General Solution Process for the Pulley Problem

Step 1: Determine the given information

On this quiz — Diameter of the winch's drum was given in inches — Diameter/2 = radius

Number of winch drum revolutions per minute

Distance needed to pull jeep out given, but given in feet

Step 2: Use given to determine distance to pull jeep out in inches not feet

12 feet given = distance in inches

Step 3: Convert pulley rpm to radians

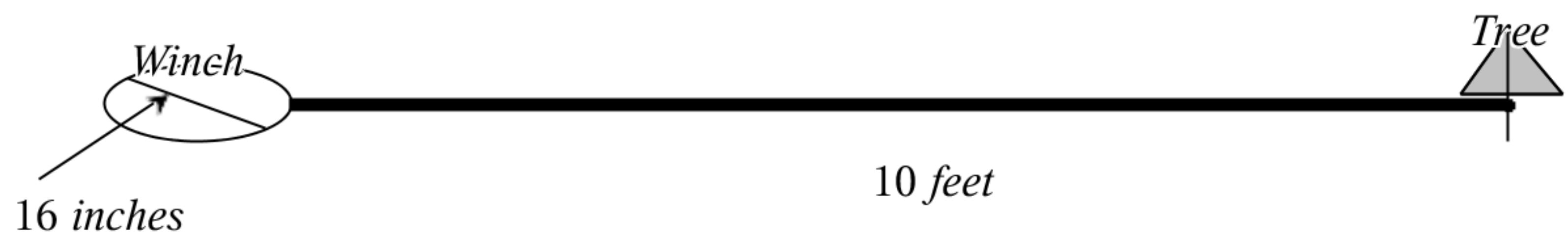
θ=(rpm)($2\cdot\pi$)=radians

Step 4: Determine arc length and linear speed of the drum

s = θ radius and linear speed = $\frac{s}{t}$

Step 5: Set up equation with linear speed and distance to pull jeep out

(Linear Speed)(x minutes) =distance to pull out jeep



Given: d = 16 inches and r = 8 inches

rpm = 5 rev./min

jeep needs to move 10 feet or 120 inches

Find $\theta = \mathbf{rpm} \cdot 2 \cdot \pi = 10 \cdot \pi$ radians

Find arc length and linear speed

$s = \theta r = (10 \cdot \pi)(8) = 80 \cdot \pi$

linear speed is $80 \cdot \pi$ **in** /min

Set up equation to solve for time

(linear speed)(time) = jeep distance

$(80 \cdot \pi)(x) = 120$

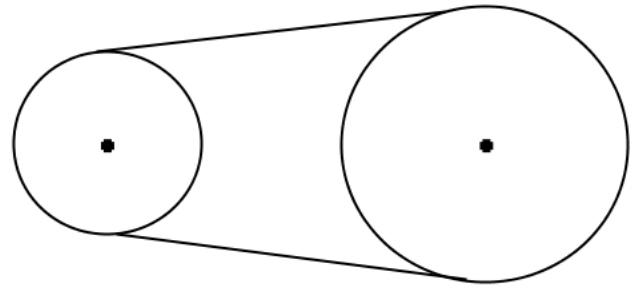
$x = 120 / 80 \cdot \pi$

$x = \dfrac{3}{2 \cdot \pi} \approx 0.477465$ minutes

a. Determine the number of revolutions that the fan is moving in the same minute.

b) Determine the angular speed of the fan in radians per minute

c. Determine the linear speed of the fan in FEET per minute.



diameter of small pulley =10 inches

radius of small pulley =5 inches

rpm of small pulley =100 rev/min

diameter of fan pulley =18 inches

radius of fan pulley =9 inches

a. Determine the number of revolutions that the fan is moving in the same minute.

Step 1 Convert rpms of small pulley into radians

θ of small pulley $= rpm\,(2\cdot\pi) = (100)(2\cdot\pi) = 200\cdot\pi$

Step 2: Determine how far the small pulley is moving the belt

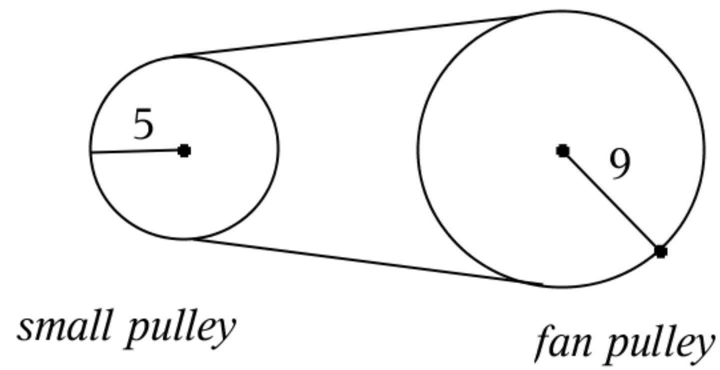
s=θr implies $(200\cdot\pi)(5) = 1000\cdot\pi$

Step 3: Convert the distance that belt moves into radians of fan using arc length equation

s=θr implies $1000\cdot\pi = \theta(9)$

$1000\cdot\pi/9 = 9\theta/9$

θ of fan $= \dfrac{1000\cdot\pi}{9}$ radians



rpm of small pulley

100

Step 4: Convert radians of fan into revolutions of fan

rev =radians/$(2\cdot\pi)$

$$=\left(\frac{1000\cdot\pi}{9}\right)/(2\cdot\pi)$$

$$=\frac{1000\cdot\pi}{9}\cdot\frac{1}{2\cdot\pi}=\frac{500}{9}=55+\frac{5}{9}$$

≈ 55.5556 **rpm of fan**

b) Determine the angular speed of the fan in radians per minute

Since the unit of time is 1 minute, angular speed is radians of fan

Angular speed of fan $=\dfrac{\theta}{t}$

$$=\frac{1000\cdot\pi}{9}\ \textbf{radians/min}\ \approx 349.066\ \textbf{radians/min}$$

c. Determine the linear speed of the fan in FEET per minute.

Linear speed = θr

$$=\frac{1000\cdot\pi}{9}(9) = 1000\cdot\pi\ \textbf{inches}$$

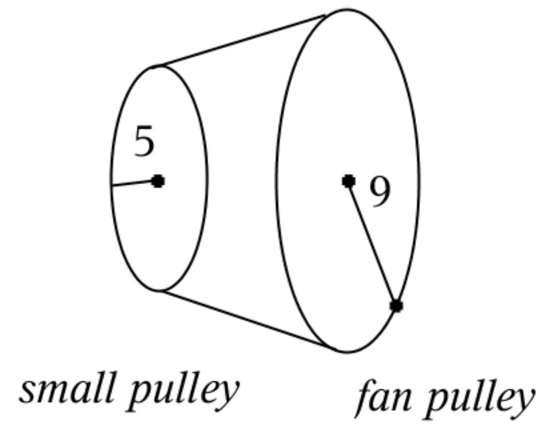
Now convert inches to feet

Linear speed $= 1000\cdot\pi$ **inches/minute**

$$=(1000\cdot\pi\ \textbf{inches})\cdot\frac{1\ foot}{12\ inches}$$

$$=(1000\cdot\pi)/12 = \frac{250\cdot\pi}{3}\ \textbf{feet/min}$$

$$\approx 261.799\ \textbf{feet/min}$$



rpm of small pulley

100

rpm of fan

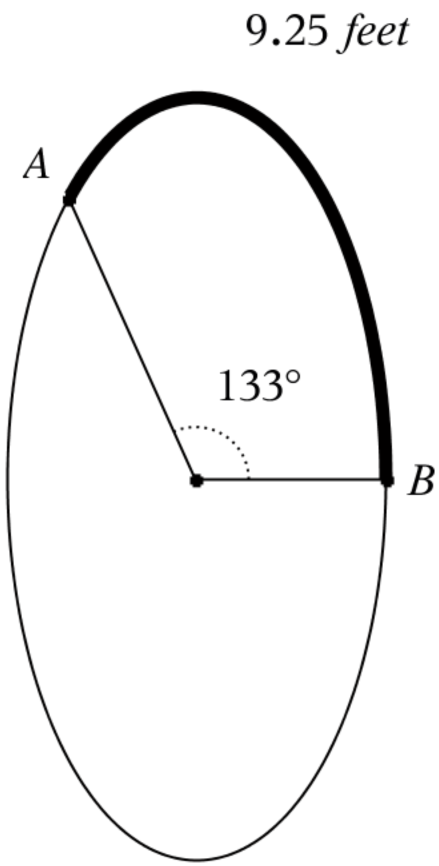
$\dfrac{500}{9}$

small pulley radians

$200\cdot\pi$

fan radians

$\dfrac{1000\cdot\pi}{9}$



Determine radius of circle

Step 1: Convert Degrees to Radians

$$\theta = 133° \cdot \frac{\pi}{180°} = \frac{133 \cdot \pi}{180} \text{ radians}$$

Step 2: Solve arc length equation for r

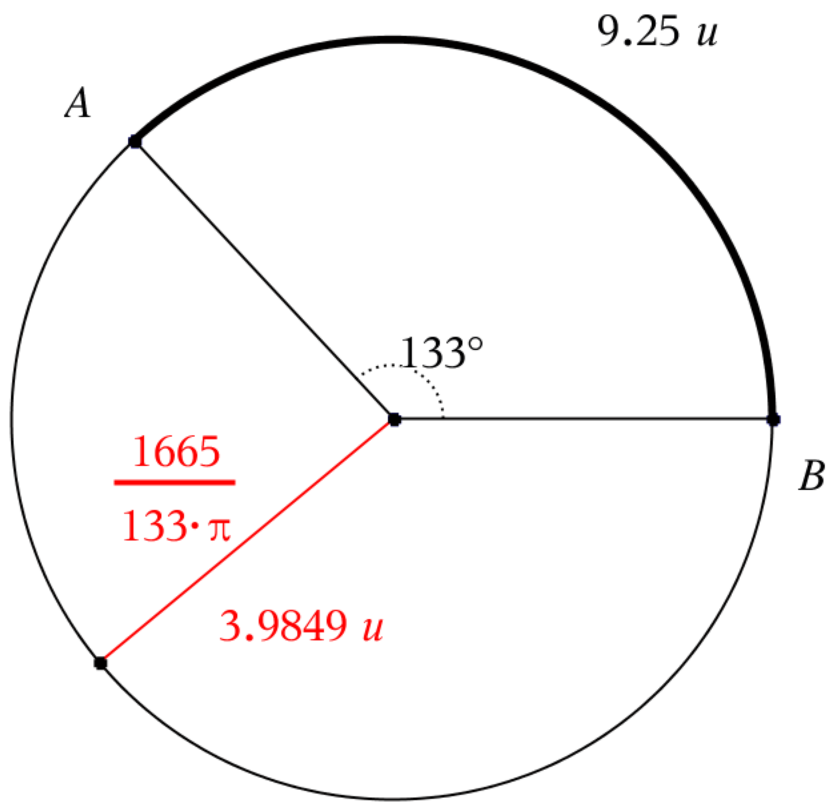
s=θr

$$9.25 = \frac{133 \cdot \pi}{180} \mathbf{r}$$

$$\frac{37}{4} = \frac{133 \cdot \pi}{180} \mathbf{r}$$

$$\frac{37}{4} \cdot \frac{180}{133 \cdot \pi} = \frac{133 \cdot \pi}{180} \cdot \frac{180}{133 \cdot \pi} \mathbf{r}$$

$$\mathbf{r} = \frac{1665}{133 \cdot \pi} = 9.25 / (\frac{133 \cdot \pi}{180}) \approx 3.98486 \text{ feet}$$


9.25 u
A
133°
B
1665
133·π
3.9849 u

Latitude Problem (assume both latitudes lie in northern hemispere)

City A is due North of city B Assume earth's radius ≈4000 miles

City A 's latitude =25°51'46" N City B's latitude =15°15'52" N

Step 1: Find the DMS difference in latitudes 25°51'46" −15°15'52" =10°35'54"

Step 2: Convert DMS difference to DD difference

10°35'54"

$$10+35/60+54/3600=\frac{6359}{600}°\approx 10.598°$$

Step 3: Convert DD difference to Radian difference

$$\frac{6359}{600}°\approx 10.598°$$

$$\textbf{EXACT}\ \frac{6359}{600}\cdot\frac{\pi}{180}=\frac{6359\cdot\pi}{108000}\ \textbf{radians}$$

$$\textbf{APPROXIMATE}\ 10.598\cdot\frac{\pi}{180}\approx\cdot\ 0.059\,\pi\ \textbf{radians}$$

$$\textbf{APPROXIMATE}\ 10.598\cdot\frac{\pi}{180}\approx\cdot\ 0.185\ \textbf{radians}$$

Depending on which version of the radian difference will determine distance between cities

Step 4: Use Arc Length to determine distance between cities

$$\textbf{EXACT}\ \frac{6359}{600}\cdot\frac{\pi}{180}=\frac{6359\cdot\pi}{108000}\ \textbf{radians}$$

distance from city A to city B

$$s=\theta r=\frac{6359\cdot\pi}{108000}(4000)=\frac{6359\cdot\pi}{27}\approx 739.903\ \textbf{miles (best)}$$

$$\textbf{APPROXIMATE}\ 10.598\cdot\frac{\pi}{180}\approx\cdot\ 0.059\,\pi\ \textbf{radians}$$

distance from city A to city B

$$s=\theta r=0.059\,\pi(4000)=235.511\,\pi\approx 739.88\ \textbf{miles (2nd best)}$$

$$\textbf{APPROXIMATE}\ 10.598\cdot\frac{\pi}{180}\approx\cdot\ 0.185\ \textbf{radians}$$

distance from city A to city B

$$s=\theta r\approx 0.185(4000)\approx 740.\ \textbf{miles (worst)}$$

Given Information

$\csc\theta = \frac{\mathbf{hyp}}{\mathbf{opp}} = \frac{r}{y} = \frac{4}{3}$

Missing adjacent leg $\sqrt{\mathbf{hyp}^2 - \mathbf{opp}^2} = \sqrt{7}$

$\theta = \sin^{-1}\left(\frac{\mathbf{opp}}{\mathbf{hyp}}\right) = \sin^{-1}\left(\frac{3}{4}\right) \approx 48.5904\ °$

Other trigonometric ratios

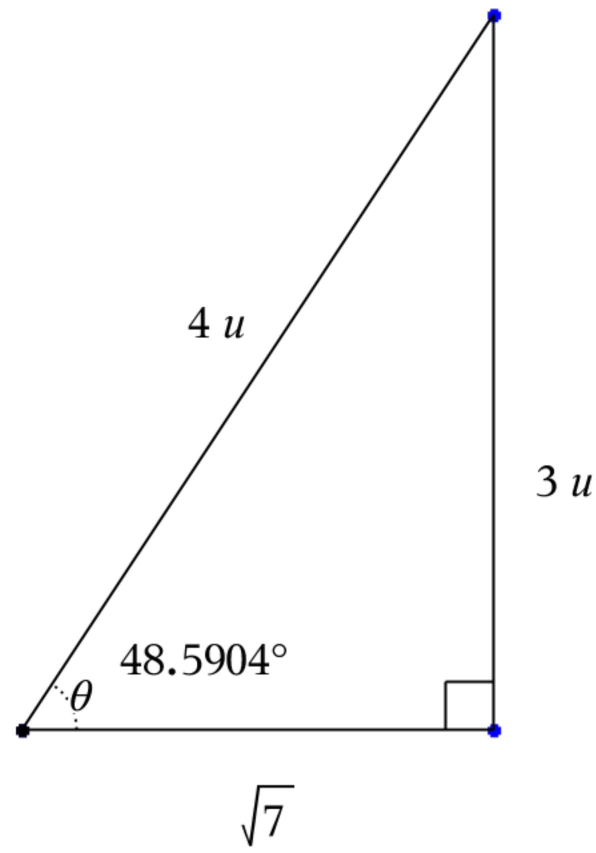
$\sin\theta = \frac{\mathbf{opp}}{\mathbf{hyp}} = \frac{y}{r} = \frac{3}{4}$

$\cos\theta = \frac{\mathbf{adj}}{\mathbf{hyp}} = \frac{x}{r} = \frac{\sqrt{7}}{4}$

$\tan\theta = \frac{\mathbf{opp}}{\mathbf{adj}} = \frac{y}{x} = 3/\sqrt{7} = \frac{3\cdot\sqrt{7}}{7}$

$\sec\theta = \frac{\mathbf{hyp}}{\mathbf{adj}} = \frac{r}{x} = 4/\sqrt{7} = \frac{4\cdot\sqrt{7}}{7}$

$\cot\theta = \frac{\mathbf{adj}}{\mathbf{opp}} = \frac{x}{y} = \frac{\sqrt{7}}{3}$



Given Information $\textbf{opp} = 12$ $\textbf{hyp} = 18$

Missing adjacent leg $\sqrt{\textbf{hyp}^2 - \textbf{opp}^2} = \sqrt{180} = 6 \cdot \sqrt{5}$

$\theta = \sin^{-1}\left(\frac{\textbf{opp}}{\textbf{hyp}}\right) = \sin^{-1}\left(\frac{2}{3}\right) \approx 41.8103\ °$

Other trigonometric ratios

$\sin\theta = \frac{\textbf{opp}}{\textbf{hyp}} = \frac{y}{r} = 12/18 = \frac{2}{3}$

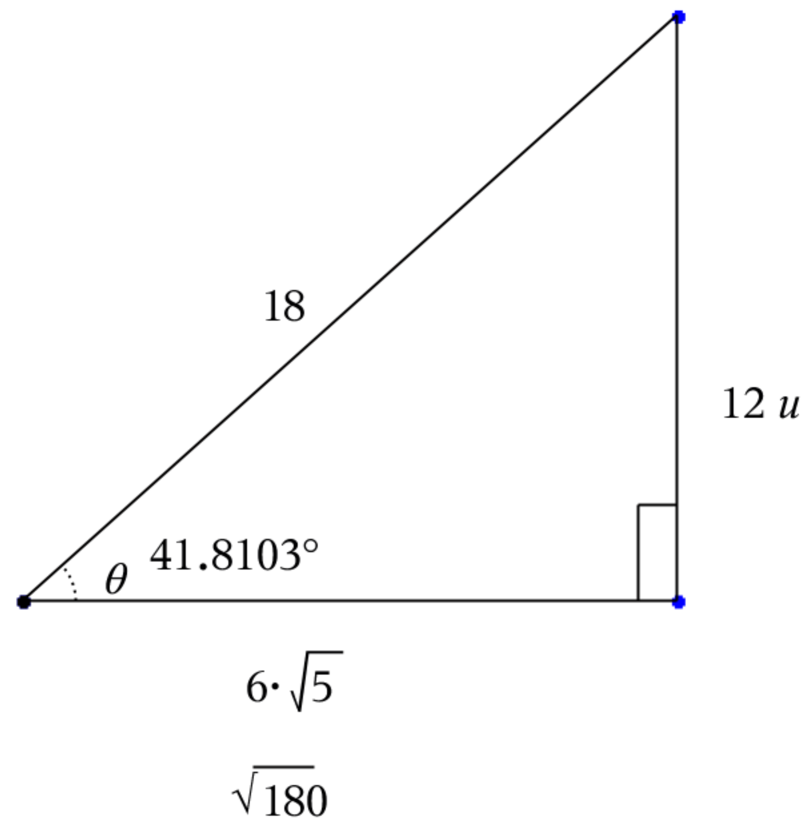
$\cos\theta = \frac{\textbf{adj}}{\textbf{hyp}} = \frac{x}{r} = \sqrt{180}/18 = \frac{\sqrt{5}}{3}$

$\tan\theta = \frac{\textbf{opp}}{\textbf{adj}} = \frac{y}{x} = 12/\sqrt{180} = \frac{2 \cdot \sqrt{5}}{5}$

$\csc\theta = \frac{\textbf{hyp}}{\textbf{opp}} = \frac{r}{y} = 18/12 = \frac{3}{2}$

$\sec\theta = \frac{\textbf{hyp}}{\textbf{adj}} = \frac{r}{x} = 18/\sqrt{180} = \frac{3 \cdot \sqrt{5}}{5}$

$\cot\theta = \frac{\textbf{adj}}{\textbf{opp}} = \frac{x}{y} = \sqrt{180}/12 = \frac{\sqrt{5}}{2}$



Given: 25 ° hypotenuse = 20

Want

x = adjacent side

y = opposite side

$\sin 25° = \frac{\textbf{opp}}{\textbf{hyp}} = \frac{y}{20}$

$y = 20 \sin 25°$

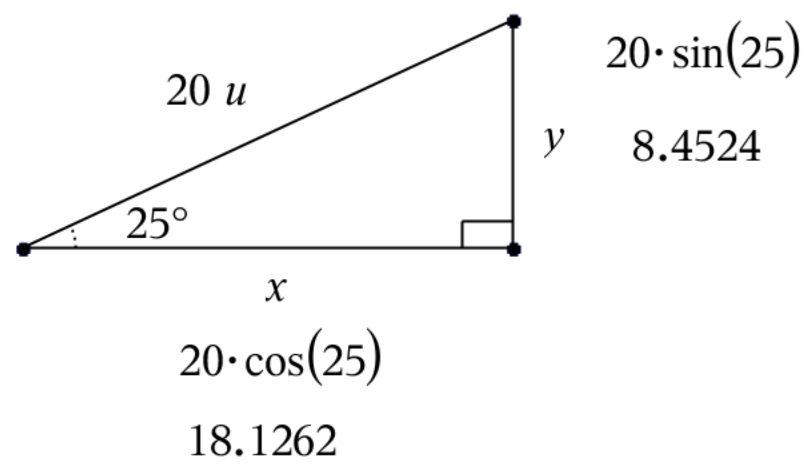
$y \approx 8.45237$

$\cos 25° = \frac{\textbf{adj}}{\textbf{hyp}} = \frac{x}{20}$

$x = 20 \cos 25°$

$x \approx 18.1262$

$\tan \theta = \frac{\textbf{opp}}{\textbf{adj}} = \frac{y}{x}$ (not enough initial information)



Given: 16 ° hypotenuse =15

Want

x = adjacent side

y = opposite side

$\sin\ 16° = \frac{\textbf{opp}}{\textbf{hyp}} = \frac{x}{w}$ (not enough initial information)

$\cos\ 16° = \frac{\textbf{adj}}{\textbf{hyp}} = \frac{15}{w}$

$\textbf{hyp} = \frac{15}{\cos(16)}$

$\textbf{hyp} \approx 15.6045$

$\tan 16° = \frac{\textbf{opp}}{\textbf{adj}} = \frac{x}{15}$

$x = 15 \cdot \tan(16)$

$x \approx 4.30118$