

Writing a sequence from two of its terms (non sequential)

Step 1) What type of sequence is given?

When given arithmetic sequence terms

Step 2) Write the definition of each term using  $a_1$  and  $d$

Step 3) Solve the resulting linear system

Step 4) Write the arithmetic sequence using  $a_1$  and  $d$

Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

Problem 4

	A	B	C	D	E	F	G
=							
1	given_term1	61					
2	term_in_seq1	3					
3	given_term2	137					
4	term_in_seq2	7					
5							
6							
7							
8							
9							
10							
11							

A1 given\_term1

So we know  $a_3=61$  and  $a_7=137$  and we also know that this is an arithmetic sequence

so all terms must follow  $a_n=a_1+(n-1)d$ , including  $a_3$  and  $a_7$

$$\text{so } a_3=a_1+(3-1)d \text{ or } 61=a_1+2d$$

$$\text{so } a_7=a_1+(7-1)d \text{ or } 137=a_1+6d$$

This system can be solved a variety of ways, but basically you can subtract the higher sequence term and lower sequence term and it yields  $d$  and  $a_1$

	A	B	C
=			
1	a_1	unknown	
2	a_2	unknown	
3	a_3	61	
4	a_4	unknown	
5	a_5	unknown	
6	a_6	unknown	
7	a_7	137	
8			
9			
10			
11			
	A1	a_1	

$$a_7 - a_3 \text{ implies } 137 - 61 = 76$$

$$a_7 - a_3 \text{ implies } (a_1 + 6d) - (a_1 + 2d) = a_1 + 6d - a_1 - 2d = 4d$$

$$\text{This allows us to find "d" } \quad 76 = 4d$$

$$76/4 = 4d/4$$

$$19 = d$$

We can now use either  $61 = a_1 + 2d$  or  $137 = a_1 + 6d$  to find  $a_1$

$$61 = a_1 + 2(19) \text{ leads to } 61 = a_1 + 38 \text{ or } a_1 = 61 - 38 = 23$$

$$137 = a_1 + 6(19) \text{ leads to } 137 = a_1 + 114 \text{ or } a_1 = 137 - 114 = 23$$

**so our arithmetic sequence is  $a_n = 23 + (n-1)(19)$**

**and our finite sequence is  $\{23, 42, 61, 80, 99, 118, 137\}$**

Writing a sequence from two of its terms (non sequential)

Step 1) What type of sequence is given?

When given geometric sequence terms

Step 2) Write the definition of each term using  $a_1$  and  $r$

Step 3) Solve the resulting proportion

(I suggest putting higher sequence term (not value) on top)

(this will require radical operations)

Step 4) Write the geometric sequence using  $a_1$  and  $r$

Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

	A	B	C
=			
1	given_term1	-3125	
2	term_in_seq1	2	
3	given_term2	1	
4	term_in_seq2	7	
5			
6			
7			
8			
9			
10			
11			
	A1	given_term1	

So we know  $a_2 = -3125$  and  $a_7 = 1$  and we also know that this is an geometric sequence

so all terms must follow  $a_n = a_1(r)^{n-1}$ , including  $a_1$  and  $a_7$

so  $a_2 = a_1 r^{(2-1)}$  or  $-3125 = a_1 \cdot r$

so  $a_7 = a_1 r^{(7-1)}$  or  $1 = a_1 \cdot r^6$

This system can be solved a variety of ways, but basically you can divide the higher sequence term and lower sequence term then apply necessary radical and it yields  $r$  and  $a_1$

	A	B	C
=			
1	a_2	-3125	
2	a_7	1	
3			
4			
5			
6			
7			
8			
9			
10			
11			
	A1	a_2	

$$a_7/a_2 \text{ implies } 1/-3125 = \frac{-1}{3125}$$

$$a_7/a_2 \text{ implies } a_1 \cdot r^6 / a_1 \cdot r = r^5$$

This allows us to find "r"  $r^5 = \frac{-1}{3125}$  which leads to  $\sqrt[5]{r^5} = \sqrt[5]{\frac{-1}{3125}}$

$$r = \frac{-1}{5}$$

I would use the smaller sequence term to use  $-3125 = a_1 \cdot r$  to find  $a_1$

$$-3125 = a_1 \left(\frac{-1}{5}\right)^1 \text{ which leads to } -3125 = \frac{-1}{5} a_1$$

$$\text{which leads to } -3125 \cdot (-5) = (-5) \left(\frac{-1}{5} a_1\right) \text{ or } a_1 = 15625$$

so our geometric sequence is  $a_n = 15625 \left(\frac{-1}{5}\right)^{(n-1)}$

and our finite sequence is  $\{ 15625, -3125, 625, -125, 25, -5, 1 \}$





So we know  $a_5 = 6$  and  $a_{15} = -60$  and we also know that this is an arithmetic sequence

so all terms must follow  $a_n = a_1 + (n-1)d$ , including  $a_3$  and  $a_{14}$

$$\text{so } a_3 = a_1 + (3-1)d \text{ or } 6 = a_1 + 2d$$

$$\text{so } a_{14} = a_1 + (14-1)d \text{ or } -60 = a_1 + 13d$$

This system can be solved a variety of ways, but basically you can subtract the higher sequence term and lower sequence term and it yields  $d$  and  $a_1$

A screenshot of a spreadsheet application showing a table with 11 rows and 3 columns labeled A, B, and C. The table contains the following data:

	A	B	C
=			
1	a_5		6
2	a_15		-60
3			
4			
5			
6			
7			
8			
9			
10			
11			

The status bar at the bottom shows the active cell is A1, containing the text "a\_5".

$$a_{14} - a_3 \text{ implies } -60 - 6 = -66$$

$$a_{14} - a_3 \text{ implies } (a_1 + 13d) - (a_1 + 2d) = a_1 + 13d - a_1 - 2d = 11d$$

$$\text{This allows us to find "d" } \quad -66 = 11d$$

$$-66/11 = 11d/11$$

$$-6 = d$$

We can now use either  $6 = a_1 + 2d$  or  $-60 = a_1 + 13d$  to find  $a_1$

$$6 = a_1 + 2(-6) \text{ leads to } 6 = a_1 - 12 \text{ or } a_1 = 6 - (-12) = 18$$

$$-60 = a_1 + 13(-6) \text{ leads to } -60 = a_1 - 78 \text{ or } a_1 = -60 - (-78) = 18$$

**so our arithmetic sequence is  $a_n = 18 + (n-1)(-6)$**

**and our finite sequence is**

$$\{ 18, 12, 6, 0, -6, -12, -18, -24, -30, -36, -42, -48, -54, -60 \}$$

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Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

	A	B	C
=			
1	given_term1	1000000	
2	term_in_seq1	7	
3	given_term2	4096	
4	term_in_seq2	13	
5			
6			
7			
8			
9			
10			
11			
	A1	given_term1	



$$a_9/a_4 \text{ implies } 4096/1000000 = \frac{64}{15625}$$

$$a_9/a_4 \text{ implies } a_1 \cdot r^{12} / a_1 \cdot r^6 = r^6$$

This allows us to find "r"  $r^6 = \frac{64}{15625}$  which leads to  $\sqrt[6]{r^6} = \sqrt[6]{\frac{64}{15625}}$

$$r = \frac{2}{5}$$

I would use the smaller sequence term to use  $1000000 = a_1 \cdot r^6$  to find  $a_1$

$$1000000 = a_1 \left(\frac{2}{5}\right)^6 \text{ which leads to } 1000000 = \frac{64}{15625} a_1$$

$$\text{which leads to } 1000000 \cdot \left(\frac{15625}{64}\right) = \left(\frac{15625}{64}\right) \left(\frac{64}{15625} a_1\right) \text{ or } a_1 = 244140625$$

so our geometric sequence is  $a_n = 244140625 \left(\frac{2}{5}\right)^{n-1}$

and our finite sequence is

$$\{ 244140625, 97656250, 39062500, 15625000, 6250000, 2500000, 1000000, 400000, 160000, 64000, 25600, 10240, 4096 \}$$