Problem 1

	A	B	C	D
=				
1	a_1	25		
2	d	4		
3	s_n	29000		
4				

B4

$$\sum_{n=1}^{n} 25+(n-1)(4)$$

We are give first three rows

$\{25,29,33\}$

We can write the sequence

an = $25+(n-1)(4)$

We can write the Sum of the Sequence

$$S_n=\frac{n}{2}(25+a_n)$$

We can rewrite the sum of the sequence

$$S_n=\frac{n}{2}(25+25+(n-1)(4))$$

$$=\frac{n}{2}(50+(n-1)(4))$$

$$=\frac{n}{2}(46+4\cdot n)$$

$$=2\cdot n^2+23\cdot n$$

$S_n=2\cdot n^2+23\cdot n$

We know we want at least 29000 **seats**

So set $S_n=29000$ and solve for n

$29000=2\cdot n^2+23\cdot n$

this can be accomplished in a variety of ways

$29000=2\cdot n^2+23\cdot n$

$29000-29000=2\cdot n^2+23\cdot n-29000$

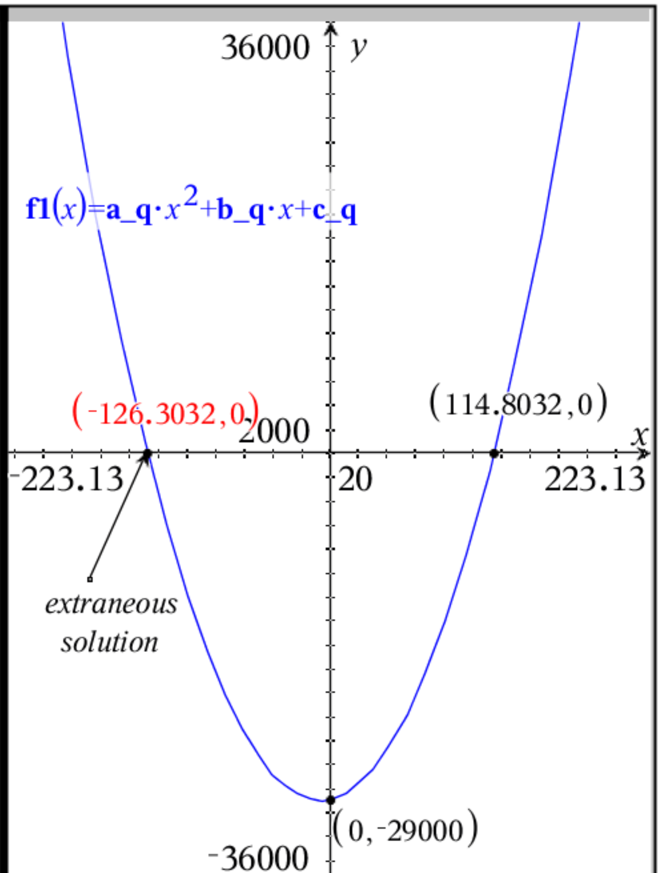
$0=2\cdot n^2+23\cdot n-29000$

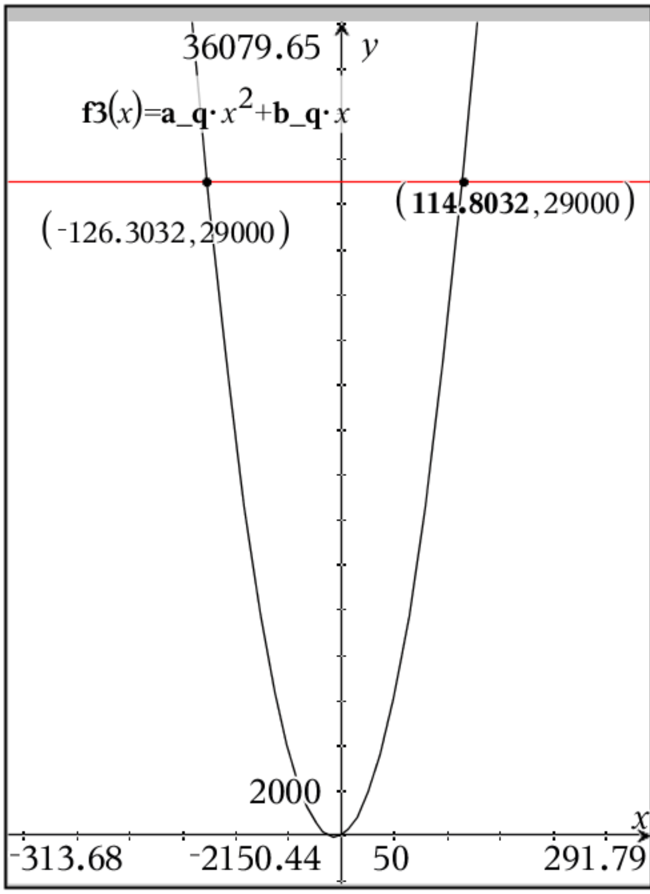
Note this has a discriminant

$D=(23)^2-4(2)(^-29000)$

$=232529$

$$n=\frac{^-25+\sqrt{232529}}{4}=114.303152178$$





Another way is to solve the quadratic equation graphically

$29000 = 2 \cdot x^2 + 23 \cdot x$

This leads to

n=114.803152178

recall n must be an integer

so n = 115.

Test

$S_n = 2 \cdot n^2 + 23 \cdot n$

$S_{115} = 2(115.)^2 + 23(115.)$

$= 29095.$

Note

$S_{114} = 2(114.)^2 + 23(114.)$

$= 28614.$

$$\sum_{n=1}^{119.} 25 + (n-1)(4) = 31059.$$

input =119.

So may choose trial and error until they can determine the number of rows

We need to build a theater with 115. rows to have at least 29000 **seats,** we will have 29095. seats which means that we will have 95. extra seats according to the given information.

	A	B	C	D
=				
1	front_row	150		
2	r	0.9		
3	max_price	50		
4				

B3 50

$$a_{12} = 150(0.9)^{12-1}$$

$$= 47.07$$

row_num = 12.

Now the issue with the cost of the tickets in each row is different type of sequence

We know that the tickets are 150 in the front row

We know that tickets in rows after the front row are 90. % of previous row until the price reaches $15, then all remaining rows will cost $15

a_n can model ticket price in each row

$$a_n = 150(0.9)^{n-1}$$

The ticket prices for the first five rows are {150.,135.,121.5,109.35,98.42 }

To answer 5b) without guessing and checking requires logarithms

$50 = 150(0.9)^{n-1}$

$50/150 = 150(0.9)^{n-1}/150$

$\frac{1}{3} = (0.9)^{n-1}$

recall $\log_b y = x$ if and only if $b^x = y$

So apply log to both sides

$\log_{0.9}(\frac{1}{3}) = n-1$

$n = \log_{0.9}(\frac{1}{3}) + 1 = (\log\frac{1}{3})/\log(0.9)+1$

$= \frac{\log\left(\frac{1}{3}\right)}{\log(0.9)}+1 \quad = 11.4271726634$ so we cannot sit any closer than row 12.

5c) Write a model that can be used to determine the revenue for this theater if it only sells out the first 5 rows

Total Revenue = Row 1 revenue+Row 2 revenue+Row 3 revenue+Row 4 revenue+Row 5 revenue

Row Revenue = seats(price)

This can also be done quickly with spreadsheets

	A	B seat	C price	D row_rev	E	F	G	H
=				=seat*pric				
1	row_1	25.	150.	3750.	19755.67			
2	row_2	29	135.	3915.				
3	row_3	33	121.5	4009.5				
4	row_4	37	109.35	4045.95				
5	row_5	41	98.42	4035.22				

A1 row_1

the fastest way to determine the revenue of the theater is to use matrices

seats*price = revenue

$$\begin{bmatrix} 25. & 29 & 33 & 37 & 41 \end{bmatrix} \begin{bmatrix} 150. \\ 135. \\ 121.5 \\ 109.35 \\ 98.42 \end{bmatrix} = \begin{bmatrix} 19755.67 \end{bmatrix}$$

Another way would to just multiply the seats in each row by the number of seats in row and add all of the revenues for each row together

Row 1 revenue 25. (150.)=3750.

Row 2 revenue 29 (135.)=3915.

Row 3 revenue 33 (121.5)=4009.5

Row 4 revenue 37 (109.35)=4045.95

Row 5 revenue 41 (98.42)=4035.22

Total revenue = 3750. +3915. +4009.5 +4045.95 +4035.22 =19755.67

Problem 2

	A term	B end_3	C end_5
=		=1003+10	=1005+10
1	1	1003	1005
2	2	1013	1015
3	3	1023	1025
4	4	1033	1035

C1 =1005

6) $a_n = 1006. + (n-1)(10)$

$= 996. + 10n$

7) $b_n = 1008. + (n-1)(10)$

$= 998. + 10n$

8) sum of all four digit numbers that end in 3

$$\sum_{n=1}^{900} (1003 + (n-1)\cdot 10) = 4948200$$

sum of all four digit numbers that end in 5

$$\sum_{n=1}^{900} (1005 + (n-1)\cdot 10) ▸ 4950000$$

Sum of all four digit numbers that end in 3 or 5

9898200