

Problem 1

$\{15, 23, 31, 39\}$

Next three terms $\{15, 23, 31, 39, 47, 55, 63\}$

Note: $39 - 31 = 8$

$31 - 23 = 8$

$23 - 15 = 8$

This means that this sequence can be written as an arithmetic sequence

$d = 8$

$a_1 = 15$

Arithmetic Sequence $a_n = a_1 + (n-1)d$
 $= 15 + (n-1)(8)$

Explicit Form of this Sequence

$$a_n = 15 + 8n - 8$$

$$a_n = 7 + 8n$$

A	seq_1	B
=		
1	15	
2	23	
3	31	
4	39	
5		
6		
7		
8		
9		
10		
11		
A1	15	

finding a_{100} given that $a_n = 15 + (n-1)(8)$

$$\begin{aligned} a_{100} &= 15 + (100-1)(8) \\ &= 807 \end{aligned}$$

finding a_{100} given that $a_n = 7 + 8n$

$$\begin{aligned} a_{100} &= 7 + (8)(100) \\ &= 807 \end{aligned}$$

now you CAN write an arithmetic sequence as a recursive sequence, but arithmetic is easier!

$$a_1 = 15$$

$$a_{n+1} = a_n + 8$$

Problem 2

$\{4096, 512, 64, 8\}$ Including the next three terms $\left\{4096, 512, 64, 8, 1, \frac{1}{8}, \frac{1}{64}\right\}$

Note: $\frac{512}{4096} = \frac{1}{8}$ $\frac{64}{512} = \frac{1}{8}$ $\frac{8}{64} = \frac{1}{8}$

This means that this sequence can be written as a geometric sequence

$r = \frac{1}{8}$ $a_1 = 4096$ Geometric Sequence $a_n = a_1 \cdot r^{n-1}$

$$= 4096 \cdot \left(\frac{1}{8}\right)^{n-1}$$

Explicit Form of this Sequence

$$a_n = 4096 \cdot \left(\frac{1}{8}\right)^{n-1} \quad \text{factor}(4096) \rightarrow 2^{12}$$

$$a_n = 2^{12} \cdot (2^{-3})^{n-1}$$

$$= 2^{12} \cdot (2^{-3n+3})$$

$$= 2^{-3n+15} = 32768 \cdot 2^{-3n}$$

	A	B
	seq_1	
=		
1	4096	
2	512	
3	64	
4	8	
5		
6		
7		
8		
9		
10		
11		
	A1 4096	

finding a_9 given that $a_n = 4096 \left(\frac{1}{8}\right)^{n-1}$

$$a_9 = 4096 \left(\frac{1}{8}\right)^{9-1}$$

$$= 4096 \left(\frac{1}{8}\right)^8$$

$$= \frac{4096}{16777216} = \frac{1}{4096}$$

now you CAN write an geometric sequence as a recursive sequence, but geometric is easier!

$$a_1 = 4096$$

$$a_{n+1} = \frac{1}{8}a_n$$

Problem 3

$$\left\{ \frac{1}{4}, \frac{-1}{16}, \frac{1}{64}, \frac{-1}{256} \right\}$$

Including the next three terms $\left\{ \frac{1}{4}, \frac{-1}{16}, \frac{1}{64}, \frac{-1}{256}, \frac{1}{1024}, \frac{-1}{4096}, \frac{1}{16384} \right\}$

Note: $\frac{-1}{16} = \frac{-1}{4} \cdot \frac{1}{4}$ $\frac{1}{64} = \frac{-1}{16} \cdot \frac{-1}{4}$ $\frac{-1}{256} = \frac{1}{64} \cdot \frac{-1}{4}$

This means that this sequence can be written as a geometric sequence

$r = \frac{-1}{4}$ $a_1 = \frac{1}{4}$ Geometric Sequence $a_n = a_1 \cdot r^{n-1}$

$$= \frac{1}{4} \cdot \left(\frac{-1}{4} \right)^{n-1}$$

	A	seq_1	B
=			
1		1/4	
2		-1/16	
3		1/64	
4		-1/256	
5			
6			
7			
8			
9			
10			
		1/4	

finding a_{10} given that $a_n = \frac{1}{4} \left(\frac{-1}{4} \right)^{n-1}$

$$a_{10} = \frac{1}{4} \left(\frac{-1}{4} \right)^{10-1}$$

$$= \frac{1}{4} \left(\frac{-1}{4} \right)^9$$

$$= \frac{-1}{1048576}$$

now you CAN write an geometric sequence as a recursive sequence, but geometric is easier!

$$a_1 = \frac{1}{4}$$

$$a_{n+1} = \frac{-1}{4} a_n$$

Problem 4

$\{11,14,18,23,29\}$ Including the next three terms $\{11,14,18,23,29,36,44,53\}$

Note: $\frac{14}{11} = \frac{14}{11}$ $\frac{18}{14} = \frac{9}{7}$ $\frac{23}{18} = \frac{23}{18}$ $\frac{29}{23} = \frac{29}{23}$

Note: $14-11 = 3$ $18-14 = 4$ $23-18 = 5$ $29-23 = 6$

So this does not have a common ratio or a common difference

This means that this sequence cannot be written as a geometric or an arithmetic sequence

So we must write a different type of sequence, recursive sequence

$$a_1 = 11$$

$$a_{n+1} = (n+2) + a_n \text{ (this means add the number of term plus 2 to the previous term)}$$

	A	B	C	D	E	F	G
1		a1	a2	a3	a4	a5	
2	a_n	11	14	18	23	29	

D11 =d10