Writing a sequence from two of its terms (non sequential)

Step 1) What type of sequence is given?

When given arthimetic sequence terms

Step 2) Write the definition of each term using $a_1$ and d

Step 3) Solve the resulting linear system

Step 4) Write the arithmetic sequence using $a_1$ and d

Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| = | | | | | | | |
| 1 | given_term1 | 63 | | | | | |
| 2 | term_in_seq1 | 3 | | | | | |
| 3 | given_term2 | 131 | | | | | |
| 4 | term_in_seq2 | 7 | | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | |
| 7 | | | | | | | |
| 8 | | | | | | | |
| 9 | | | | | | | |
| 10 | | | | | | | |
| 11 | | | | | | | |

A1 *given_term1*

So we know $a_3=63$ and $a_7=131$ and we also know that this is an arithmetic sequence

so all terms must follow $a_n=a_1+(n-1)d$, including $a_3$ and $a_7$

so $a_3=a_1+(3-1)\text{d}$ or $63=a_1+2\text{d}$

so $a_7=a_1+(7-1)\text{d}$ or $131=a_1+6\text{d}$

This system can be solved a variety of ways, but basically you can subtract the higher sequence term and lower sequence term and it yields d and $a_1$

| | A | B |
|---|---|---|
| 1 | a_1 | unknown |
| 2 | a_2 | unknown |
| 3 | a_3 | 63 |
| 4 | a_4 | unknown |
| 5 | a_5 | unknown |
| 6 | a_6 | unknown |
| 7 | a_7 | 131 |
| 8 | | |
| 9 | | |
| 10 | | |
| 11 | | |

A1 a_1

$a_7 - a_3$ implies $131 - 63 = 68$

$a_7 - a_3$ implies $(a_1 + 6d) - (a_1 + 2d) = a_1 + 6d - a_1 - 2d = 4d$

This allows us to find "d" $68 = 4d$

$68/4 = 4d/4$

$17 = \mathbf{d}$

We can now use either $63 = a_1 + 2d$ or $131 = a_1 + 6d$ to find $a_1$

$63 = a_1 + 2(17)$ leads to $63 = a_1 + 34$ or $a_1 = 63 - 34 = 29$

$131 = a_1 + 6(17)$ leads to $131 = a_1 + 102$ or $a_1 = 131 - 102 = 29$

**so our arithmetic sequence is** $a_n = 29 + (n-1)(17)$

**and our finite sequence is** $\{29, 46, 63, 80, 97, 114, 131\}$

Writing a sequence from two of its terms (non sequential)

Step 1) What type of sequence is given?

When given geometric sequence terms

Step 2) Write the definition of each term using $a_1$ and r

Step 3) Solve the resulting proportion

(I suggest putting higher sequence term (not value) on top)

(this will require radical operations)

Step 4) Write the geometric sequence using $a_1$ and r

Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

| | A | B | C |
|---|---|---|---|
| = | | | |
| 1 | given_term1 | ⁻32768 | |
| 2 | term_in_seq1 | 2 | |
| 3 | given_term2 | 1 | |
| 4 | term_in_seq2 | 7 | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |

A1 *given_term1*

So we know $a_2 = {}^{-}32768$ and $a_7 = 1$ and we also know that this is an geometic sequence

so all terms must follow $a_n = a_1 (r)^{n-1}$, including $a_1$ and $a_7$

so $a_2 = a_1$ r^(2 − 1) or $^{-}32768 = a_1 \cdot r$

so $a_7 = a_1$ r^(7 − 1) or $1 = a_1 \cdot r^6$

This system can be solved a variety of ways, but basically you can divide the higher sequence term and lower sequence term then apply necessary radical and it yields r and $a_1$

| | A | B | C |
|---|---|---|---|
| = | | | |
| 1 | a_2 | -32768 | |
| 2 | a_7 | 1 | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |

A3

$a_7/a_2$ implies $1/{}^{-}32768 = \frac{{}^{-}1}{32768}$

$a_7/a_2$ implies $a_1 \cdot r^6 / a_1 \cdot r = r^5$

This allows us to find "r" $\quad r^5 = \frac{{}^{-}1}{32768}$ which leads to $\sqrt[5]{r^5} = \sqrt[5]{\frac{{}^{-}1}{32768}}$

$$r = \frac{{}^{-}1}{8}$$

I would use the smaller sequence term to use ${}^{-}32768 = a_1 \cdot r$ to find $a_1$

${}^{-}32768 = a_1 \left(\frac{{}^{-}1}{8}\right)$^1 which leads to ${}^{-}32768 = \frac{{}^{-}1}{8} a_1$

which leads to ${}^{-}32768 \cdot ({}^{-}8) = ({}^{-}8)\left(\frac{{}^{-}1}{8} a_1\right)$ or $a_1 = 262144$

**so our geometric sequence is $a_n = 262144 \left(\frac{{}^{-}1}{8}\right)$^(n−1)**

**and our finite sequence is** $\{262144, {}^{-}32768, 4096, {}^{-}512, 64, {}^{-}8, 1\}$

Writing a sequence from two of its terms

(non sequential)

Step 1) What type of sequence is given?

When given arthimetic sequence terms

Step 2) Write the definition of each term using $a_1$ and d

Step 3) Solve the resulting linear system

Step 4) Write the arithmetic sequence using $a_1$ and d

Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

| | A | B | C |
|---|---|---|---|
| = | | | |
| 1 | given_term1 | -42 | |
| 2 | term_in_seq1 | 5 | |
| 3 | given_term2 | -212 | |
| 4 | term_in_seq2 | 15 | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |

A1 given_term1

So we know $a_5 = {}^{-}42$ and $a_{15} = {}^{-}212$ and we also know that this is an arithmetic sequence

so all terms must follow $a_n = a_1 + (n-1)d$, including $a_5$ and $a_{15}$

so $a_5 = a_1 + (5-1)\mathrm{d}$ or $\quad {}^{-}42 = a_1 + 4\mathrm{d}$

so $a_{15} = a_1 + (15-1)\mathrm{d}$ or ${}^{-}212 = a_1 + 14\mathrm{d}$

This system can be solved a variety of ways, but basically you can subtract the higher sequence term and lower sequence term and it yields d and $a_1$

| | A | B | C |
|---|---|---|---|
| = | | | |
| 1 | a_5 | ⁻42 | |
| 2 | a_15 | ⁻212 | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |

A1 $a\_5$

$a_{15}-a_5$ implies $^-212 - ^-42 = ^-170$

$a_{15}-a_5$ implies $(a_1+14\,d)-(a_1+4\,d) = a_1+14\,d-a_1-4\,d=10\,d$

This allows us to find "d" $^-170 = 10\,d$

$^-170/10 = 10\,d/10$

$^-17 = \mathbf{d}$

We can now use either $^-42 = a_1+4\,d$ or $^-212 = a_1+14\,d$ to find $a_1$

$^-42 = a_1+4(^-17)$ leads to $^-42 = a_1 + ^-68$ or $a_1 = ^-42 - ^-68 = 26$

$^-212 = a_1+14(^-17)$ leads to $^-212 = a_1 + ^-238$ or $a_1 = ^-212 - ^-238 = 26$

**so our arithmetic sequence is $a_n = 26 + (n-1)(^-17)$**

**and our finite sequence is**

$\{26, 9, ^-8, ^-25, ^-42, ^-59, ^-76, ^-93, ^-110, ^-127, ^-144, ^-161, ^-178, ^-195, ^-212\}$

Writing a sequence from two of its terms (non sequential)

Step 1) What type of sequence is given?

When given geometric sequence terms

Step 2) Write the definition of each term using $a_1$ and r

Step 3) Solve the resulting proportion

(I suggest putting higher sequence term (not value) on top)

(this will require radical operations)

Step 4) Write the geometric sequence using $a_1$ and r

Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

| | A | B | C |
|---|---|---|---|
| = | | | |
| 1 | given_term1 | 200000 | |
| 2 | term_in_seq1 | 4 | |
| 3 | given_term2 | 65536 | |
| 4 | term_in_seq2 | 9 | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |

A1 *given_term1*

## Problem 7

So we know $a_4$=200000 and $a_9$=65536 and we also know that this is an geometic sequence

so all terms must follow $a_n = a_1(r)^{n-1}$ , including $a_4$ and $a_9$

so $a_4 = a_1$ r^(4−1) or $200000 = a_1 \cdot r^3$

so $a_9 = a_1$ r^(9−1) or $65536 = a_1 \cdot r^8$

This system can be solved a variety of ways, but basically you can divide the higher sequence term and lower sequence term then apply necessary radical and it yields r and $a_1$

| | A | B | C |
|---|---|---|---|
| = | | | |
| 1 | a_2 | 200000 | |
| 2 | a_7 | 65536 | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| 11 | | | |

A1 *a_2*

$a_9/a_4$ implies $65536/200000 = \frac{1024}{3125}$

$a_9/a_4$ implies $a_1 \cdot r^8 / a_1 \cdot r^3 = r^5$

This allows us to find "r" $r^5 = \frac{1024}{3125}$ which leads to $\sqrt[5]{r^5} = \sqrt[5]{\frac{1024}{3125}}$

$r = \frac{4}{5}$

I would use the smaller sequence term to use $200000 = a_1 \cdot r^3$ to find $a_1$

$200000 = a_1 (\frac{4}{5})$^3 which leads to $200000 = \frac{64}{125} a_1$

which leads to $200000 \cdot (\frac{125}{64}) = (\frac{125}{64})(\frac{64}{125} a_1)$ or $a_1 = 390625$

**so our geometric sequence is $a_n = 390625 (\frac{4}{5})$^(n−1)**

**and our finite sequence is** {390625,312500,250000,200000,160000,128000,102400,81920,65536 }