

Problem 1

A	
=	
1	a_1
2	d
3	n_1
4	n_2
5	
6	
7	
8	
9	
10	
11	

S Method  $a_n = 275 + (n-1)(-9.5)$

$a_{1000} = 275 + (1000-1)(-9.5)$   
 $= -9215.5$

$S_{1000} = (1000/2)[275 + -9215.5]$   
 $= -4470250.$   
 $= -4.47025E6$  (scientific notation)

**infinite sum** `inf_sum` ▶  $-\infty$

---


$$\sum_{n=1}^{1000} 275 + (n-1)(-9.5) = -4.47025E6$$

A1 a\_1

Problem 2

A		B
=		
1	a_1	
2	d	
3	n_1	
4	n_2	
5		
6		
7		
8		
9		
10		
11		

S Method  $a_n = 926 + (n-1)(11.4)$

$a_{75} = 926 + (75-1)(11.4)$   
 $= 1769.6$

$S_{75} = (75/2)[926 + 1769.6]$   
 $= 101085.$

**infinite sum**  $\infty$

---


$$\sum_{n=1}^{75} 926 + (n-1)(11.4) = 101085.$$

A1 a\_1

Problem 3

A	B
=	
1 a_1	
2	
3 n_1	
4 n_2	
5	
6	
7	
8	
9	
10	
11	

$$S_{37} = \frac{1 - \left(\frac{8}{9}\right)^{37}}{1 - \frac{8}{9}} = \frac{4003188949503703117845921654166874420}{22528399544939174411840147874772641} \approx 177.6952$$

$$S_8 = \frac{1 - \left(\frac{8}{9}\right)^8}{1 - \frac{8}{9}} = \frac{525390100}{4782969} \approx 109.846$$

$$S_{37} - S_8 = \frac{1528533962244628819401976799842795520}{22528399544939174411840147874772641} \approx 67.8492$$

$$n_1 \sum_{n=9}^{37} 20 \left(\frac{8}{9}\right)^{n-1} = \frac{1528533962244628819401976799842795520}{22528399544939174411840147874772641} \approx 67.8492$$

Problem 4

A	B
=	
1 a_1	
2 d	
3 n_1	
4 n_2	
5	
6	
7	
8	
9	
10	
11	

S Method  $a_n = 542 + (n-1)(-7.2)$

$a_{55} = 542 + (55-1)(-7.2) = 153.2$

$S_{55} = (55/2)[542 + 153.2] = 19118.$

$a_{26} = 542 + (26-1)(-7.2) = 362.$

$S_{26} = (26/2)[542 + 362.] = 11752.$

$S_{55} - S_{26} = 19118. - 11752. = 7366.$

Infinite Sum =  $-\infty$

$$\sum_{n=27}^{55} 542 + (n-1)(-7.2) = 7366.$$

Problem 5

A	B
=	
1	a_1
2	d
3	n_1
4	n_2
5	
6	
7	
8	
9	
10	
11	
A1	a_1

S Method  $a_n = 6000 + (n-1)(-5)$

$$a_{17} = 6000 + (17-1)(-5)$$

$$= 5920$$

$$S_{17} = (17/2)[6000 + 5920]$$

$$= 101320$$

**infinite sum**  $-\infty$

$$\sum_{n=1}^{17} 6000 + (n-1)(-5) = 101320$$

Problem 6

A	B
=	
1	a_1
2	r
3	n_1
4	n_2
5	
6	
7	
8	
9	
10	
11	
A1	a_1

$$a_n = 50 \left(\frac{5}{6}\right)^{n-1}$$

$$S_9 = \frac{1 - \left(\frac{5}{6}\right)^9}{1 - \frac{5}{6}} = \frac{203114275}{839808} \approx 241.858$$

$$\sum_{n=1}^9 50 \left(\frac{5}{6}\right)^{n-1} = \frac{203114275}{839808} \approx 241.858$$

*infinite sum* 300

Problem 7

A	B
=	
1 a_1	
2	
3 n_1	
4 n_2	
5	
6	
7	
8	
9	
10	
11	

$$a_n = 40 \left(\frac{-5}{4}\right)^{n-1}$$

$$S_{25} = \frac{1 - \left(\frac{-5}{4}\right)^{25}}{1 - \frac{-5}{4}} = \frac{166193957657664305}{3518437208832} \approx 4723.5164$$

$$\sum_{n=1}^{25} 40 \left(\frac{-5}{4}\right)^{n-1} = \frac{166193957657664305}{3518437208832} \approx 4723.5164$$

*infinite sum impossible to determine*

Problem 8

A	B	C	D	E	F	G	H
=							
1 a_1		71 differenc...		15 ratio		86/71	

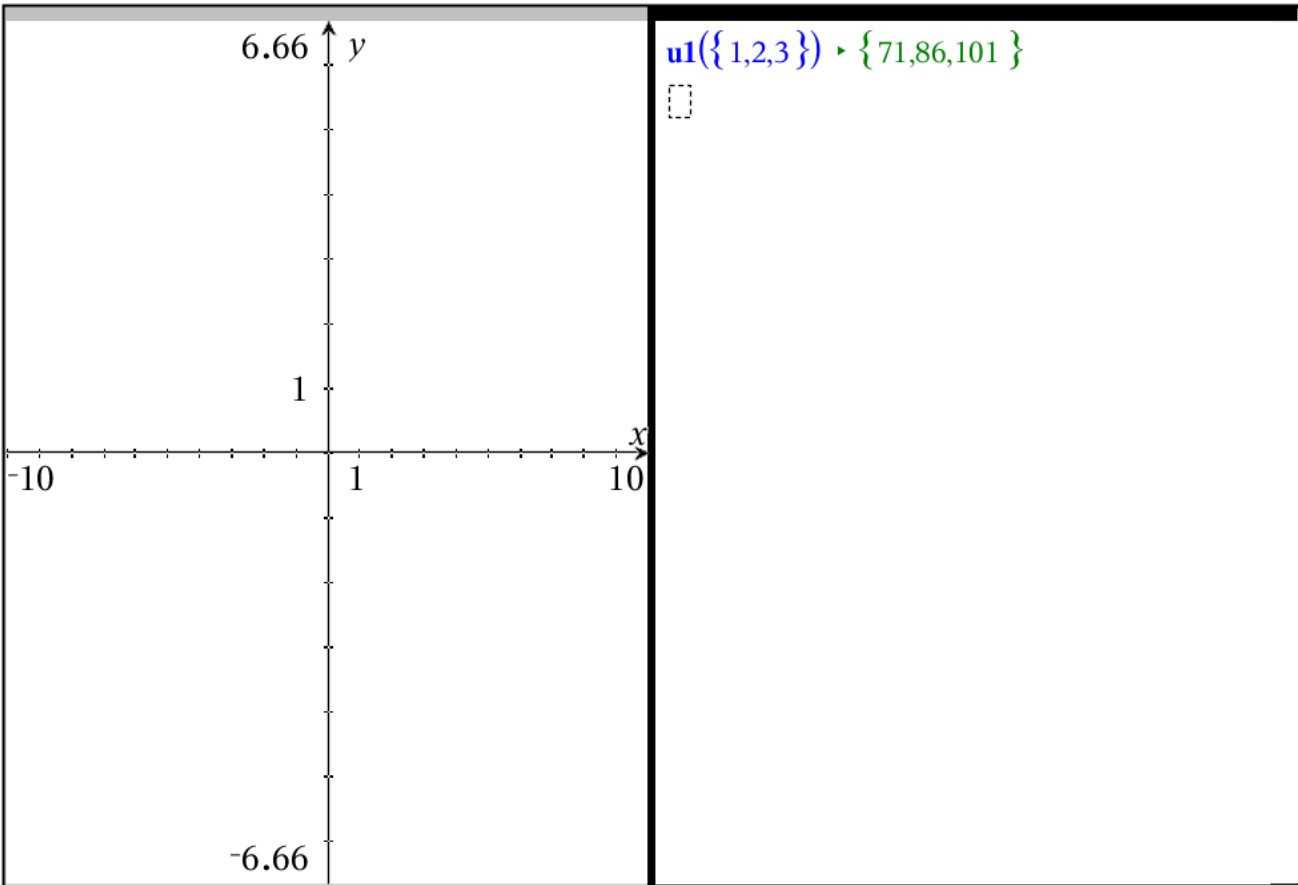
{71,86,116 }

This is a recursive pattern The difference in 86 and 71 is 15  
 The difference in 116 and 86 is 30

So a pattern for the change can be written as: Add 15  
 Add 15 more than the last time to each term

a <sub>1</sub> = 71	a <sub>2</sub> = 116 + (15 + (4-2)(15)) = 116 + 45 = 161
a <sub>n</sub> = a <sub>n-1</sub> + (15 + (n-2)(15))	a <sub>3</sub> = 161 + (15 + (5-2)(15)) = 161 + 60 = 221
	a <sub>4</sub> = 221 + (15 + (6-2)(15)) = 221 + 90 = 311
a <sub>1</sub> = 71	a <sub>5</sub> = 311 + (15 + (7-2)(15)) = 311 + 105 = 416
a <sub>n</sub> = a <sub>n-1</sub> + (15 + (n-1)(15))	a <sub>6</sub> = 416 + (15 + (8-2)(15)) = 416 + 135 = 551

This is a "fancier" version a<sub>1</sub> = 71 a<sub>n</sub> = 71 +  $\sum_{k=1}^{n-1} (15 + (k-1)(15))$



Problem 9

# TYPO

Problem 10

	A	B	C	D	E	F	G	H
=								
1	a_1	4050000	differenc...	-3105000	ratio		7/30	

AI a\_1

{ 4050000,945000,220500 }

This is a geometric pattern The ratio is 945000 divided by 4050000 is  $\frac{7}{30}$

The ratio is 220500 divided by 945000 is  $\frac{7}{30}$

$a_1 = 4050000$   $r = \frac{7}{30}$   $a_n = 4050000 \left(\frac{7}{30}\right)^{n-1}$

$a_4 = 4050000 \left(\frac{7}{30}\right)^{4-1} = 51450$   $a_5 = 4050000 \left(\frac{7}{30}\right)^{5-1} = 12005$   $a_6 = 4050000 \left(\frac{7}{30}\right)^{6-1} = \frac{16807}{6} = 2801.16666667$

$a_9 = 4050000 \left(\frac{7}{30}\right)^{9-1} = \frac{5764801}{162000} = 35.585191358$

Problem 11

	A	B	C	D	E	F	G	H
=								
1	a_1	92	ratio	31	diff	6		

AI a\_1

{ 92,2852,105524,4537532 }

This is a recursive pattern The ratio between 2852 and 92 is 31 The ratio between 105524 and 2852 is 37  
The ratio between 4537532 and 105524 is 43

So a pattern for the change can be written as: multiply by 31  
Add 6 more to the multiplier than the last time to each term

$a_1 = 92$   $a_5 = 4537532(31 + (5-2)(6)) = 4537532 [49] = 222339068$

$a_n = a_{n-1}(31 + (n-2)(6))$   $a_6 = 222339068(31 + (6-2)(6)) = 222339068 + 55 = 12228648740$   
 $a_7 = 12228648740 + (31 + (7-2)(6)) = 12228648740 + 61 = 745947573140$

$a_1 \rightarrow 92$   $a_8 = 745947573140 + (31 + (8-2)(6)) = 745947573140 + 67 = 49978487400380$

$a_n = a_{n-1}(6 \cdot n + 19)$   $a_9 = 49978487400380 + (31 + (9-2)(6)) = 49978487400380 + 73 = 3648429580227740$

This is a "fancier" version  $a_1 = 92$   $a_n = 92 + \prod_{k=1}^{n-1} (31 + (k-1)(6))$

Problem 12

	A	B	C	D	E	F	G	H
	=							
1	a_1	1536	differenc...	8448	ratio	13/2		

AI a\_1

{ 1536,9984,64896 }

This is a geometric pattern The ratio is 9984 divided by 1536 is  $\frac{13}{2}$

The ratio is 64896 divided by 9984 is  $\frac{a_3}{a_2} = \frac{13}{2}$

$a_1 = 1536$   $r = \frac{13}{2}$   $a_n = 1536 \left(\frac{13}{2}\right)^{n-1}$

$a_4 = 1536 \left(\frac{13}{2}\right)^{4-1} = 421824$   $a_5 = 1536 \left(\frac{13}{2}\right)^{5-1} = 2741856$   $a_6 = 1536 \left(\frac{13}{2}\right)^{6-1} = 17822064$

$a_{12} = 1536 \left(\frac{13}{2}\right)^{12-1} = \frac{5376481182111}{4} = 1.34412029553E12$

	A	B	C	D	E	F	G	H
	=							
1	1536							
2	9984							
3	64896							
4								
5								
6								
7								
8								
9								
10								
11								

AI 1536

Problem 13

Writing a sequence from two of its terms  
(non sequential)

Step 1) What type of sequence is given?

When given arithmetic sequence terms

Step 2) Write the definition of each term using  $a_1$  and  $d$

Step 3) Solve the resulting linear system

Step 4) Write the arithmetic sequence using  $a_1$  and  $d$

Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

	A	B
=		
1	given_term1	
2	term_in_seq1	
3	given_term2	
4	term_in_seq2	
AI given_term1		

So we know  $a_{13}=480$  and  $a_{17}=-36$  and we also know that this is an arithmetic sequence

so all terms must follow  $a_n=a_1+(n-1)d$ , including  $a_{13}$  and  $a_{17}$

so  $a_{13}=a_1+(13-1)d$  or  $480=a_1+12d$

so  $a_{17}=a_1+(17-1)d$  or  $-36=a_1+16d$

This system can be solved a variety of ways, but basically you can subtract the higher sequence term and lower sequence term and it yields  $d$  and  $a_1$

	A	B	C
=			
1	a_5	480	
2	a_15	-36	
3			
4			
5			
6			
7			
8			
9			
10			
11			
AI a_5			



$$a_{17} - a_{13} \text{ implies } -36 - 480 = -516$$

$$a_{17} - a_{13} \text{ implies } (a_1 + 16d) - (a_1 + 12d) = a_1 + 16d - a_1 - 12d = 4d$$

$$\text{This allows us to find "d" } -516 = 4d$$

$$-516 / 4 = 4d / 4$$

$$-129 = d$$

We can now use either  $480 = a_1 + 12d$  or  $-36 = a_1 + 16d$  to find  $a_1$

$$480 = a_1 + 12(-129) \text{ leads to } 480 = a_1 - 1548 \text{ or } a_1 = 480 - (-1548) = 2028$$

$$-36 = a_1 + 16(-129) \text{ leads to } -36 = a_1 - 2064 \text{ or } a_1 = -36 - (-2064) = 2028$$

so our arithmetic sequence is  $a_n = 2028 + (n-1)(-129)$

and our finite sequence is

$$\{2028, 1899, 1770, 1641, 1512, 1383, 1254, 1125, 996, 867, 738, 609, 480, 351, 222\}$$

#### Problem 14

Writing a sequence from two of its terms (non sequential)

Step 1) What type of sequence is given?

When given geometric sequence terms

Step 2) Write the definition of each term using  $a_1$  and  $r$

Step 3) Solve the resulting proportion

(I suggest putting higher sequence term (not value) on top)

(this will require radical operations)

Step 4) Write the geometric sequence using  $a_1$  and  $r$

Step 5) Complete the missing terms in the sequence to check to see that rule actually works for each term

	A	B	C
=			
1	given_term1	7776/16...	
2	term_in_seq1		4
3	given_term2	1679616...	
4	term_in_seq2		7
5			
6			
7			
8			
9			
10			
11			

AI given\_term1

So we know  $a_2 = \frac{7776}{16807}$  and  $a_7 = \frac{1679616}{5764801}$  and we also know that this is an geometric sequence so all terms must follow  $a_n = a_1(r)^{n-1}$ , including  $a_1$  and  $a_7$

so  $a_2 = a_1 r^{(4-1)}$  or  $\frac{7776}{16807} = a_1 \cdot r^3$

so  $a_7 = a_1 r^{(7-1)}$  or  $\frac{1679616}{5764801} = a_1 \cdot r^6$

This system can be solved a variety of ways, but basically you can divide the higher sequence term and lower sequence term then apply necessary radical and it yields  $r$  and  $a_1$

	A	B	C
=			
1	ate_term	7776/16...	
2	early_te...	1679616...	
3			
4			
5			
6			
7			
8			
9			
10			
11			

A1 late\_term

$a_7/a_4$  implies  $\frac{1679616}{5764801} / \frac{7776}{16807} = \frac{216}{343}$

$a_7/a_4$  implies  $a_1 \cdot r^6 / a_1 \cdot r^3 = r^3$

This allows us to find "r"  $r^3 = \frac{216}{343}$  which leads to  $\sqrt[3]{r^3} = \sqrt[3]{\frac{343}{216}}$

$r = \frac{7}{6}$

I would use the smaller sequence term to use  $\frac{7776}{16807} = a_1 \cdot r^3$  to find  $a_1$

$\frac{7776}{16807} = a_1 \left(\frac{6}{7}\right)^3$  which leads to  $\frac{7776}{16807} = \frac{216}{343} a_1$

which leads to  $\frac{7776}{16807} \cdot \left(\frac{343}{216}\right) = \left(\frac{343}{216}\right) \left(\frac{216}{343}\right) a_1$  or  $a_1 = \frac{36}{49} \approx 0.734693877551$

so our geometric sequence is  $a_n = \frac{36}{49} \left(\frac{6}{7}\right)^{(n-1)}$

and our finite sequence is  $\left\{ \frac{36}{49}, \frac{216}{343}, \frac{1296}{2401}, \frac{7776}{16807}, \frac{46656}{117649}, \frac{279936}{823543}, \frac{1679616}{5764801} \right\}$

Problem 15

A	seq_1	
=		$42 + 63 + 84 + 105 + 126 + 147 + 168 + 189 + 210 + 231$ $+ 252 + 273 + 294 + 315 + 336 + 357 + 378$
1	42	this is the sum of the following arithmetic sequence
2	63	$\{42, 63, 84, 105, 126, 147, 168, 189, 210, 231, 252, 273, 294, 315, 336, 357, 378, 399, 420\}$
3	84	Method 1: count terms (this is n) $n = 20$
4	105	add first and last terms
5	126	First term ( $a_1$ ) = 42 Last term ( $a_{17}$ ) = 378
6	147	Sum of first and last term $42 + 378 = 483$
7	168	multiply sum of first and last terms by 1/2 the number of
		terms
AI	42	$(20/2)(42+441) = 10(483) = 4830$

$$\sum_{n=1}^{20} 42 + (n-1)(21) = 4830 \quad \text{This is how Riemann would have applied}$$

Problem 16

A	seq_1	
=		$\frac{8}{27} + \frac{16}{81} + \frac{78134919483}{593337044824} + \frac{64}{729} + \frac{128}{2187} + \frac{1953405025}{50063634254} + \frac{976563403}{37542377854}$
1	8/27	this is the sum of the following geometric sequence
2	16/81	$\left\{ \frac{8}{27}, \frac{16}{81}, \frac{32}{243}, \frac{64}{729}, \frac{128}{2187}, \frac{256}{6561}, \frac{512}{19683}, \frac{1024}{59049}, \frac{2048}{177147}, \frac{4096}{531441} \right\}$
3	32/243	Method 1: Count terms (this is n) $n = 10$
4	64/729	Determine $a_1$ and $r$ First term ( $a_1$ ) = $\frac{8}{27}$ $r = \frac{2}{3}$
5	128/2187	Use $S_n = a_1 \cdot \frac{1-(r)^n}{1-r}$
6	256/6561	
AI	$\frac{8}{27}$	$S_{10} = \frac{8}{27} \cdot (1 - (\frac{2}{3})^{10}) / (1 - \frac{2}{3}) = a_1 \cdot \frac{1-r_1^{n-1}}{1-r_1} = \frac{464200}{531441}$

$$\sum_{n=1}^{10} \frac{8}{27} \left(\frac{2}{3}\right)^{(n-1)} = \frac{464200}{531441} \quad \text{This is how Riemann would have applied}$$

$0.87347419563$   
 $1.0 \cdot p$

Problem 17

	A missed	B raw	C percent	D	E	F	G	H
=		=140-mis	=raw/(1.4)					
1	0	140	100.					
2	0.5	139.5	99.6428...					
3	1.	139.	99.2857...					
4	1.5	138.5	98.9285...					
5	2.	138.	98.5714...					
6	2.5	137.5	98.2142...					
7	3.	137.	97.8571...					
8	3.5	136.5	97.5					
9	4.	136.	97.1428...					
10	4.5	135.5	96.7857...					
11	5.	135.	96.4285...					

Problem 18

$7^{10}$	▶ 282475249
$\sqrt{2304}$	▶ 48
$4^2$	▶ 16
$(4 \cdot 3)^2$	▶ 144
$(4 \cdot 3 \cdot 4)^2$	▶ 2304
$(4 \cdot 3 \cdot 4 \cdot 5)^2$	▶ 57600
$(4 \cdot 3 \cdot 4 \cdot 5 \cdot 6)^2$	▶ 2073600
$(4 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7)^2$	▶ 101606400
$(4 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8)^2$	▶ 6502809600
$(4 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9)^2$	▶ 526727577600
$(4 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10)^2$	▶ 52672757760000

$101+90 \cdot 10^{2-1}$	1001
$1001+90 \cdot 10^{3-1}$	10001
$10001+90 \cdot 10^{4-1}$	100001
$100001+90 \cdot 10^{5-1}$	1000001
$\square$	