

Problem 1

	A	B	C	D	E	F	G
=							
1	seq_num	value	difference	diff_diff			
2	1	12	5	2			
3	2	17	7	2			
4	3	24	9	2			
5	4	33	11				
6	5	44					
7							
8							
9							
10							
11							

B7

$\{12, 17, 24, 33, 44\}$ 

this sequence is NOT arithmetic nor geometric

this sequence can be written as a recursive pattern

The pattern changes after the second term to an arithmetic pattern

THE PATTERN

So we need to get a sequence to add 5 to its first term and then only and then after that only add 2 to each successive term

$a_1 = 12$  (this is always just stated in a recursive sequence)

$$a_2 = 17 = 12 + [5 + 0(2)]$$

$$a_3 = 24 = 17 + [5 + 1(2)]$$

$$a_4 = 33 = 24 + [5 + 2(2)]$$

$$a_5 = 44 = 33 + [5 + 3(2)]$$

$a_1 = 12$  (this is always just stated in a recursive sequence)

**notice that the highlighted counters are off by 2**

$$a_2 = 17 = 12 + [5 + 0(2)]$$

$$a_3 = 24 = 17 + [5 + 1(2)]$$

$$a_4 = 33 = 24 + [5 + 2(2)]$$

$$a_5 = 44 = 33 + [5 + 3(2)]$$

$a_1 = 12$  (this is always just stated in a recursive sequence)

Since the highlighted counters are off by 2,

lets make that adjustment by subtracting 2 from sequence number,  $n$ .

$$a_2 = 17 = 12 + [5 + (n-2)(2)]$$

$$a_3 = 24 = 17 + [5 + (n-2)(2)]$$

$$a_4 = 33 = 24 + [5 + (n-2)(2)]$$

$$a_5 = 44 = 33 + [5 + (n-2)(2)]$$

$a_1 = 12$  (this is always just stated in a recursive sequence)

**Use the pattern to find the next two terms**

$$a_2 = 17 = 12 + [5 + (2-2)(2)]$$

$$a_3 = 24 = 17 + [5 + (3-2)(2)]$$

$$a_4 = 33 = 24 + [5 + (4-2)(2)]$$

$$a_5 = 44 = 33 + [5 + (5-2)(2)]$$

$$a_6 = 57 = 44 + [5 + (6-2)(2)]$$

$$a_7 = 72 = 57 + [5 + (7-2)(2)]$$

So now we can write a rule for the sequence  $\{12,17,24,33,44\}$

$$a_1 = 12$$

$$a_n = a_{n-1} + [5 + (n-2)(2)]$$

Now the second part of the rule has an EXPLICIT formula that all versions of this rule must eventually become

$$a_n = a_{n-1} + 2 \cdot n + 1$$

$$\left\{ 1024, 128, 16, 2, \frac{1}{4}, \frac{1}{32}, \frac{1}{256}, \frac{1}{2048}, \frac{1}{16384} \right\}$$

Writing a sequence from the first couple of terms

Step 1) What type of sequence is given?

Look at how the terms change from each term

Can you show it has a common difference?

OR

Can you show it has a common ratio?

OR

Do you need to use the previous term to reach the next term?

When given geometric sequence terms

Step 2) You already know  $a_1$  Find  $r = \frac{a_{n+1}}{a_n}$

Step 3) Write the geometric sequence using  $a_1$  and  $r$

	A	B
=		
1	given_term1	1024
2	term_in_seq1	1
3	given_term2	128
4	term_in_seq2	2
5		
6		
7		
8		
9		
10		
11		
B4		2

So we know  $a_1 = 1024$  and  $a_2 = 128$  and we also can show that this is an geometric sequence

so all terms must follow  $a_n = a_1(r)^{n-1}$

	A	B
=		=seq_1
1		1024
2		128
3		16
4		2
5		1/4
6		1/32
7		1/256
8		1/2048
9		1/16384
10		
11		

AI



We were given  $a_1 = 1024$

$$r = a_2/a_1 \text{ implies } 128/1024 = \frac{1}{8}$$

so our geometric sequence is  $a_n = 1024 \left(\frac{1}{8}\right)^{(n-1)}$

and our finite sequence is  $\left\{ 1024, 128, 16, 2, \frac{1}{4}, \frac{1}{32}, \frac{1}{256}, \frac{1}{2048}, \frac{1}{16384} \right\}$

Problem 3

$$\left\{ \frac{2}{9}, \frac{-2}{81}, \frac{2}{729}, \frac{-2}{6561}, \frac{2}{59049}, \frac{-2}{531441}, \frac{2}{4782969}, \frac{-2}{43046721}, \frac{2}{387420489} \right\}$$

Writing a sequence from the first couple of terms

Step 1) What type of sequence is given?

Look at how the terms change from each term

Can you show it has a common difference?

OR

Can you show it has a common ratio?

OR

Do you need to use the previous term to reach the next term?

When given geometric sequence terms

Step 2) You already know  $a_1$  Find  $r = \frac{a_{n+1}}{a_n}$

Step 3) Write the geometric sequence using  $a_1$  and  $r$

	A	B
=		
1	given_term1	2/9
2	term_in_seq1	1
3	given_term2	-2/81
4	term_in_seq2	2
5		
6		
7		
8		
9		
10		
11		
B4		2

So we know  $a_1 = \frac{2}{9}$  and  $a_2 = \frac{-2}{81}$  and we also can

show that this is an geometric sequence

so all terms must follow  $a_n = a_1(r)^{n-1}$

	A	B
=		=seq_1
1		2/9
2		-2/81
3		2/729
4		-2/6561
5		2/59049
6		-2/5314...
7		2/47829...
8		-2/4304...
9		2/38742...
10		
11		

AI

We were given  $a_1 = \frac{2}{9}$

$r = a_2/a_1$  implies  $\frac{-2}{81} / \frac{2}{9} = \frac{-1}{9}$

so our geometric sequence is  $a_n = \frac{2}{9} \left(\frac{-1}{9}\right)^{(n-1)}$

and our finite sequence is

$$\left\{ \frac{2}{9}, \frac{-2}{81}, \frac{2}{729}, \frac{-2}{6561}, \frac{2}{59049}, \frac{-2}{531441}, \frac{2}{4782969}, \frac{-2}{43046721}, \frac{2}{387420489} \right\}$$

Problem 4

{ 70,59,48,37,26,15,4 }

Writing a sequence from the first couple of terms

Step 1) What type of sequence is given?

Look at how the terms change from each term

Can you show it has a common difference?

OR

Can you show it has a common ratio?

OR

Do you need to use the previous term to reach the next term?

When given arithmetic sequence terms

Step 2) You already know  $a_1$  Find  $d =$

$$a_{n-1} - a_n$$

Step 3) Write the arithmetic sequence using

$a_1$  and  $d$

	A	B	C
=			
1	given_term1	70	
2	term_in_seq1	1	
3	given_term2	59	
4	term_in_seq2	2	
5			
6			
7			
8			
9			
10			
11			
<div style="border: 1px solid black; padding: 2px; display: inline-block;"> <span style="font-family: monospace;">B4</span> 2         </div>			

So we know  $a_1 = \text{given\_1}$  and  $a_2 = \text{given\_2}$  and we can show that this is an arithmetic sequence, by proving that there is a common difference so all terms must follow  $a_n = a_1 + (n-1)d$ ,

	A	B
=		=seq_1
1	a_1	70
2	a_2	59
3	a_3	48
4	a_4	37
5	a_5	26
6	a_6	15
7	a_7	4
8		
9		
10		
11		
	A1	a_1

We actually know  $a_1 = 70$

$d = a_2 - a_1$  implies  $59 - 70 = -11$

so our arithmetic sequence is  $a_n = 70 + (n-1)(-11)$   
 $= 70 + 11 - 11 \cdot n$

$$a_n = 81 - 11 \cdot n$$

and our finite sequence is seq\_1

To find  $a_{100}$ , simply replace  $n = 100$  into sequence

$$a_{100} = 70 + (100-1)(-11)$$

$$= a_1 + (99)(-11)$$

$$= a_1 + -99$$

$$= -29$$

Problem 5

	A	B	C	D	E	F	G
1	seq_num	value	ratio	diff_ratio			
2	1	50	4	1			
3	2	200	5	1			
4	3	1000	6	1			
5	4	6000	7				
6	5	42000					
7							
8							
9							
10							
11							

B7



$$\{50, 200, 1000, 6000, 42000\}$$

this sequence is NOT arithmetic nor geometric

this sequence can be written as a recursive pattern

The pattern changes after the second term to an arithmetic pattern

THE PATTERN

So we need to get a sequence to multiply by 4 to its first term and then only and then after that multiply by 1 more than the previous term to each successive term

$a_1 = 50$  (this is always just stated in a recursive sequence)

$$a_2 = 200 = 50 [4 + 0(1)]$$

$$a_3 = 1000 = 200 [4 + 1(1)]$$

$$a_4 = 6000 = 1000 [4 + 2(1)]$$

$$a_5 = 42000 = 6000 [4 + 3(1)]$$

$a_1 = 50$  (this is always just stated in a recursive sequence)

**notice that the highlighted counters are off by 2**

$$a_2 = 200 = 50 [4 + 0(1)]$$

$$a_3 = 1000 = 200 [4 + 1(1)]$$

$$a_4 = 6000 = 1000 [4 + 2(1)]$$

$$a_5 = 42000 = 6000 [4 + 3(1)]$$

$a_1 = 50$  (this is always just stated in a recursive sequence)

Since the highlighted counters are off by 2,  
lets make that adjustment by subtracting 2 from sequence  
number, n.

$$a_2 = 200 = 50 [4 + (n-2)(1)]$$

$$a_3 = 1000 = 200 [4 + (n-2)(1)]$$

$$a_4 = 6000 = 1000 [4 + (n-2)(1)]$$

$$a_5 = 42000 = 6000 [4 + (n-2)(1)]$$

$a_1 = 50$  (this is always just stated in a recursive sequence)

**Use the pattern to find the next two terms**

$$a_2 = 200 = 50 + [4 + (2-2)(1)]$$

$$a_3 = 1000 = 200 + [4 + (3-2)(1)]$$

$$a_4 = 6000 = 1000 + [4 + (4-2)(1)]$$

$$a_5 = 42000 = 6000 + [4 + (5-2)(1)]$$

$$a_6 = 336000 = 42000 + [4 + (6-2)(1)]$$

$$a_7 = 3024000 = 336000 + [4 + (7-2)(1)]$$

So now we can write a rule for the sequence

$$\{50, 200, 1000, 6000, 42000\}$$

$$a_1 = 50$$

$$a_n = a_{n-1} [4 + (n-2)(1)]$$

Now the second part of the rule has an EXPLICIT formula that all versions of this rule must eventually become

$$a_n = a_{n-1} [n+2]$$