Problem 1

	A	B	C	D
=				
1	a_1	27		
2	d	4		
3	s_n	31000		
4				

A1 a_1

$$\sum_{n=1}^{n} 27+(n-1)(4)$$

We are give first three rows

$\{27,31,35\}$

We can write the sequence

an = $27+(n-1)(4)$

We can write the Sum of the Sequence

$$S_n=\frac{n}{2}(27+a_n)$$

We can rewrite the sum of the sequence

$$S_n=\frac{n}{2}(27+27+(n-1)(4))$$

$$=\frac{n}{2}(54+(n-1)(4))$$

$$=\frac{n}{2}(50+4\cdot n)$$

$$=2\cdot n^2+25\cdot n$$

$S_n=2\cdot n^2+25\cdot n$

We know we want at least 31000 **seats**

So set $S_n=31000$ and solve for n

$31000=2\cdot n^2+25\cdot n$

this can be accomplished in a variety of ways

$31000=2\cdot n^2+25\cdot n$

$31000-31000=2\cdot n^2+25\cdot n-31000$

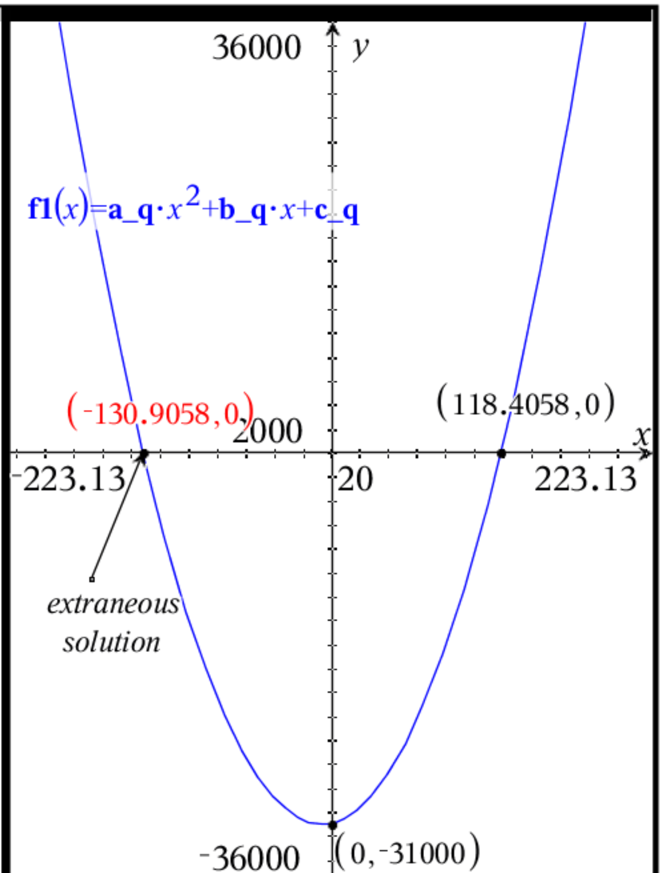
$0=2\cdot n^2+25\cdot n-31000$

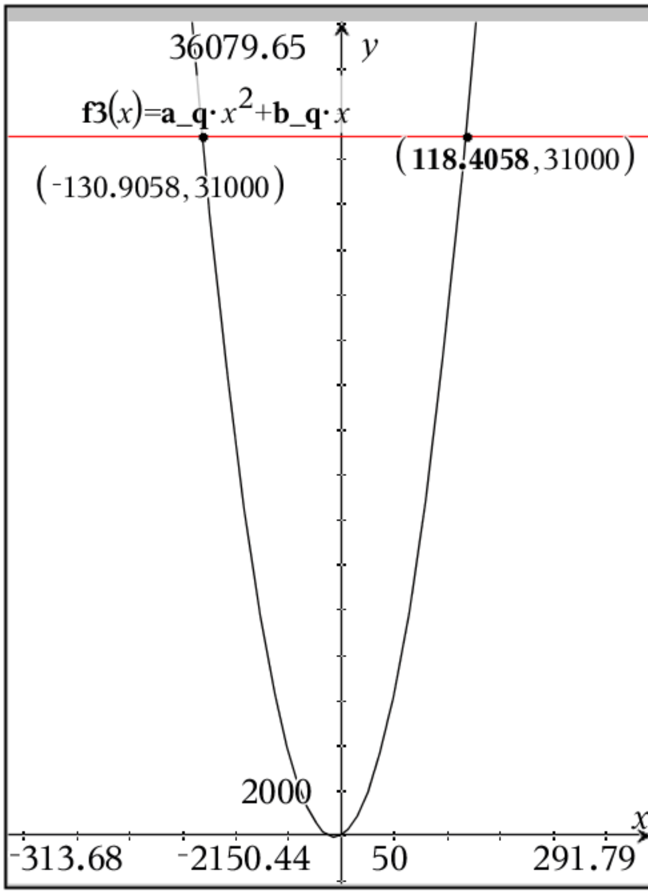
Note this has a discriminant

$D=(25)^2-4(2)(^-31000)$

$=248625$

$$n=\frac{^-25+\sqrt{248625}}{4}=118.405776039$$





Another way is to solve the quadratic equation graphically

$31000 = 2 \cdot x^2 + 25 \cdot x$

This leads to

n=118.405776039

recall n must be an integer

so n = 119.

Test

$S_n = 2 \cdot n^2 + 25 \cdot n$

$S_{119} = 2(119.)^2 + 25(119.)$

$= 31297.$

Note

$S_{118} = 2(118.)^2 + 25(118.)$

$= 30798.$

$$\sum_{n=1}^{119.} \left(27 + (n-1)(4)\right) = 31297.$$

input =119.

So may choose trial and error until they can determine the number of rows

We need to build a theater with 119. rows to have at least 31000 **seats,** we will have 31297. seats which means that we will have 297. extra seats according to the given information.

	A	B	C	D
=				
1	front_row	250		
2	r	0.75		
3	max_price	50		
4				

A1 **front_row**

$$a_7 = 250\,(0.75)^{7-1}$$

$$= 44.49$$

row_num =7.

Now the issue with the cost of the tickets in each row is different type of sequence

We know that the tickets are 250 in the front row

We know that tickets in rows after the front row are 75. % of previous row until the price reaches $15, then all remaining rows will cost $15

a_n can model ticket price in each row

$$a_n = 250(0.75)^{n-1}$$

The ticket prices for the first five rows are
{250.,187.5,140.63,105.47,79.1}

To answer 5b) without guessing and checking requires logarithms

$50 = 250(0.75)^{n-1}$

$50/250 = 250(0.75)^{n-1}/250$

$\frac{1}{5} = (0.75)^{n-1}$

recall $\log_b y = x$ if and only if $b^x = y$

So apply log to both sides

$\log_{0.75}\left(\frac{1}{5}\right) = n-1$

$n = \log_{0.75}\left(\frac{1}{5}\right) + 1 = \left(\log\frac{1}{5}\right)/\log(0.75) + 1$

$= \frac{\log\left(\frac{1}{5}\right)}{\log(0.75)} + 1$ $= 6.59450194$ so we cannot sit any closer than row 7.

5c) Write a model that can be used to determine the revenue for this theater if it only sells out the first 5 rows

Total Revenue = Row 1 revenue+Row 2 revenue+Row 3 revenue+Row 4 revenue+Row 5 revenue

Row Revenue = seats(price)

This can also be done quickly with spreadsheets

	A	B seat	C price	D row_rev	E	F	G	H
=				=seat*pric				
1	row_1	27.	250.	6750.	24999.18			
2	row_2	31	187.5	5812.5				
3	row_3	35	140.63	4922.05				
4	row_4	39	105.47	4113.33				
5	row_5	43	79.1	3401.3				

A1 row_1

the fastest way to determine the revenue of the theater is to use matrices

seats*price = revenue

$$\begin{bmatrix} 27. & 31 & 35 & 39 & 43 \end{bmatrix} \begin{bmatrix} 250. \\ 187.5 \\ 140.63 \\ 105.47 \\ 79.1 \end{bmatrix} = \begin{bmatrix} 24999.18 \end{bmatrix}$$

Another way would to just multiply the seats in each row by the number of seats in row and add all of the revenues for each row together

Row 1 revenue 27. (250.)=6750.

Row 2 revenue 31 (187.5)=5812.5

Row 3 revenue 35 (140.63)=4922.05

Row 4 revenue 39 (105.47)=4113.33

Row 5 revenue 43 (79.1)=3401.3

Total revenue =6750. +5812.5 +4922.05 +4113.33 +3401.3 =24999.18

Problem 2

	A term	B end_6	C end_8
=		=1006+10	=1008+10
1	1	1006	1008
2	2	1016	1018
3	3	1026	1028
4	4	1036	1038

A1 1

6) $a_n = 1006. + (n-1)(10)$

$= 996. + 10n$

7) $b_n = 1008. + (n-1)(10)$

$= 998. + 10n$

8) sum of all four digit numbers that end in 6

$$\sum_{n=1}^{900} (1006 + (n-1) \cdot 10) = 4950900$$

sum of all four digit numbers that end in 8

$$\sum_{n=1}^{900} (1008 + (n-1) \cdot 10) \blacktriangleright 4952700$$

Sum of all four digit numbers that end in 6 or 8

9903600