Problem 1

	A	B	C	D
=				
1	a_1	20		
2	d	4		
3	s_n	25000		
4				

A1 a_1

$$\sum_{n=1}^{n} 20+(n-1)(4)$$

We are give first three rows

$\{20,24,28\}$

We can write the sequence

$a_n = 20+(n-1)(4)$

We can write the Sum of the Sequence

$$S_n = \frac{n}{2}(20+a_n)$$

We can rewrite the sum of the sequence

$$S_n = \frac{n}{2}(20+20+(n-1)(4))$$

$$= \frac{n}{2}(40+(n-1)(4))$$

$$= \frac{n}{2}(36+4\cdot n)$$

$$= 2\cdot n^2+18\cdot n$$

$S_n = 2\cdot n^2+18\cdot n$

We know we want at least 25000 **seats**

So set $S_n = 25000$ and solve for n

$25000=2\cdot n^2+18\cdot n$

this can be accomplished in a variety of ways

$25000=2\cdot n^2+18\cdot n$

$25000-25000=2\cdot n^2+18\cdot n-25000$

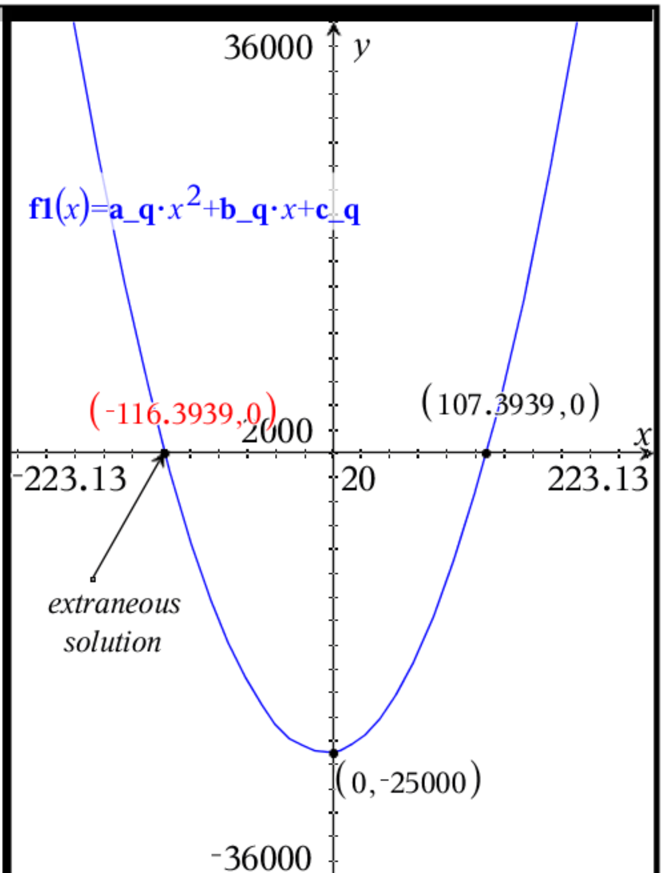
$0=2\cdot n^2+18\cdot n-25000$

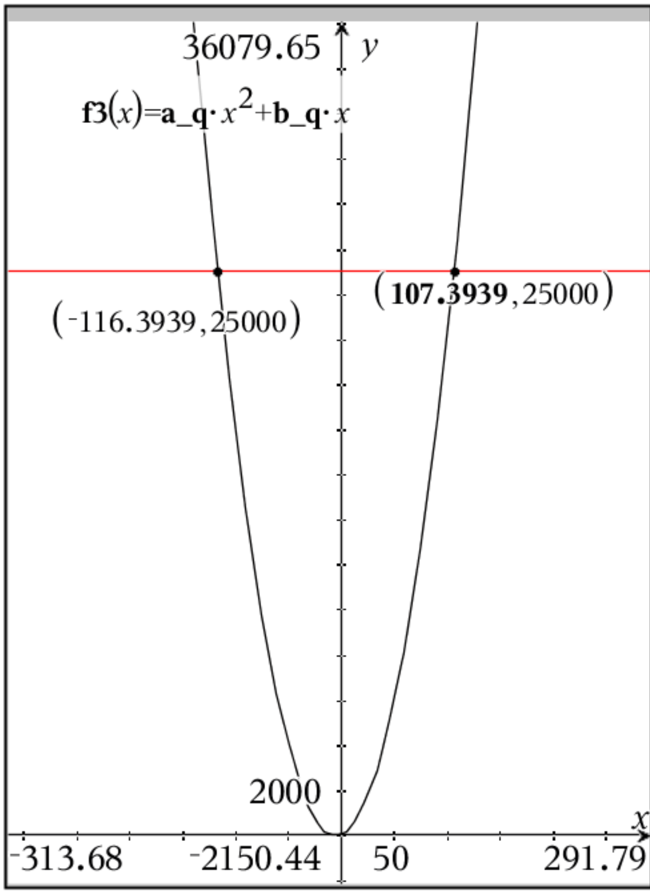
Note this has a discriminant

$D=(18)^2-4(2)(^-25000)$

$=200324$

$$n=\frac{^-18+\sqrt{200324}}{4}=107.393922981$$





Another way is to solve the quadratic equation graphically

$25000 = 2 \cdot x^2 + 18 \cdot x$

This leads to

n=107.393922981

recall n must be an integer

so n = 108.

Test

$S_n = 2 \cdot n^2 + 18 \cdot n$

$S_{108} = 2(108.)^2 + 18(108.)$

$= 25272.$

Note

$S_{107} = 2(107.)^2 + 18(107.)$

$= 24824.$

$$\sum_{n=1}^{108.} 20 + (n-1)(4) = 25272.$$

input =108.

So may choose trial and error until they can determine the number of rows

We need to build a theater with 108. rows to have at least 25000 **seats,** we will have 25272. seats which means that we will have 272. extra seats according to the given information.

	A	B	C	D
=				
1	front_row	175		
2	r	0.85		
3	max_price	50		
4				

A1 **front_row**

$a_9 = 175\,(0.85)^{9-1}$

$= 47.69$

row_num =9.

Now the issue with the cost of the tickets in each row is different type of sequence

We know that the tickets are 175 in the front row

We know that tickets in rows after the front row are 85. % of previous row until the price reaches \$15, then all remaining rows will cost \$15

a_n can model ticket price in each row

$a_n = 175(0.85)^{n-1}$

The ticket prices for the first five rows are
{175.,148.75,126.44,107.47,91.35}

To answer 5b) without guessing and checking requires logarithms

$50 = 175(0.85)^{n-1}$

$50/175 = 175(0.85)^{n-1}/175$

$\frac{2}{7} = (0.85)^{n-1}$

recall $\log_b y = x$ if and only if $b^x = y$

So apply log to both sides

$\log_{0.85}\left(\frac{2}{7}\right) = n-1$

$n = \log_{0.85}\left(\frac{2}{7}\right) + 1 = \left(\log\frac{2}{7}\right)/\log(0.85) + 1$

$= \frac{\log\left(\frac{2}{7}\right)}{\log(0.85)} + 1 \quad = 8.7084126284$ so we cannot sit any closer than row 9.

5c) Write a model that can be used to determine the revenue for this theater if it only sells out the first 5 rows

Total Revenue = Row 1 revenue+Row 2 revenue+Row 3 revenue+Row 4 revenue+Row 5 revenue

Row Revenue = seats(price)

This can also be done quickly with spreadsheets

	A	B seat	C price	D row_rev	E	F	G	H
=				=seat*pric				
1	row_1	20.	175.	3500.	17337.96			
2	row_2	24	148.75	3570.				
3	row_3	28	126.44	3540.32				
4	row_4	32	107.47	3439.04				
5	row_5	36	91.35	3288.6				

A1 row_1

the fastest way to determine the revenue of the theater is to use matrices

seats*price = revenue

$$\begin{bmatrix} 20. & 24 & 28 & 32 & 36 \end{bmatrix} \begin{bmatrix} 175. \\ 148.75 \\ 126.44 \\ 107.47 \\ 91.35 \end{bmatrix} = \begin{bmatrix} 17337.96 \end{bmatrix}$$

Another way would to just multiply the seats in each row by the number of seats in row and add all of the revenues for each row together

Row 1 revenue $20.(175.)=3500.$

Row 2 revenue $24(148.75)=3570.$

Row 3 revenue $28(126.44)=3540.32$

Row 4 revenue $32(107.47)=3439.04$

Row 5 revenue $36(91.35)=3288.6$

Total revenue $=3500.+3570.+3540.32+3439.04+3288.6=17337.96$

Problem 2

	A term	B end_4	C end_7
=		=1004+10	=1007+10
1	1	1004	1007
2	2	1014	1017
3	3	1024	1027
4	4	1034	1037

A1 1

6) $a_n=1004.+(n-1)(10)$

$=994.+10n$

7) $b_n=1007.+(n-1)(10)$

$=997.+10n$

8) sum of all four digit numbers that end in 4

$$\sum_{n=1}^{900}(1004+(n-1)\cdot 10)=4949100$$

sum of all four digit numbers that end in 7

$$\sum_{n=1}^{900}(1007+(n-1)\cdot 10) ▸ 4951800$$

Sum of all four digit numbers that end in 4 or 7

9900900