Problem 1

	A runs	B diff	C
=			
1	5	_	week_num
2	6.5	1.5	
3	8	1.5	
4	9.5	1.5	
5	11.	1.5	

A1 5

1. A runner begins training by running 5 mi. one week. The second week she runs a total of 6.5 mi. The third week she runs 8 mi. Assume this pattern continues.
   - How far will she run in the tenth week?
   - At the end of the tenth week, what will be the total distance she has run since she started training?
   - Express the total distance with summation notation (∑).

first three distances ran

$\{5.,6.5,8.\}$

first ten distances ran

$\{5.,6.5,8.,9.5,11.,12.5,14.,15.5,17.,18.5\}$

Related Arthimetic sequence

$a_n = 5+(n-1)(1.5)$

Related Arthimetic Sequence Sum

$$S_n = \frac{n}{2}(a_1+a_n)$$

$$= \frac{n}{2}(\text{a\_1} \blacktriangleright 5 + a_n)$$

Finding the first ten distances ran

Recall Related Arthimetic sequence

$a_n = 5+(n-1)(1.5)$

$a_1 = 5+(1-1)(1.5) = 5.$

$a_2 = 5+(2-1)(1.5) = 6.5$

$a_3 = 5+(3-1)(1.5) = 8.$

$a_4 = 5+(4-1)(1.5) = 9.5$

$a_5 = 5+(5-1)(1.5) = 11.$

$a_6 = 5+(6-1)(1.5) = 12.5$

$a_7 = 5+(7-1)(1.5) = 14.$

$a_8 = 5+(8-1)(1.5) = 15.5$

$a_9 = 5+(9-1)(1.5) = 17.$

$a_{10} = 5+(10-1)(1.5) = 18.5$

Finding the sum of the first ten weeks of distances ran

Related Arthimetic Sequence Sum

$$S_n = \frac{n}{2}(5+a_n)$$

$S_1 = \frac{1}{2}(5+5.) = 5.$ $\quad$ $S_6 = \frac{6}{2}(5+12.5) = 52.5$

$S_2 = \frac{2}{2}(5+6.5) = 11.5$ $\quad$ $S_7 = \frac{7}{2}(5+14.) = 66.5$

$S_3 = \frac{3}{2}(5+8.) = 19.5$ $\quad$ $S_8 = \frac{8}{2}(5+15.5) = 82.$

$S_4 = \frac{4}{2}(5+9.5) = 29.$ $\quad$ $S_9 = \frac{9}{2}(5+17.) = 99.$

$S_5 = \frac{5}{2}(5+11.) = 40.$ $\quad$ $S_{10} = \frac{10}{2}(5+18.5) = 117.5$

Problem 2

	A height	B ratio	C	D
=				
1	2	_	bounce_num	
2	1.8	0.9		
3	1.62	0.9		
4	1.458	0.9		

A1 2

first five heights

{2.,1.8,1.62,1.458,1.3122}

next five heights

{1.18098,1.062882,0.9565938,0.86093442,0.774840978}

Related Geometric sequence

$a_n = 2(0.9)^{(n-1)}$

2. A superball is dropped from a height of 2 m and bounces 90% of its original height on each bounce.
   - When it hits the ground for the eighth time, how far has it traveled?
   - How high off the floor is the ball at the top of the eighth bounce?

Related Sum *of Geometric Sequence*

$$S_n = 2\left(\frac{1-0.9^n}{1-0.9}\right)$$

Finding the first nine heights   Related Geometric sequence $a_n = 2(0.9)^{(n-1)}$

$a_1 = 2(0.9)^{(1-1)} = 2.$ before 1st bounce

$a_2 = 2(0.9)^{(2-1)} = 1.8$ after 1st bounce

$a_3 = 2(0.9)^{(3-1)} = 1.62$ after 2nd bounce

$a_4 = 2(0.9)^{(4-1)} = 1.458$ after 3rd bounce

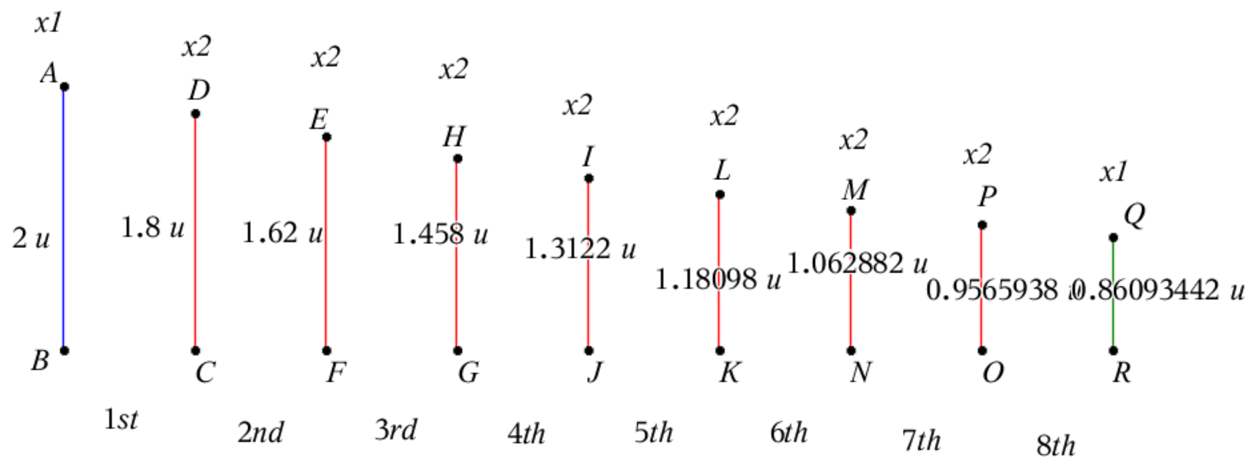
$a_5 = 2(0.9)^{(5-1)} = 1.3122$ after 4th bounce

$a_6 = 2(0.9)^{(6-1)} = 1.18098$ after 5th bounce

$a_7 = 2(0.9)^{(7-1)} = 1.062882$ after 6th bounce

$a_8 = 2(0.9)^{(8-1)} = 0.9565938$ after 7th bounce

$a_9 = 2(0.9)^{(9-1)} = 0.86093442$ after 8th bounce



n_given =16.

First and last are different than all in the middle

UP / DOWN

2 + 1.8 + 1.8 + 1.62 + 1.62 + 1.458 + 1.458 +

1st bounce — 2nd bounce — 3rd bounce — 4th bounce

1.3122 + 1.3122 + 1.18098 + 1.18098 +

5th bounce — 6th bounce

TOP of 8th

on way up from 8th bounce

1.062882 + 1.062882 + 0.9565938 + 0.9565938 + 0.86093442

7th bounce — 8th bounce

Answer to Question 2a) When it hits the ground for the eighth time how far has it traveled?

$2+2(1.8)+2(1.62)+2(1.458)+2(1.3122)+2(1.18098)+2(1.062882)+2(0.9565938)=$
20.7813116

$2+2[1.8+1.62+1.458+1.3122+1.18098+1.062882+0.9565938]=$

$$2+2\sum_{n=2}^{8}\left(2\cdot(0.9)^{n-1}\right)=\frac{51953279}{2500000}=20.7813116$$

Answer to Question 2b)

The height after the eighth bounce is actually a_9

$$a_9=2\cdot(0.9)^{9-1}=\frac{43046721}{50000000}=0.86093442$$

Problem 3

	A snail_t...	B ratio	C
=			
1	16	_	n_given
2	12	3/4	
3	9	3/4	
4	6.75	0.75	

A1 16

3. A snail is crawling straight up a wall. The first hour it climbs 16 inches, the second hour it climbs 12 inches, and each succeeding hour, it climbs only three-fourths the distance it climbed the previous hour. Assume the pattern continues.
   - How far does the snail climb during the seventh hour?
   - What is the total distance the snail has climbed in seven hours?
   - Express the total distance with summation notation (∑).

1st seven snail trips

$$\left\{16,12,9,\frac{27}{4},\frac{81}{16},\frac{243}{64},\frac{729}{256}\right\}$$

$$\{16.,12.,9.,6.75,5.0625,3.796875,2.84765625\}$$

3a) How far does the snail travel in its 7th trip?

This is $a_7=16\left(\frac{3}{4}\right)^{7-1}=\frac{729}{256}\approx 2.84765625$

3b) How far does the snail travel total in its 7 trips?

This is $S_7=16\cdot\frac{1-\left(\frac{3}{4}\right)^{7}}{1-\frac{3}{4}}=\frac{14197}{256}\approx 55.45703125$

3c) $\sum_{n=1}^{7}\left(16\cdot\left(\frac{3}{4}\right)^{n-1}\right)=\frac{14197}{256}=55.45703125$

Problem 4

	A deposits	B	C	D
=				
1	1	n	52	
2	1.5			
3	2			
4	2.5			

A1 1

4. Suppose on Jan. 1 you deposit $1.00 in an empty piggy bank. On Jan. 8 you deposit $1.50; on Jan. 15 you deposit $2.00; and each week thereafter you deposit $0.50 more than the previous week.
   - What kind of sequence do these deposits generate?
   - What amount will you deposit in the 52nd week?
   - What is the total in the piggy bank at the end of these 52 weeks?

4a) These deposits are arithmetic

So $a_n = 1 + (n-1)(0.5)$

$S_n = (n/2)(1 + a_n)$

4b) What is the deposit amount at week 52

This is $a_{52} = 1 + (52-1)(0.5) = 26.5$

4c) What is the total amount in the piggy bank?

This is $S_{52} = (52/2)(1 + 26.5)$

$= 715.$

OR

$$\sum_{n=1}^{52} \left(1 + 0.5 \cdot (n-1)\right) = 715.$$

Finding the first ten deposits made

Recall Related Arthimetic sequence

$a_n = 1 + (n-1)(0.5)$

$a_1 = 1 + (1-1)(0.5) = 1.$

$a_2 = 1 + (2-1)(0.5) = 1.5$

$a_3 = 1 + (3-1)(0.5) = 2.$

$a_4 = 1 + (4-1)(0.5) = 2.5$

$a_5 = 1 + (5-1)(0.5) = 3.$

$a_6 = 1 + (6-1)(0.5) = 3.5$

$a_7 = 1 + (7-1)(0.5) = 4.$

$a_8 = 1 + (8-1)(0.5) = 4.5$

$a_9 = 1 + (9-1)(0.5) = 5.$

$a_{10} = 1 + (10-1)(0.5) = 5.5$

Finding the sum of the first ten weeks of deposits made

Related Arthimetic Sequence Sum

$S_n = \frac{n}{2}(1 + a_n)$

$S_1 = \frac{1}{2}(1 + 1.) = 1.$ $\quad S_6 = \frac{6}{2}(1 + 3.5) = 13.5$

$S_2 = \frac{2}{2}(1 + 1.5) = 2.5$ $\quad S_7 = \frac{7}{2}(1 + 4.) = 17.5$

$S_3 = \frac{3}{2}(1 + 2.) = 4.5$ $\quad S_8 = \frac{8}{2}(1 + 4.5) = 22.$

$S_4 = \frac{4}{2}(1 + 2.5) = 7.$ $\quad S_9 = \frac{9}{2}(1 + 5.) = 27.$

$S_5 = \frac{5}{2}(1 + 3.) = 10.$ $\quad S_{10} = \frac{10}{2}(1 + 5.5) = 32.5$

Problem 5

	A profits	B	C
=			
1	3000	n	
2	4500.		
3	6750.		
4	10125.		

A1 3000

5. Carla's Clothing Shop opened eight years ago. The first year she made $3,000 profit. Each year thereafter her profits averaged 50% greater than the previous year.
   - How much profit did Carla earn during her 18th year of business?
   - What was the total amount of profit Carla earned over her first 18 years?

5a) How much profit was earned during her 18th year?

this is $a_{18}=3000(1.5)\wedge(18-1)$

$=3000(1.5)\wedge(17)$

$=2955783.76007$

5b) How much profit was earned during her 18 years in business?

This S18 =

$$3000\cdot\frac{1-(1.5)^{18}}{1-1.5}=8861351.28021$$

OR

$$\sum_{n=1}^{18}\left(3000\cdot(1.5)^{n-1}\right)=8861351.28021$$

Problem 6

	A swing	B	C	D
=				
1	50	n	6	
2	45			
3	40.5			
4	36.45			

A1 50

6. A ball on a pendulum moves 50 cm on its first swing. Each succeeding swing it moves 0.9 the distance of the previous swing.
   - Write the first six terms of the sequence generated.
   - Assuming the pattern continues, how far will the ball travel before coming to rest?

6a) Determine the first six terms of a_n

$$a_n=50\left(\frac{9}{10}\right)^{n-1}$$

$$\left\{50,45,\frac{81}{2},\frac{729}{20},\frac{6561}{200},\frac{59049}{2000}\right\}$$

$$\{50.,45.,40.5,36.45,32.805,29.5245\}$$

6b) How far will the ball travel before coming to rest?

Theoretically, it won't stop moving

$$S_\infty=\frac{50}{1-\frac{9}{10}}=500\text{ cm}$$

(check for the first 100 terms)

I entered the first 100 terms in a column of the spreadsheet! 499.986719301

Problem 7

	A integers	B ball_1	C ball_2	D
=		=25.*(3/4)	=18.*(4/5)	
1	1	25.	18.	n_ba
2	2	18.75	14.4	n_ba
3	3	14.0625	11.52	
4	4	10.5468...	9.216	
5	5	7.91015...	7.3728	
6	6	5.93261...	5.89824	
7	7	4.44946...	4.718592	
8	8	3.33709...	3.77487...	
9	9	2.50282...	3.01989...	
10	10	1.87711...	2.41591...	
11	11	1.40783...	1.93273...	

A1 1

Question 1) See table

Question 2)

Ball 2 will exceed the height of Ball 1 on the 7th Bounce

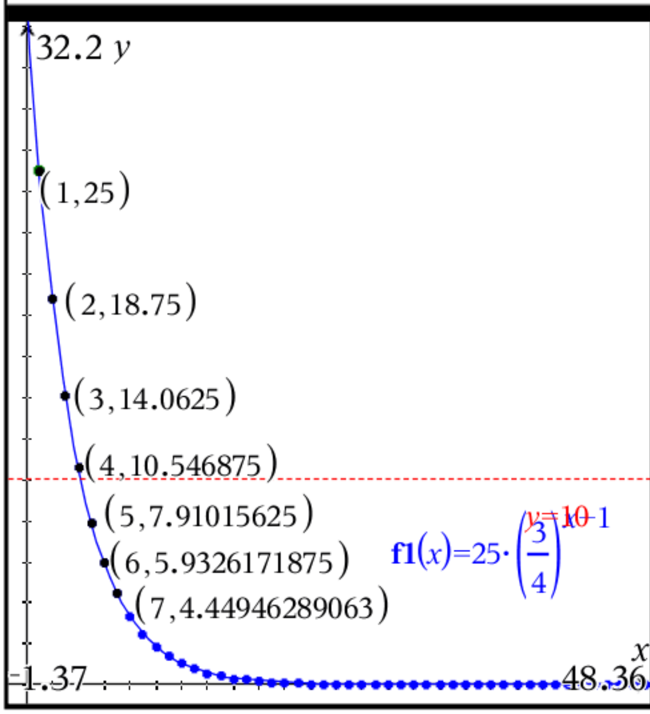
Question 3) See next slide

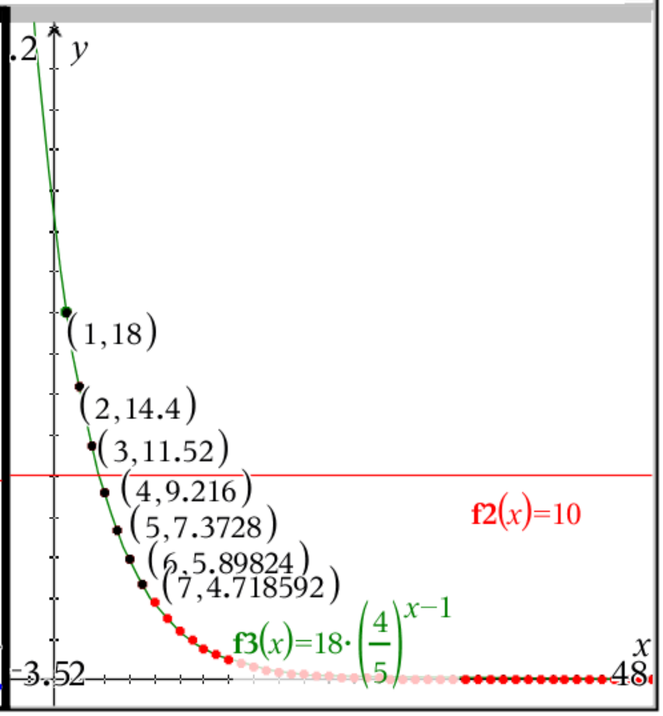
Question 4) When do the balls stop bouncing (over at least when do the balls' bounce height fall below 3 inches NOTE $\frac{3.}{12}$ ▸ 0.25)

See slide after next slide

Ball 1: Will bounce to a height over 10 ft four times {25.,18.75,14.0625,10.546875 }

Ball 2: Will bounce to a height over 10 ft three times {18.,14.4,11.52 }



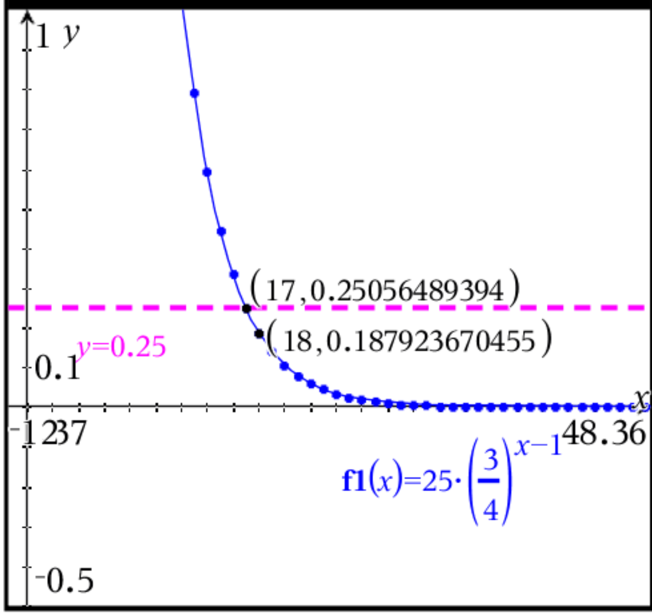


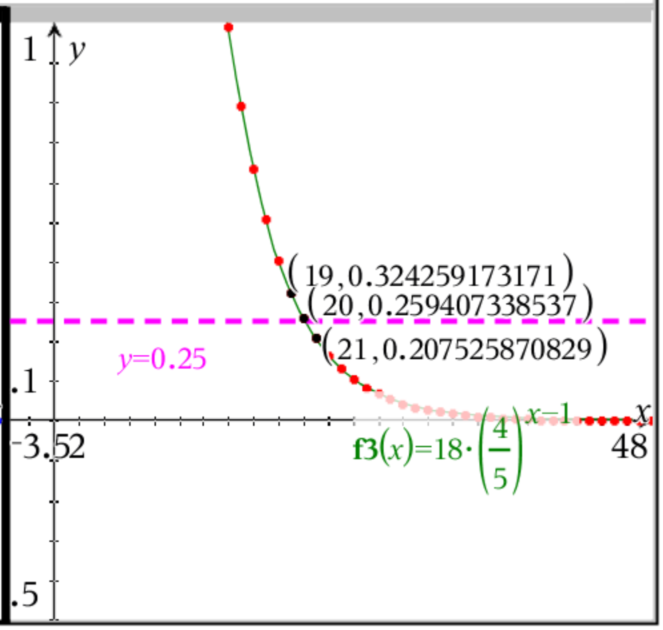
Ball 1: Will bounce to a height under 3 inches (0.25 ft) after 17 bounces

This represents bounce 17 and 18 heights $\{0.25056489394, 0.187923670455\}$

Ball 2: Will bounce to a height under 3 inches (0.25 ft) after 20 bounces

This represents bounce 20 and 21 heights $\{0.259407338537, 0.207525870829\}$





Problem 5) What initial bounce height (to nearest foot) would you have to have to ensure for each ball in order to guarantee that it bounces at least 8 feet high by the sixth bounce?

This is just an inequality to solve!

Ball 1 $a_n = a_1 \left(\frac{3}{4}\right)^{n-1}$

We know that we want $a_6 > 8$

$8 < a_1 \left(\frac{3}{4}\right)^{6-1}$ implies $8 < \frac{243}{1024} a_1$

which implies $8 \cdot \frac{1024}{243} < a_1$

$\frac{8192}{243} < a_1$

$a_1 > 33.7119341564$

Test $a_n = 34 \left(\frac{3}{4}\right)^{n-1}$ $a_6 = 34 \left(\frac{3}{4}\right)^{6-1} = \frac{4131}{512} = 8.068359375$

Problem 5) What initial bounce height (to nearest foot) would you have to have to ensure for each ball in order to guarantee that it bounces at least 8 feet high by the sixth bounce?

This is just an inequality to solve!

Ball 2 $b_n = b_1 (\frac{4}{5})^{n-1}$

We know that we want $b_6 > 8$

$8 < b_1 (\frac{4}{5})^{6-1}$ implies $8 < \frac{1024}{3125} a_1$

which implies $8 \cdot \frac{3125}{1024} < a_1$

$\frac{3125}{128} < a_1$

$b_1 > 24.4140625$

Test $b_n = 25 (\frac{4}{5})^{n-1}$ $b_6 = 25 (\frac{4}{5})^{6-1} = \frac{1024}{125} = 8.192$

$b_n = 24 (\frac{4}{5})^{n-1}$ $b_6 = 24 (\frac{4}{5})^{6-1} = \frac{24576}{3125} = 7.86432$

Problem 8

Problem 6) Determine the sum of all the bounce heights for both balls

This is a very different problem from #2 on the front side of the work sheet

Note: Ball 1 has sequence $a_n = 25 \cdot \left(\frac{3}{4}\right)^{n-1}$ Note Ball 2 has sequence $b_n = 18 \cdot \left(\frac{4}{5}\right)^{n-1}$

$$S_\infty = \frac{25}{1-\frac{3}{4}} = 100 \quad S_\infty = \frac{18}{1-\frac{4}{5}} = 90$$