

## Writing a sequence from two of its terms (non sequential)

Step 1) What type of sequence is given?

When given arithmetic sequence terms

Step 2) Write the definition of each term using  $a_1$  and  $d$

Step 3) Solve the resulting linear system

Step 4) Write the arithmetic sequence using  $a_1$  and  $d$

Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

Problem 4

	A	B	C	D	E	F	G
=							
1	given_term1	63					
2	term_in_seq1	3					
3	given_term2	135					
4	term_in_seq2	7					
5							
6							
7							
8							
9							
10							
11							

B4 7

So we know  $a_3=63$  and  $a_7=135$  and we also know that this is an arithmetic sequence

so all terms must follow  $a_n=a_1+(n-1)d$ , including  $a_3$  and  $a_7$

$$\text{so } a_3=a_1+(3-1)d \text{ or } 63=a_1+2d$$

$$\text{so } a_7=a_1+(7-1)d \text{ or } 135=a_1+6d$$

This system can be solved a variety of ways, but basically you can subtract the higher sequence term and lower sequence term and it yields  $d$  and  $a_1$

	A	B
=		
1	a_1	unknown
2	a_2	unknown
3	a_3	63
4	a_4	unknown
5	a_5	unknown
6	a_6	unknown
7	a_7	135
8		
9		
10		
11		
	A1	a_1

$a_7 - a_3$  implies  $135 - 63 = 72$

$a_7 - a_3$  implies  $(a_1 + 6d) - (a_1 + 2d) = a_1 + 6d - a_1 - 2d = 4d$

This allows us to find "d"  $72 = 4d$

$$72/4 = 4d/4$$

$$18 = d$$

We can now use either  $63 = a_1 + 2d$  or  $135 = a_1 + 6d$  to find  $a_1$

$$63 = a_1 + 2(18) \text{ leads to } 63 = a_1 + 36 \text{ or } a_1 = 63 - 36 = 27$$

$$135 = a_1 + 6(18) \text{ leads to } 135 = a_1 + 108 \text{ or } a_1 = 135 - 108 = 27$$

**so our arithmetic sequence is  $a_n = 27 + (n-1)(18)$**

**and our finite sequence is  $\{27, 45, 63, 81, 99, 117, 135\}$**

Problem 5

Writing a sequence from two of its terms (non sequential)

Step 1) What type of sequence is given?

When given geometric sequence terms

Step 2) Write the definition of each term using  $a_1$  and  $r$

Step 3) Solve the resulting proportion

(I suggest putting higher sequence term (not value) on top) (this will require radical operations)

Step 4) Write the geometric sequence using  $a_1$  and  $r$

Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

	A	B
=		
1	given_term1	-7776
2	term_in_seq1	2
3	given_term2	1
4	term_in_seq2	7
5		
6		
7		
8		
9		
10		
11		
< >		
B2	2	

So we know  $a_2 = -7776$  and  $a_7 = 1$  and we also know that this is an geometric sequence

so all terms must follow  $a_n = a_1 (r)^{n-1}$ , including  $a_1$  and  $a_7$

$$\text{so } a_2 = a_1 r^{(2-1)} \text{ or } -7776 = a_1 \cdot r$$

$$\text{so } a_7 = a_1 r^{(7-1)} \text{ or } 1 = a_1 \cdot r^6$$

This system can be solved a variety of ways, but basically you can divide the higher sequence term and lower sequence term then apply necessary radical and it yields  $r$  and  $a_1$

	Y	Z seq_1
=		
1		46656
2		-7776
3		1296
4		-216
5		36
6		-6
7		1
8		
9		
10		
11		
Z1	=a_1	

$$a_7/a_2 \text{ implies } 1/-7776 = \frac{-1}{7776}$$

$$a_7/a_2 \text{ implies } a_1 \cdot r^6 / a_1 \cdot r = r^5$$

This allows us to find "r"  $r^5 = \frac{-1}{7776}$  which leads to  $\sqrt[5]{r^5} = \sqrt[5]{\frac{-1}{7776}}$

$$r = \frac{-1}{6}$$

I would use the smaller sequence term to use  $-7776 = a_1 \cdot r$  to find  $a_1$

$$-7776 = a_1 \left(\frac{-1}{6}\right)^1 \text{ which leads to } -7776 = \frac{-1}{6} a_1$$

$$\text{which leads to } -7776 \cdot (-6) = (-6) \left(\frac{-1}{6} a_1\right) \text{ or } a_1 = 46656$$

**so our geometric sequence is**  $a_n = 46656 \left(\frac{-1}{6}\right)^{(n-1)}$

**and our finite sequence is**  $\{46656, -7776, 1296, -216, 36, -6, 1\}$

Writing a sequence from two of its terms  
(non sequential)

Step 1) What type of sequence is given?

When given arithmetic sequence terms

Step 2) Write the definition of each term using  $a_1$  and  $d$

Step 3) Solve the resulting linear system

Step 4) Write the arithmetic sequence using  $a_1$  and  $d$

Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

	A	B
=		
1	given_term1	6
2	term_in_seq1	3
3	given_term2	-90
4	term_in_seq2	15
5		
6		
7		
8		
9		
10		
11		
B5		



So we know  $a_3 = 6$  and  $a_{15} = -90$  and we also know that this is an arithmetic sequence

so all terms must follow  $a_n = a_1 + (n-1)d$ , including  $a_3$  and  $a_{15}$

$$\text{so } a_3 = a_1 + (3-1)d \text{ or } 6 = a_1 + 2d$$

$$\text{so } a_{15} = a_1 + (15-1)d \text{ or } -90 = a_1 + 14d$$

This system can be solved a variety of ways, but basically you can subtract the higher sequence term and lower sequence term and it yields  $d$  and  $a_1$

	X	Y
=		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

Y15

$$a_{15} - a_3 \text{ implies } -90 - 6 = -96$$

$$a_{15} - a_3 \text{ implies } (a_1 + 14d) - (a_1 + 2d) = a_1 + 14d - a_1 - 2d = 12d$$

$$\text{This allows us to find "d" } \quad -96 = 12d$$

$$-96/12 = 12d/12$$

$$-8 = d$$

We can now use either  $6 = a_1 + 2d$  or  $-90 = a_1 + 14d$  to find  $a_1$

$$6 = a_1 + 2(-8) \text{ leads to } 6 = a_1 + -16 \text{ or } a_1 = 6 - -16 = 22$$

$$-90 = a_1 + 14(-8) \text{ leads to } -90 = a_1 + -112 \text{ or } a_1 = -90 - -112 = 22$$

**so our arithmetic sequence is  $a_n = 22 + (n-1)(-8)$**

**and our finite sequence is**

$$\{ 22, 14, 6, -2, -10, -18, -26, -34, -42, -50, -58, -66, -74, -82, -90 \}$$

Writing a sequence from two of its terms (non sequential)

Step 1) What type of sequence is given?

When given geometric sequence terms

Step 2) Write the definition of each term using  $a_1$  and  $r$

Step 3) Solve the resulting proportion

(I suggest putting higher sequence term (not value) on top)

(this will require radical operations)

Step 4) Write the geometric sequence using  $a_1$  and  $r$

Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

	A	B
=		
1	given_term1	759375
2	term_in_seq1	6
3	given_term2	59049
4	term_in_seq2	11
5		
6		
7		
8		
9		
10		
11		

B5

So we know  $a_6 = 759375$  and  $a_{11} = 59049$  and we also know that this is an geometric sequence

so all terms must follow  $a_n = a_1(r)^{n-1}$ , including  $a_6$  and  $a_{11}$

$$\text{so } a_6 = a_1 r^{(6-1)} \text{ or } 759375 = a_1 \cdot r^5$$

$$\text{so } a_{11} = a_1 r^{(11-1)} \text{ or } 59049 = a_1 \cdot r^{10}$$

This system can be solved a variety of ways, but basically you can divide the higher sequence term and lower sequence term then apply necessary radical and it yields  $r$  and  $a_1$

	Y	Z seq_1
=		
4		2109375
5		1265625
6		759375
7		455625
8		273375
9		164025
10		98415
11		59049
12		
13		
14		

Z11 = z10 · r\_given

Problem 7

$$a_{11}/a_6 \text{ implies } 59049/759375 = \frac{243}{3125}$$

$$a_{11}/a_6 \text{ implies } a_1 \cdot r^{10} / a_1 \cdot r^5 = r^5$$

This allows us to find "r"  $r^5 = \frac{243}{3125}$  which leads to  $\sqrt[5]{r^5} = \sqrt[5]{\frac{59049}{759375}}$

$$r = \frac{3}{5}$$

I would use the smaller sequence term to use  $759375 = a_1 \cdot r^5$  to find  $a_1$

$$759375 = a_1 \left(\frac{3}{5}\right)^5 \text{ which leads to } 759375 = \frac{243}{3125} a_1$$

$$\text{which leads to } 759375 \cdot \left(\frac{3125}{243}\right) = \left(\frac{3125}{243}\right) \left(\frac{243}{3125} a_1\right) \text{ or } a_1 = 9765625$$

so our geometric sequence is  $a_n = 9765625 \left(\frac{3}{5}\right)^{(n-1)}$

and our finite sequence is  $\{9765625, 5859375, 3515625, 2109375, 1265625, 759375, 455625, 273375, 164025, 98415, 59049\}$