

Writing a sequence from two of its terms (non sequential)

Step 1) What type of sequence is given?

When given arithmetic sequence terms

Step 2) Write the definition of each term using a_1 and d

Step 3) Solve the resulting linear system

Step 4) Write the arithmetic sequence using a_1 and d

Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

Problem 6

	A	B	C	D	E	F	G
=							
1	given_term1	128					
2	term_in_seq1	3					
3	given_term2	210					
4	term_in_seq2	7					
5							
6							
7							
8							
9							
10							
11							

B4 7

So we know $a_3 = 128$ and $a_7 = 210$ and we also know that this is an arithmetic sequence

so all terms must follow $a_n = a_1 + (n-1)d$, including a_3 and a_7

$$\text{so } a_3 = a_1 + (3-1)d \text{ or } 128 = a_1 + 2d$$

$$\text{so } a_7 = a_1 + (7-1)d \text{ or } 210 = a_1 + 6d$$

This system can be solved a variety of ways, but basically you can subtract the higher sequence term and lower sequence term and it yields d and a_1

	A	B
=		
1	a_1	unknown
2	a_2	unknown
3	a_3	128
4	a_4	unknown
5	a_5	unknown
6	a_6	unknown
7	a_7	210
8		
9		
10		
11		
A1		a_1

$$a_7 - a_3 \text{ implies } 210 - 128 = 82$$

$$a_7 - a_3 \text{ implies } (a_1 + 6d) - (a_1 + 2d) = a_1 + 6d - a_1 - 2d = 4d$$

$$\text{This allows us to find "d" } \quad 82 = 4d$$

$$82/4 = 4d/4$$

$$\frac{41}{2} = d$$

We can now use either $128 = a_1 + 2d$ or $210 = a_1 + 6d$ to find a_1

$$128 = a_1 + 2\left(\frac{41}{2}\right) \text{ leads to } 128 = a_1 + 41 \text{ or } a_1 = 128 - 41 = 87$$

$$210 = a_1 + 6\left(\frac{41}{2}\right) \text{ leads to } 210 = a_1 + 123 \text{ or } a_1 = 210 - 123 = 87$$

so our arithmetic sequence is $a_n = 87 + (n-1)\left(\frac{41}{2}\right)$

and our finite sequence is $\left\{ 87, \frac{215}{2}, 128, \frac{297}{2}, 169, \frac{379}{2}, 210 \right\}$

Writing a sequence from two of its terms (non sequential)

Step 1) What type of sequence is given?

When given geometric sequence terms

Step 2) Write the definition of each term using a_1 and r

Step 3) Solve the resulting proportion

(I suggest putting higher sequence term (not value) on top)

(this will require radical operations)

Step 4) Write the geometric sequence using a_1 and r

Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

	A	B
=		
1	given_term1	-500
2	term_in_seq1	2
3	given_term2	128/25
4	term_in_seq2	7
5		
6		
7		
8		
9		
10		
11		
B4		7

So we know $a_2 = -500$ and $a_7 = \frac{128}{25}$ and we also know that this is an geometric sequence

so all terms must follow $a_n = a_1(r)^{n-1}$, including a_1 and a_7

$$\text{so } a_2 = a_1 r^{(2-1)} \text{ or } -500 = a_1 \cdot r$$

$$\text{so } a_7 = a_1 r^{(7-1)} \text{ or } \frac{128}{25} = a_1 \cdot r^6$$

This system can be solved a variety of ways, but basically you can divide the higher sequence term and lower sequence term then apply necessary radical and it yields r and a_1

	A	B
=		
1	a_2	-500
2	a_7	128/25
3		
4		
5		
6		
7		
8		
9		
10		
11		
B2		=given_2

$$a_7/a_2 \text{ implies } \frac{128}{25} / -500 = \frac{-32}{3125}$$

$$a_7/a_2 \text{ implies } a_1 \cdot r^6 / a_1 \cdot r = r^5$$

This allows us to find "r" $r^5 = \frac{-32}{3125}$ which leads to $\sqrt[5]{r^5} = \sqrt[5]{\frac{-32}{3125}}$ leads to $r = \frac{-2}{5}$

I would use the smaller sequence term to use $-500 = a_1 \cdot r$ to find a_1

$$-500 = a_1 \left(\frac{-2}{5}\right)^1 \text{ which leads to } -500 = \frac{-2}{5} a_1$$

$$\text{which leads to } -500 \cdot \left(\frac{-5}{2}\right) = \left(\frac{-5}{2}\right) \left(\frac{-2}{5} a_1\right) \text{ or } a_1 = 1250$$

so our geometric sequence is $a_n = 1250 \left(\frac{-2}{5}\right)^{(n-1)}$

and our finite sequence is $\left\{ 1250, -500, 200, -80, 32, \frac{-64}{5}, \frac{128}{25} \right\}$

Writing a sequence from two of its terms

(non sequential)

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	A	B
=		
1	given_term1	64
2	term_in_seq1	3
3	given_term2	-20
4	term_in_seq2	17
5		
6		
7		
8		
9		
10		
11		

A8

So we know $a_5 = 64$ and $a_{15} = -20$ and we also know that this is an arithmetic sequence

so all terms must follow $a_n = a_1 + (n-1)d$, including a_3 and a_{14}

$$\text{so } a_3 = a_1 + (3-1)d \text{ or } 64 = a_1 + 2d$$

$$\text{so } a_{14} = a_1 + (17-1)d \text{ or } -20 = a_1 + 16d$$

This system can be solved a variety of ways, but basically you can subtract the higher sequence term and lower sequence term and it yields d and a_1

	A	B
=		
1	a_5	64
2	a_15	-84
3		
4		
5		
6		
7		
8		
9		
10		
11		
B2		-84

$$a_{14} - a_3 \text{ implies } -20 - 64 = -84$$

$$a_{14} - a_3 \text{ implies } (a_1 + 16d) - (a_1 + 2d) = a_1 + 16d - a_1 - 2d = 14d$$

$$\text{This allows us to find "d" } \quad -84 = 14d$$

$$-84/14 = 14d/14$$

$$-6 = d$$

We can now use either $64 = a_1 + 2d$ or $-20 = a_1 + 16d$ to find a_1

$$64 = a_1 + 2(-6) \text{ leads to } 64 = a_1 - 12 \text{ or } a_1 = 64 - (-12) = 76$$

$$-20 = a_1 + 16(-6) \text{ leads to } -20 = a_1 - 96 \text{ or } a_1 = -20 - (-96) = 76$$

so our arithmetic sequence is $a_n = 76 + (n-1)(-6)$

and our finite sequence is

$$\{ 76, 70, 64, 58, 52, 46, 40, 34, 28, 22, 16, 10, 4, -2 \}$$

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Step 4) Write the geometric sequence using a_1 and r

Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

	A	B
=		
1	given_term1	200
2	term_in_seq1	7
3	given_term2	6075/128
4	term_in_seq2	12
5		
6		
7		
8		
9		
10		
11		
B4		12

So we know $a_7 = 200$ and $a_{13} = \frac{6075}{128}$ and we also

know that this is an geometric sequence

so all terms must follow $a_n = a_1(r)^{n-1}$, including a_7 and a_{13}

$$\text{so } a_7 = a_1 r^{(7-1)} \text{ or } 200 = a_1 \cdot r^6$$

$$\text{so } a_{13} = a_1 r^{(12-1)} \text{ or } \frac{6075}{128} = a_1 \cdot r^{11}$$

This system can be solved a variety of ways, but basically you can divide the higher sequence term and lower sequence term then apply necessary radical and it yields r and a_1

	A	B
=		
1	a_2	200
2	a_7	6075/128
3		
4		
5		
6		
7		
8		
9		
10		
11		
A1	a_2	

Problem 9

$$a_9/a_4 \text{ implies } \frac{6075}{128} / 200 = \frac{243}{1024}$$

$$a_9/a_4 \text{ implies } a_1 \cdot r^{11} / a_1 \cdot r^6 = r^5$$

This allows us to find "r" $r^5 = \frac{243}{1024}$ which leads to $\sqrt[5]{r^5} = \sqrt[5]{\frac{243}{1024}}$

$$r = \frac{3}{4}$$

I would use the smaller sequence term to use $200 = a_1 \cdot r^6$ to find a_1

$$200 = a_1 \left(\frac{3}{4}\right)^6 \text{ which leads to } 200 = \frac{729}{4096} a_1$$

$$\text{which leads to } 200 \cdot \left(\frac{4096}{729}\right) = \left(\frac{4096}{729}\right) \left(\frac{729}{4096} a_1\right) \text{ or } a_1 = \frac{819200}{729}$$

so our geometric sequence is $a_n = \frac{819200}{729} \left(\frac{3}{4}\right)^{n-1}$

and our finite sequence is $\left\{ \frac{819200}{729}, \frac{204800}{243}, \frac{51200}{81}, \frac{12800}{27}, \frac{3200}{9}, \frac{800}{3}, 200, 150, \frac{225}{2}, \frac{675}{8}, \frac{2025}{32}, \frac{6075}{128}, \frac{18225}{512} \right\}$