

General Solution Process for the Pulley Problem

Step 1: Determine the given information

On this quiz Diameter of the winch's drum was given in inches Diameter/2 = radius
Number of winch drum revolutions per minute
Distance needed to pull jeep out given, but given in feet

Step 2: Use given to determine distance to pull jeep out in inches not feet

12 feet given = distance in inches

Step 3: Convert pulley rpm to radians

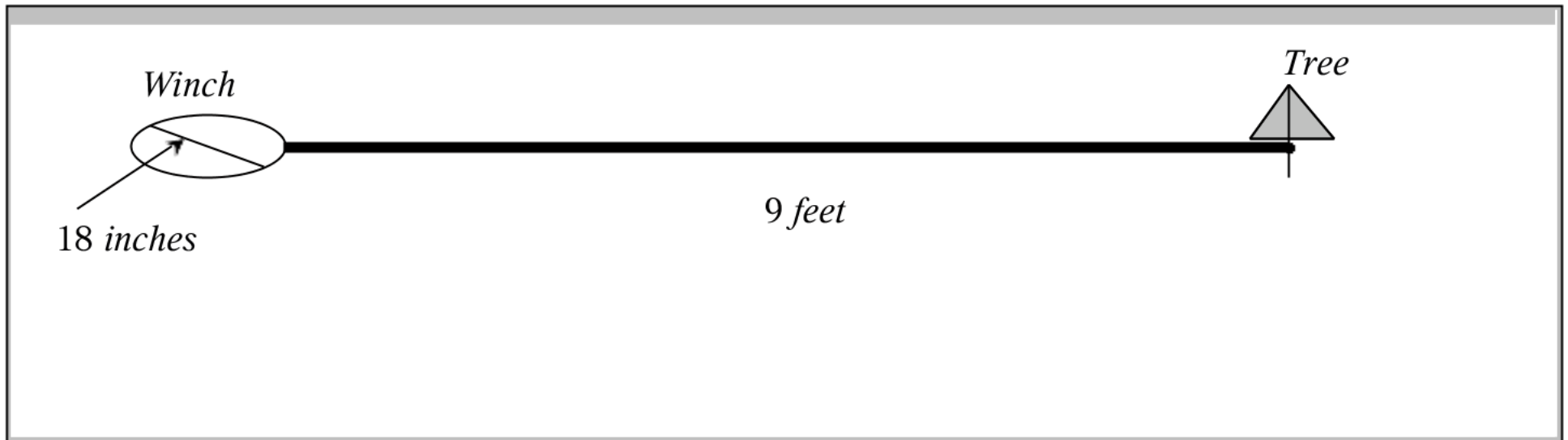
$$\theta = (\text{rpm})(2 \cdot \pi) = \text{radians}$$

Step 4: Determine arc length and linear speed of the drum

$$s = \theta \text{ radius and linear speed} = \frac{s}{t}$$

Step 5: Set up equation with linear speed and distance to pull jeep out

$$(\text{Linear Speed})(x \text{ minutes}) = \text{distance to pull out jeep}$$



Given: $d = 18$ inches and $r = 9$ inches

$\text{rpm} = 7$ rev./min

jeep needs to move 12 feet or 144 inches

Find $\theta = \text{rpm} \cdot 2 \cdot \pi = 14 \cdot \pi$ radians

Find arc length and linear speed

$$s = \theta r = (14 \cdot \pi)(9) = 126 \cdot \pi$$

linear speed is $126 \cdot \pi$ /min

Set up equation to solve for time

(linear speed)(time) = jeep distance

$$(126 \cdot \pi)(x) = 144$$

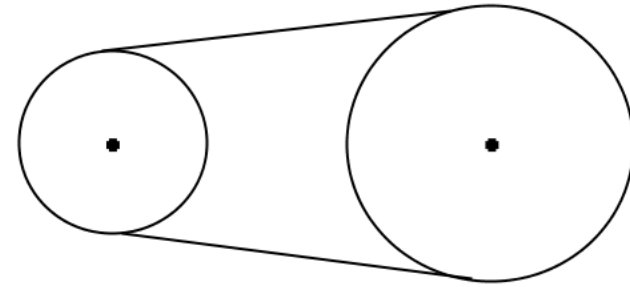
$$x = 144 / 126 \cdot \pi$$

$$x = \frac{8}{7 \cdot \pi} \approx 0.363783 \text{ minutes}$$

a. Determine the number of revolutions that the fan is moving in the same minute.

b) Determine the angular speed of the fan in radians per minute

c. Determine the linear speed of the fan in FEET per minute.



diameter of small pulley = 14 inches

radius of small pulley = 7 inches

rpm of small pulley = 200 rev/min

diameter of fan pulley = 24 inches

radius of fan pulley = 12 inches

a. Determine the number of revolutions that the fan is moving in the same minute.

Step 1 Convert rpms of small pulley into radians

$$\theta \text{ of small pulley} = \text{rpm} (2 \cdot \pi) = (200)(2 \cdot \pi) = 400 \cdot \pi$$

Step 2: Determine how far the small pulley is moving the belt

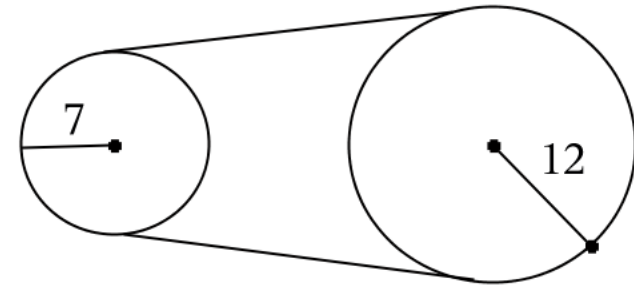
$$s = \theta r \text{ implies } (400 \cdot \pi)(7) = 2800 \cdot \pi$$

Step 3: Convert the distance that belt moves into radians of fan using arc length equation

$$s = \theta r \text{ implies } 2800 \cdot \pi = \theta(12)$$

$$2800 \cdot \pi / 12 = 12 \theta / 12$$

$$\theta \text{ of fan} = \frac{700 \cdot \pi}{3} \text{ radians}$$



small pulley

fan pulley

rpm of small pulley

200

Step 4: Convert radians of fan into revolutions of fan

$$\text{rev} = \text{radians} / (2 \cdot \pi)$$

$$= \left(\frac{700 \cdot \pi}{3} \right) / (2 \cdot \pi)$$

$$= \frac{700 \cdot \pi}{3} \cdot \frac{1}{2 \cdot \pi} = \frac{350}{3} = 116 + \frac{2}{3}$$

≈ 116.667 rpm of fan

b) Determine the angular speed of the fan in radians per minute

Since the unit of time is 1 minute, angular speed is radians of fan

$$\begin{aligned} \text{Angular speed of fan} &= \frac{\theta}{t} \\ &= \frac{700 \cdot \pi}{3} \text{ radians/min} \approx 733.038 \text{ radians/min} \end{aligned}$$

c. Determine the linear speed of the fan in FEET per minute.

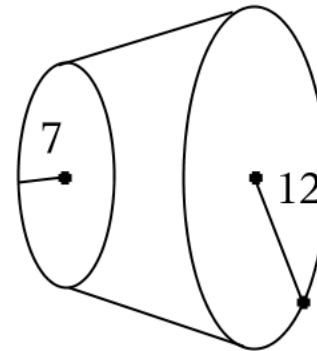
Linear speed = θr

$$= \frac{700 \cdot \pi}{3} (12) = 2800 \cdot \pi \text{ inches}$$

Now convert inches to feet

Linear speed = $2800 \cdot \pi$ inches/minute

$$\begin{aligned} &= (2800 \cdot \pi \text{ inches}) \cdot \frac{1 \text{ foot}}{12 \text{ inches}} \\ &= (2800 \cdot \pi) / 12 = \frac{700 \cdot \pi}{3} \text{ feet/min} \\ &\approx 733.038 \text{ feet/min} \end{aligned}$$



small pulley

fan pulley

rpm of small pulley

200

rpm of fan

$\frac{350}{3}$

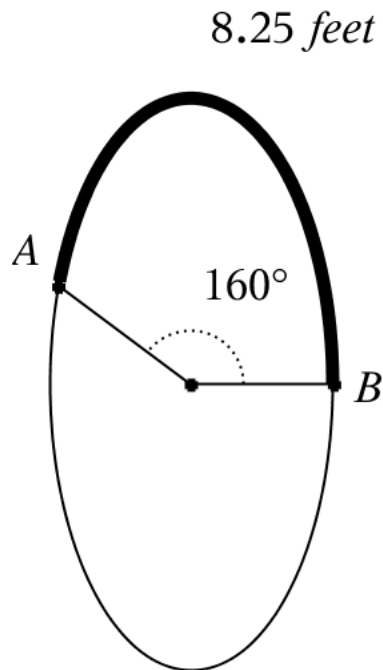
small pulley radians

$400 \cdot \pi$

fan radians

$\frac{700 \cdot \pi}{3}$

6.1.2 arc length to radius



Determine radius of circle

Step 1: Convert Degrees to Radians

$$\theta = 160^\circ \cdot \frac{\pi}{180^\circ} = \frac{8 \cdot \pi}{9} \text{ radians}$$

Step 2: Solve arc length equation for r

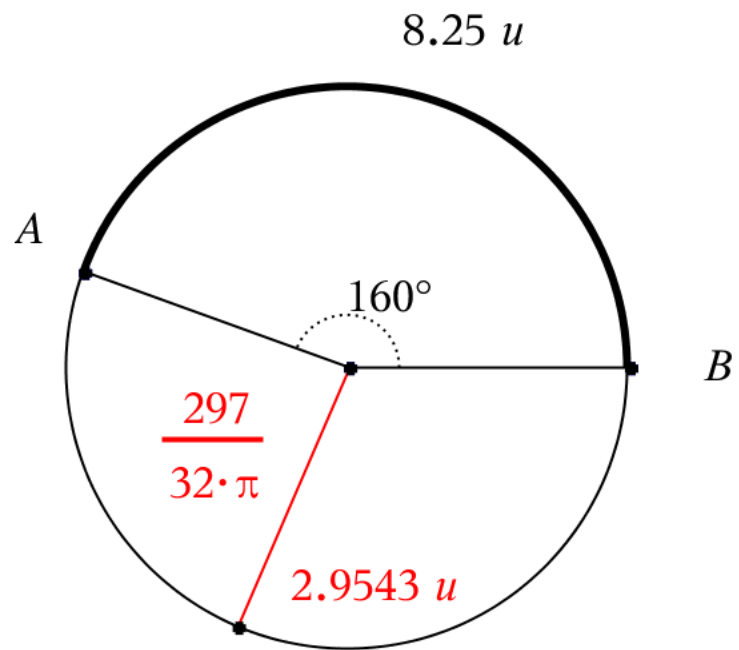
$$s = \theta r$$

$$8.25 = \frac{8 \cdot \pi}{9} r$$

$$\frac{33}{4} = \frac{8 \cdot \pi}{9} r$$

$$\frac{33}{4} \cdot \frac{9}{8 \cdot \pi} = \frac{8 \cdot \pi}{9} \cdot \frac{9}{8 \cdot \pi} r$$

$$r = \frac{297}{32 \cdot \pi} = 8.25 / \left(\frac{8 \cdot \pi}{9} \right) \approx 2.95431$$



6.1.2 latitude

Latitude Problem (assume both latitudes lie in northern hemisphere)

City A is due North of city B Assume earth's radius ≈ 4000 miles

City A's latitude = $18^\circ 21' 40''$ N City B's latitude = $7^\circ 45' 32''$ N

Step 1: Find the DMS difference in latitudes $18^\circ 21' 40'' - 7^\circ 45' 32'' = 10^\circ 36' 8''$

Step 2: Convert DMS difference to DD difference

$$10^\circ 36' 8''$$

$$10 + 36/60 + 8/3600 = \frac{4771}{450} \approx 10.602^\circ$$

Step 3: Convert DD difference to Radian difference

$$\frac{4771}{450} \approx 10.602^\circ \quad \text{EXACT} \quad \frac{4771}{450} \cdot \frac{\pi}{180} = \frac{4771 \cdot \pi}{81000} \text{ radians}$$

$$\text{APPROXIMATE } 10.602 \cdot \frac{\pi}{180} \approx 0.059 \pi \text{ radians}$$

$$\text{APPROXIMATE } 10.602 \cdot \frac{\pi}{180} \approx 0.185 \text{ radians}$$

Depending on which version of the radian difference will determine distance between cities

Step 4: Use Arc Length to determine distance between cities

$$\text{EXACT } \frac{4771}{450} \cdot \frac{\pi}{180} = \frac{4771 \cdot \pi}{81000} \text{ radians}$$

distance from city A to city B

$$s = \theta r = \frac{4771 \cdot \pi}{81000} (4000) = \frac{19084 \cdot \pi}{81} \approx 740.175 \text{ miles (best)}$$

$$\text{APPROXIMATE } 10.602 \cdot \frac{\pi}{180} \approx 0.059 \pi \text{ radians}$$

distance from city A to city B

$$s = \theta r = 0.059 \pi (4000) = 235.6 \pi \approx 740.159 \text{ miles (2nd best)}$$

$$\text{APPROXIMATE } 10.602 \cdot \frac{\pi}{180} \approx 0.185 \text{ radians}$$

distance from city A to city B

$$s = \theta r \approx 0.185 (4000) \approx 740 \text{ miles (worst)}$$

6.2.2 reciprocal identity problem

Given Information

$$\sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x} = \frac{6}{5}$$

$$\text{Missing opposite leg } \sqrt{\text{hyp}^2 - \text{adj}^2} = \sqrt{11}$$

$$\theta = \cos^{-1}\left(\frac{\text{adj}}{\text{hyp}}\right) = \cos^{-1}\left(\frac{5}{6}\right) \approx 33.5573^\circ$$

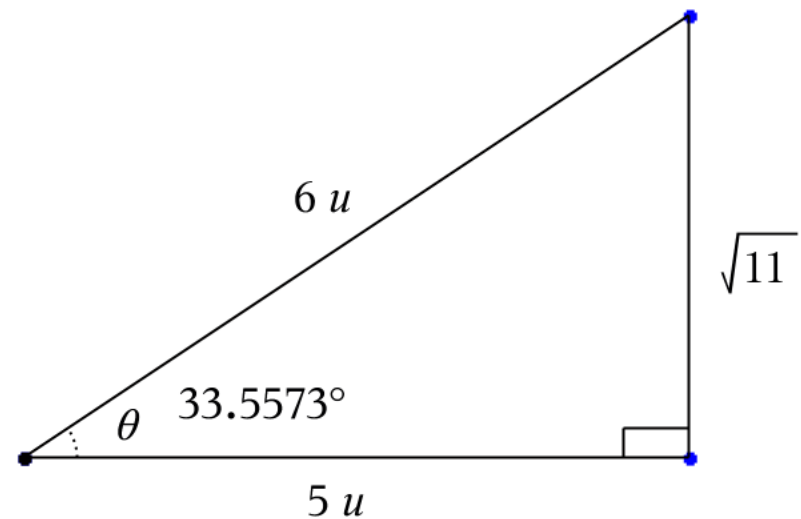
Other trigonometric ratios

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} = \frac{\sqrt{11}}{6}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} = \frac{5}{6}$$

$$\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y} = 6/\sqrt{11} = \frac{6 \cdot \sqrt{11}}{11}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{x}{y} = 5/\sqrt{11} = \frac{5 \cdot \sqrt{11}}{11}$$



6.2.2 opp hyp

Given Information **opp** = 21 **hyp** = 24

Missing adjacent leg $\sqrt{\text{hyp}^2 - \text{opp}^2} = \sqrt{135} = 3 \cdot \sqrt{15}$

$$\theta = \sin^{-1}\left(\frac{\text{opp}}{\text{hyp}}\right) = \sin^{-1}\left(\frac{7}{8}\right) \approx 61.045^\circ$$

Other trigonometric ratios

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} = 21/24 = \frac{7}{8}$$

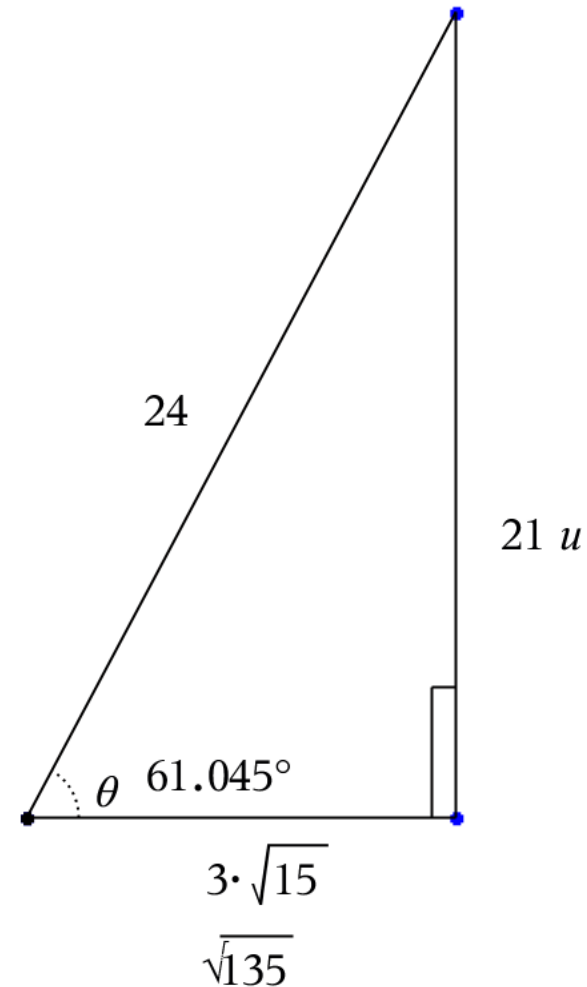
$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} = \sqrt{135}/24 = \frac{\sqrt{15}}{8}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x} = 21 / \sqrt{135} = \frac{7 \cdot \sqrt{15}}{15}$$

$$\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y} = 24/21 = \frac{8}{7}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x} = 24 / \sqrt{135} = \frac{8 \cdot \sqrt{15}}{15}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{x}{y} = \sqrt{135}/21 = \frac{\sqrt{15}}{7}$$



6.2.2 hyp angle

Given: 24° hypotenuse = 38

Want

x = adjacent side

y = opposite side

$$\sin 24^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{y}{38}$$

$$y = 38 \sin 24^\circ$$

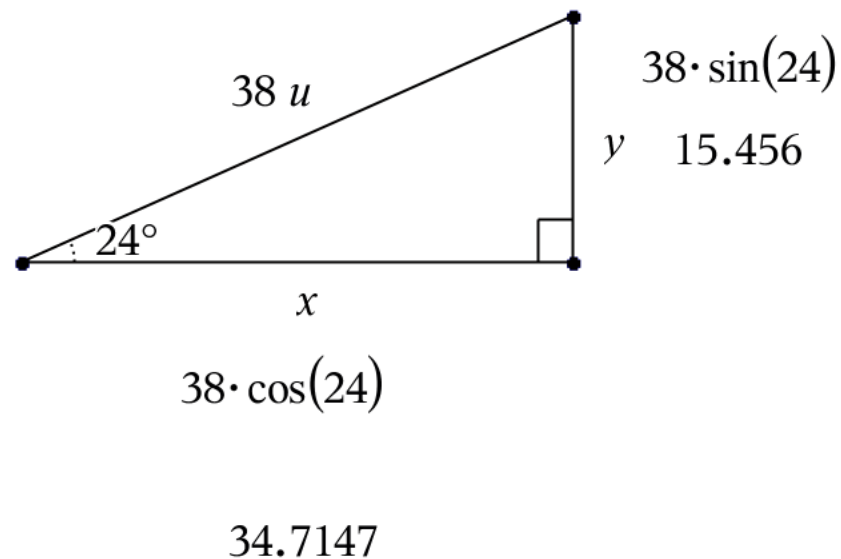
$$y \approx 15.456$$

$$\cos 24^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{x}{38}$$

$$x = 38 \cos 24^\circ$$

$$x \approx 34.7147$$

$$\tan 24^\circ = \frac{\text{opp}}{\text{adj}} = \frac{y}{x} \quad (\text{not enough initial information})$$



6.2.2 adj angle

Given: 27° hypotenuse = 20

Want

x = adjacent side

y = opposite side

$$\sin 27^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{x}{w} \quad (\text{not enough initial information})$$

$$\cos 27^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{20}{w}$$

$$\text{hyp} = \frac{20}{\cos(27)}$$

$$\text{hyp} \approx 22.4465$$

$$\tan 27^\circ = \frac{\text{opp}}{\text{adj}} = \frac{x}{20}$$

$$\text{op} = 20 \cdot \tan(27)$$

$$\text{op} \approx 10.1905$$

