use inverse cos to find θ

$\theta = \cos^{-1}(1 / (\sqrt{170})) = 85.6013\,°$

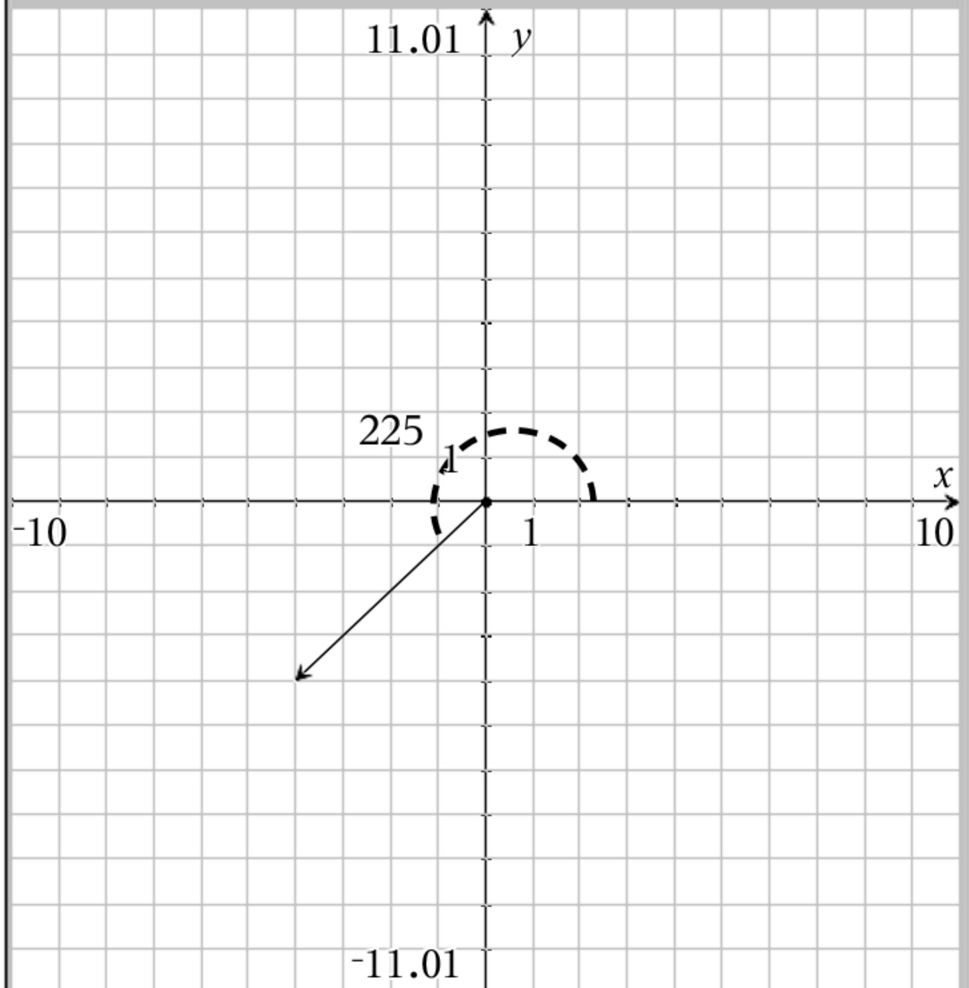
Question 21

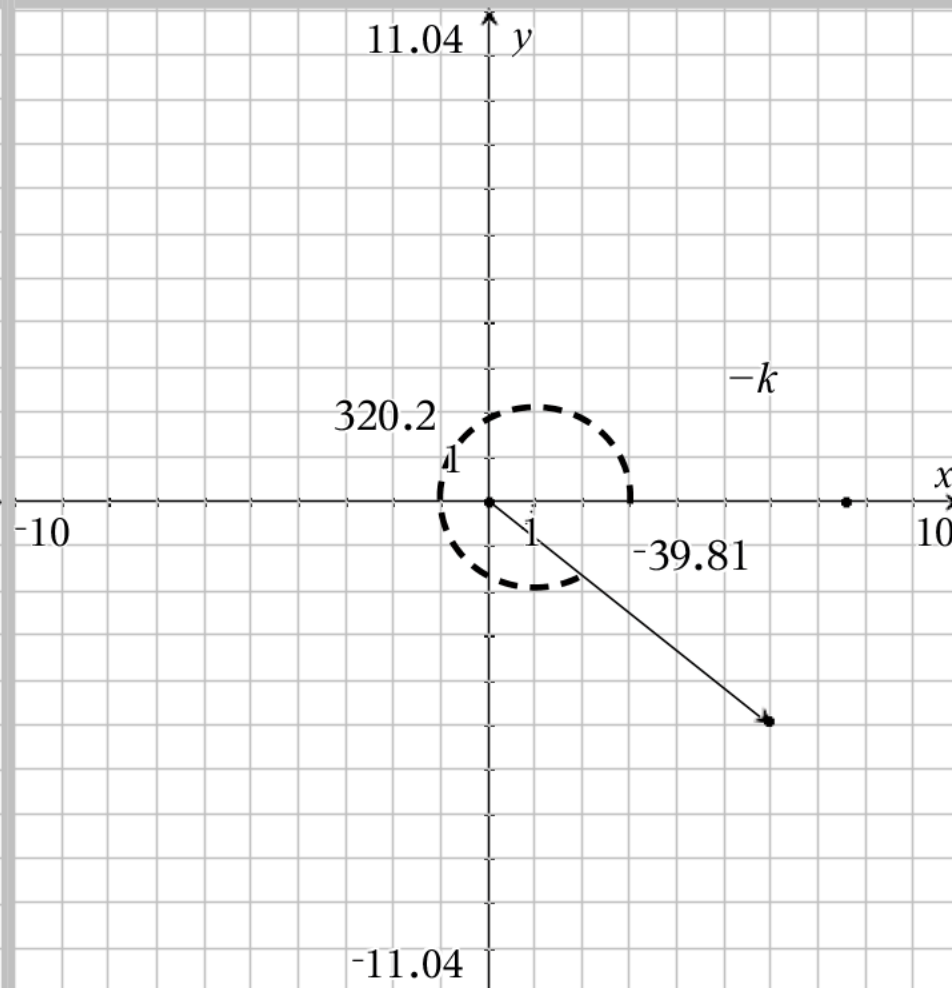
What is the difference between a directional angle and angle formed by two vectors?	These are examples of quadrants 1 and 2 directions angles formed by the left hand side of the x axes and the given vectors



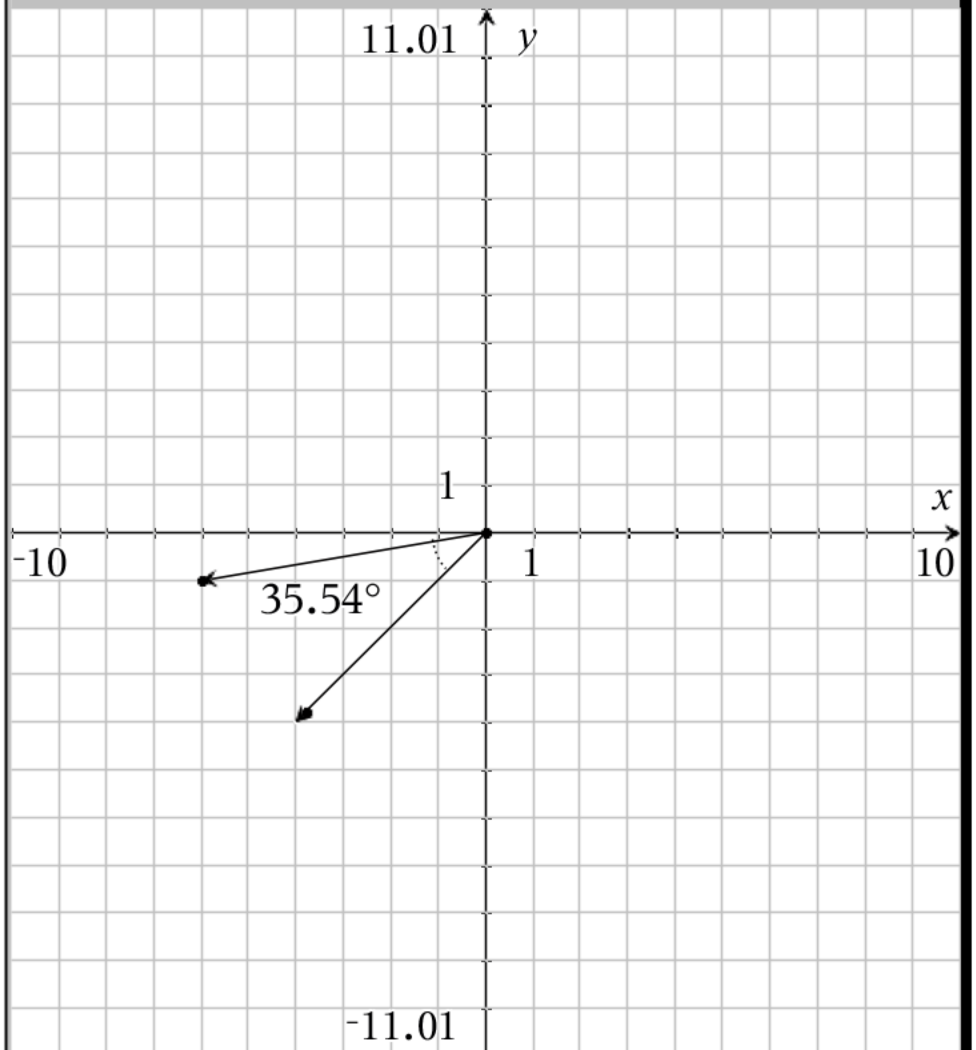
What is the difference between a directional angle and angle formed by two vectors?

These are examples of quadrants 3 and 4 directions angles formed by the left hand side of the x axes and the given vectors





What is the difference between a directional angle and angle formed by two vectors?



The angle between two vectors is any angle created by two vectors that have the same vertex

These angles can be found using

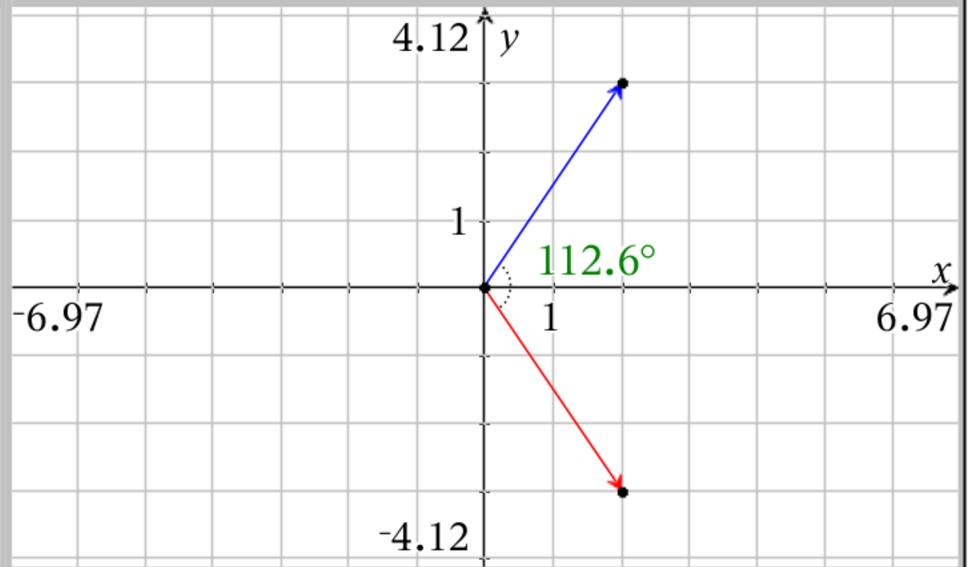
$\cos(\theta)=\frac{a\cdot b}{|a|\cdot|b|}$

Vector a= $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and Vector b = $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$

note $|a| = \sqrt{13}$

note $|b| = \sqrt{13}$

These vectors have the same magnitude



Vector a and b do not lie on the same line nor are they parallel.

no value of n satisfies $\begin{bmatrix} 2n \\ 3n \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

Vector a and vector b have a dot product that is NON ZERO, so these vectors are NOT perpendicular

$\begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = 4-9 = {}^{-}5$

$\cos(\theta) = \frac{{}^{-}5}{\sqrt{13} \cdot \sqrt{13}}$ implies

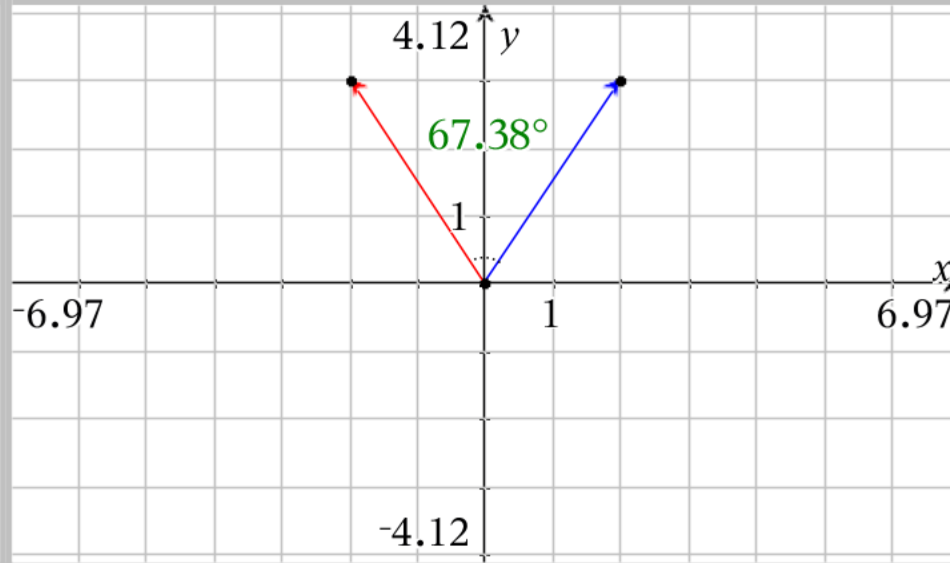
$\theta = \arccos\left(\frac{{}^{-}5.}{\sqrt{13} \cdot \sqrt{13}}\right) \blacktriangleright 112.62$

Vector a= $\begin{bmatrix}2\\3\end{bmatrix}$ and Vector c = $\begin{bmatrix}-2\\3\end{bmatrix}$

note $|a|=\sqrt{13}$

note $|c|=\sqrt{13}$

These vectors have the same magnitude



Vector a and b do not lie on the same line nor are they parallel.

no value of n satisfies $\begin{bmatrix}2n\\3n\end{bmatrix}=\begin{bmatrix}2\\-3\end{bmatrix}$

Vector a and vector b have a dot product that is NON ZERO, so these vectors are NOT perpendicular

$\begin{bmatrix}2\\3\end{bmatrix}\begin{bmatrix}-2\\3\end{bmatrix}=-4+9=5$

$\cos(\theta)=\dfrac{5}{\sqrt{13}\cdot\sqrt{13}}$ implies

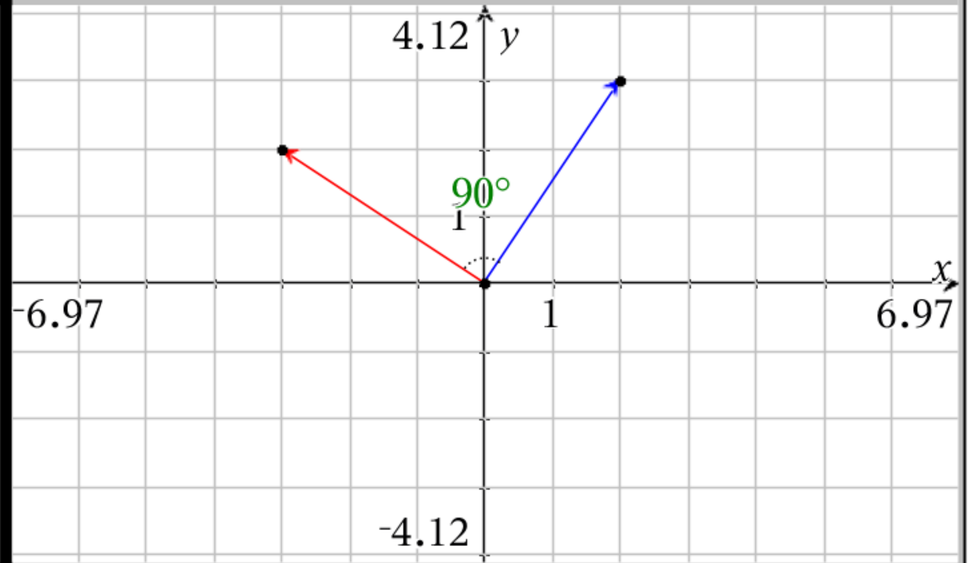
$\theta=\arccos\left(\dfrac{5.}{\sqrt{13}\cdot\sqrt{13}}\right)$ ▸ 67.3801

Vector a= $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and Vector d = $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$

note $|a| = \sqrt{13}$

note $|d| = \sqrt{13}$

These vectors have the same magnitude



Vector a and b do not lie on the same line nor are they parallel.

no value of n satisfies $\begin{bmatrix} 2n \\ 3n \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

Vector a and vector b have a dot product that is ZERO, so these vectors are perpendicular

$\begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \end{bmatrix} = {}^{-}6+6 = 0$

$\cos(\theta) = \frac{0}{\sqrt{13} \cdot \sqrt{13}}$ implies

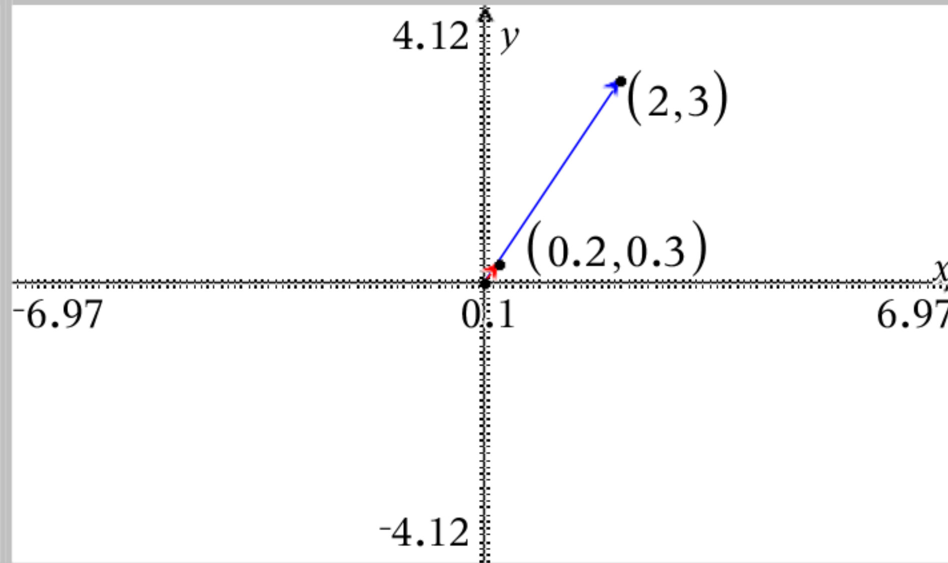
$\theta = \arccos\left(\frac{0.}{\sqrt{13} \cdot \sqrt{13}}\right) \blacktriangleright 90.$

Vector a= $\begin{bmatrix}2\\3\end{bmatrix}$ and Vector e = $\begin{bmatrix}0.2\\0.3\end{bmatrix}$

note $|a| = \sqrt{13}$

note $|e| = \dfrac{\sqrt{13}}{10} = 0.331662$

These vectors do NOT have the same magnitude



Vector a and b do lie on the same line and they parallel.

If n = 0.1 then satisfies $\begin{bmatrix}2n\\3n\end{bmatrix} = \begin{bmatrix}0.2\\0.3\end{bmatrix}$

Vector a and vector b have a dot product that is NON ZERO, so these vectors are NOT perpendicular

$\begin{bmatrix}2\\3\end{bmatrix}\begin{bmatrix}0.2\\0.3\end{bmatrix} = 0.4+0.9 = 1.3$

$\cos(\theta) = \dfrac{1.3}{\dfrac{\sqrt{13}\cdot\sqrt{13}}{10}} \blacktriangleright 1.$ implies

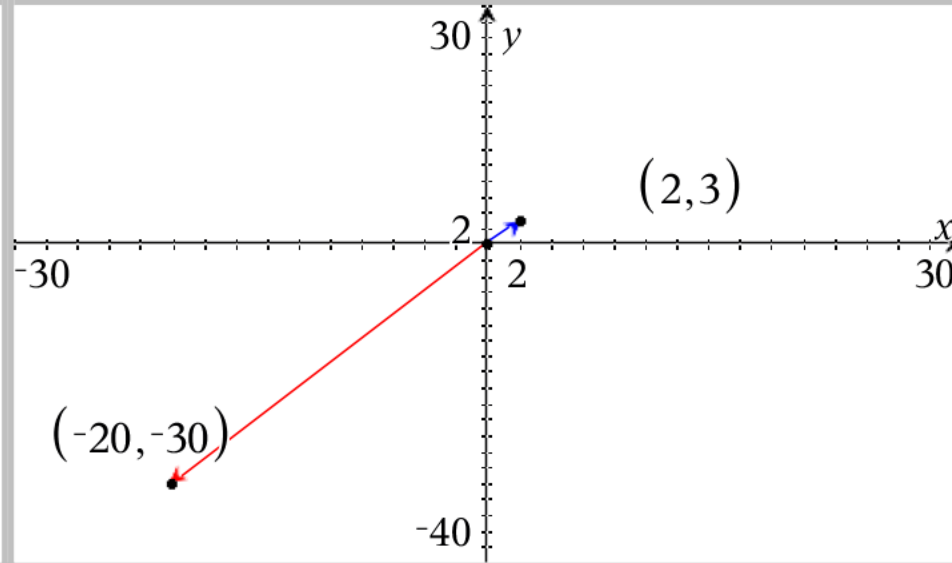
$\theta = \arccos\left(\dfrac{1.3}{\dfrac{\sqrt{13}\cdot\sqrt{13}}{10}}\right) \blacktriangleright 0.$

Vector a= $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and Vector f = $\begin{bmatrix} -20 \\ -30 \end{bmatrix}$

note $|a| = \sqrt{13}$

note $|f| = 10 \cdot \sqrt{13}$

These vectors do NOT have the same magnitude



Vector a and b do lie on the same line and they parallel.

If n = −10 then satisfies $\begin{bmatrix} 2n \\ 3n \end{bmatrix} = \begin{bmatrix} -20 \\ -30 \end{bmatrix}$

These vectors are in opposite directions on the same line

Vector a and vector b have a dot product that is NON ZERO, so these vectors are NOT perpendicular

$\begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} -20 \\ -30 \end{bmatrix} = {}^{-}40 + {}^{-}90 = {}^{-}130$

$\cos(\theta) = \dfrac{{}^{-}130}{\sqrt{13} \cdot 10 \cdot \sqrt{13}}$ implies

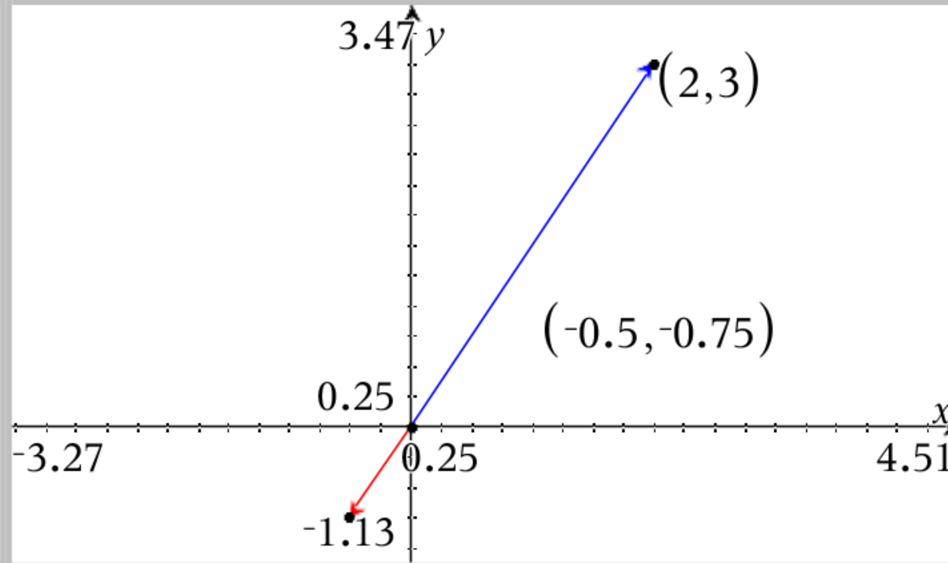
$\theta = \arccos\left(\dfrac{-130}{\sqrt{13} \cdot 10\sqrt{13}}\right) \blacktriangleright 180$

Vector a= $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and Vector g = $\begin{bmatrix} -0.5 \\ -0.75 \end{bmatrix}$

note $|a| = \sqrt{13}$

note abs(c) $= \frac{\sqrt{13}}{4} = 0.360555$

These vectors do NOT have the same magnitude



Vector a and b do lie on the same line and they parallel.

If $n = \frac{-1}{4}$ then satisfies $\begin{bmatrix} 2n \\ 3n \end{bmatrix} = \begin{bmatrix} -0.5 \\ -0.75 \end{bmatrix}$

Vectors a and g are parallel and travel in opposite directions

Vector a and vector b have a dot product that is NON ZERO, so these vectors are NOT perpendicular

$\begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} -0.5 \\ -0.75 \end{bmatrix} = {}^{-}1 + {}^{-}2.25 = {}^{-}3.25$

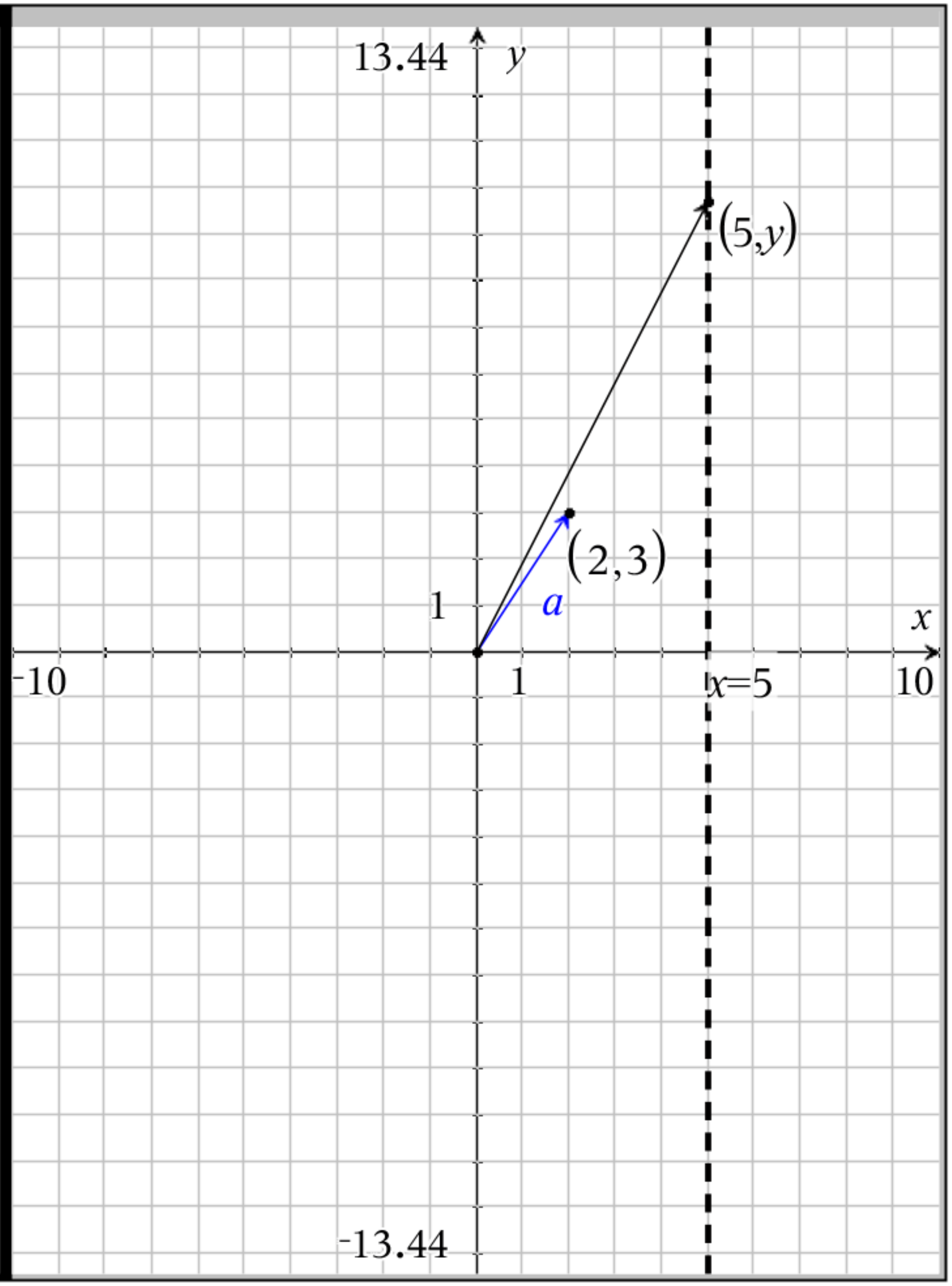
$\cos(\theta) = \frac{{}^{-}3.25}{\frac{\sqrt{13} \cdot \sqrt{13}}{4}} \blacktriangleright {}^{-}1.$ implies

$\theta = \arccos\left(\frac{-3.25}{\frac{\sqrt{13} \cdot \sqrt{13}}{4}}\right) \blacktriangleright 180.$

if we think as h $=\begin{bmatrix}5\\y\end{bmatrix}$ as any vector who has an endpoint on the line at x = 5 as shown to the right, then there is exact one value for y that will make this vector parallel (in this case collinear) to vector a

Since all vectors that are parallel are scalar multiples of each other we can say that $\begin{bmatrix}2n\\3n\end{bmatrix}$ is the collection of all vectors that are parallel to a= $\begin{bmatrix}2\\3\end{bmatrix}$ including the one that will make h be parallel to a

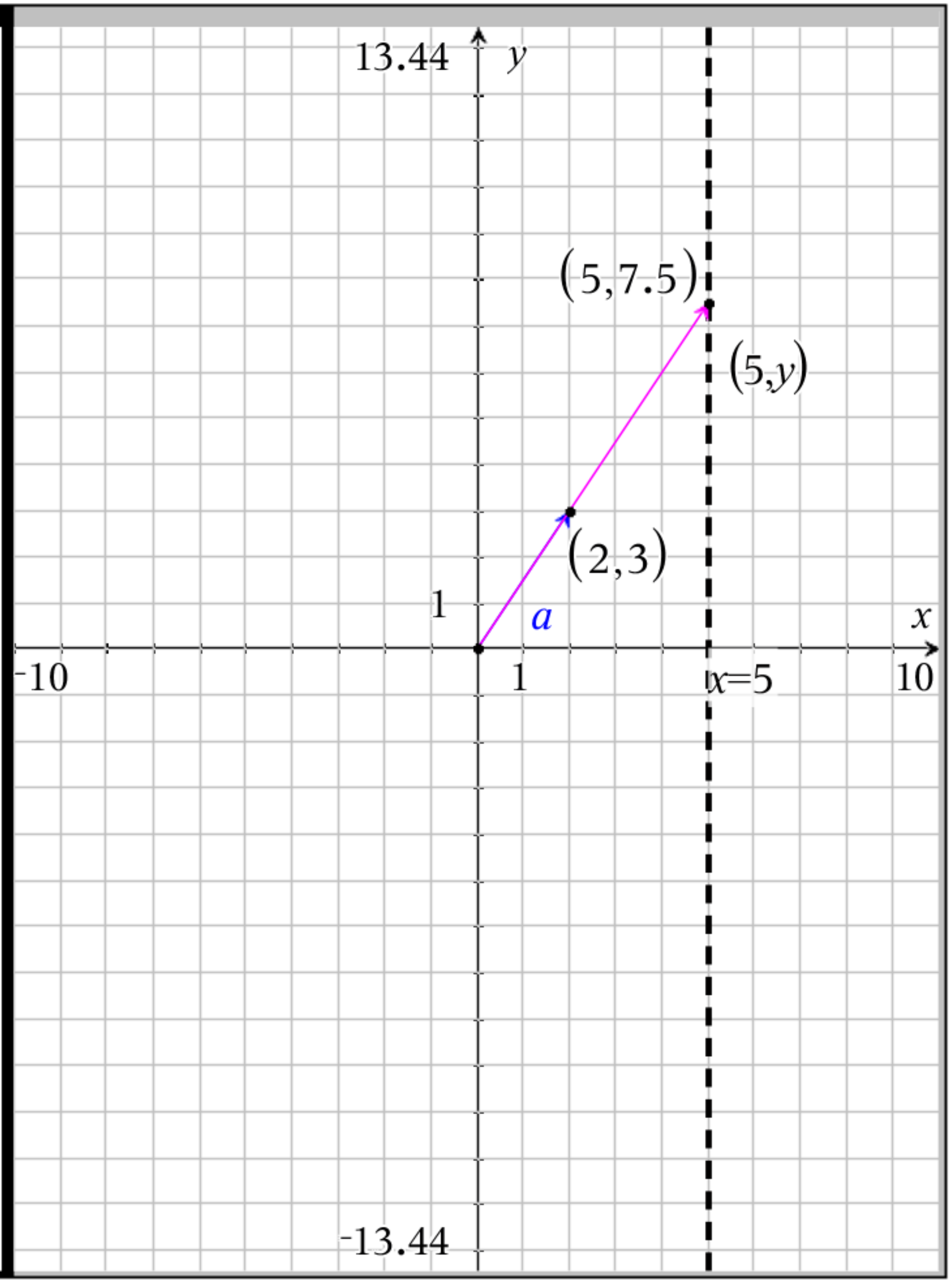
So we set $\begin{bmatrix}2n\\3n\end{bmatrix}=\begin{bmatrix}5\\y\end{bmatrix}$ and solve for n and ultimately y



$$\begin{bmatrix} 2n \\ 3n \end{bmatrix} = \begin{bmatrix} 5 \\ y \end{bmatrix}$$ implies 2n=5 and 3n = y

2n = 5 implies that $n = \frac{5}{2} = 2.5$

So 3(2.5)=y =7.5



For vectors a and m to be perpendicular to each other their dot product must be zero.

We can also think of this as looking for the one point that lies on line y = 8 that will make $m=\begin{bmatrix} w \\ 8 \end{bmatrix}$ have a dot product of 0

note: $a=\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $m=\begin{bmatrix} w \\ 8 \end{bmatrix}$

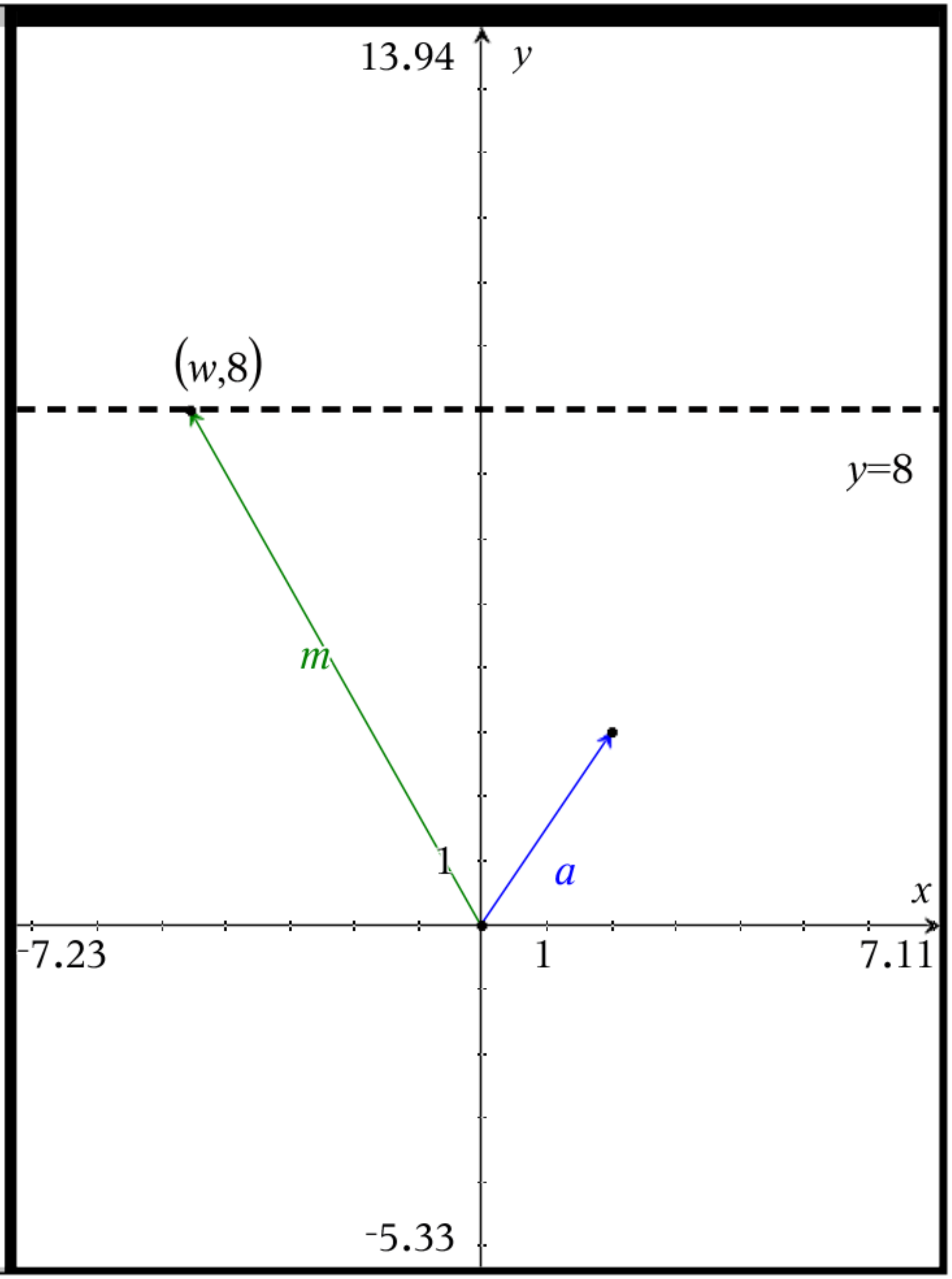
these has a dot product of $2w+24$

Set the dot product equal to 0 and solve for w

$2w+24=0$ leads to $2w+24-24=0-24$

$2w=-24$

$\frac{2w}{2}=\frac{-24}{2}$ implies $w = -12$



For vectors a and m to be perpendicular to each other their dot product must be zero.

We can also think of this as looking for the one point that lies on line y = 8 that will make m= $\begin{bmatrix} w \\ 8 \end{bmatrix}$ have a dot product of 0

note: a= $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and m= $\begin{bmatrix} w \\ 8 \end{bmatrix}$

these has a dot product of $2w+24$

Set the dot product equal to 0 and solve for w

$2w+24=0$ leads to $2w+24-24=0-24$

2w=−24

$\frac{2w}{2}=\frac{-24}{2}$ implies w = −12