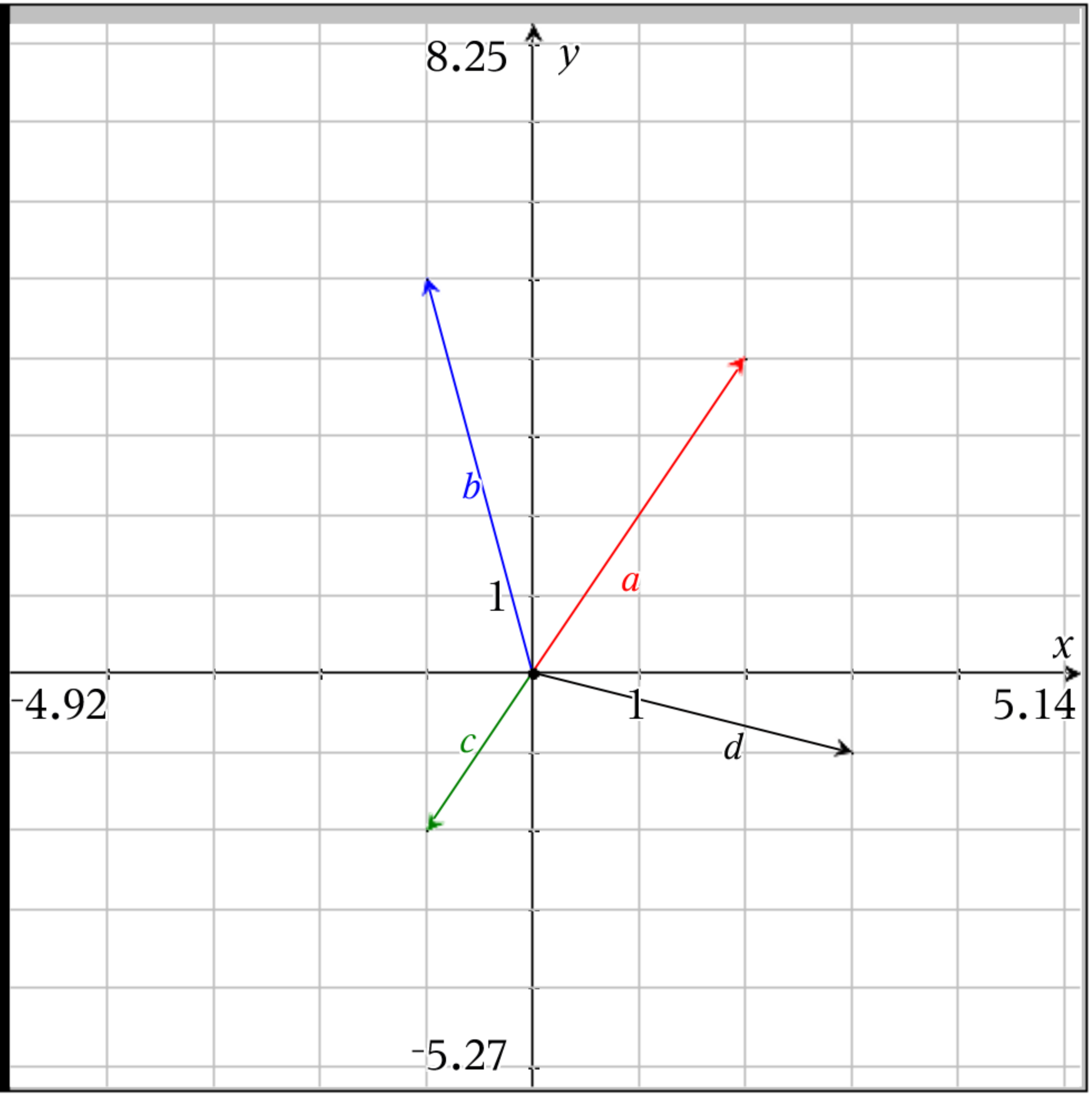
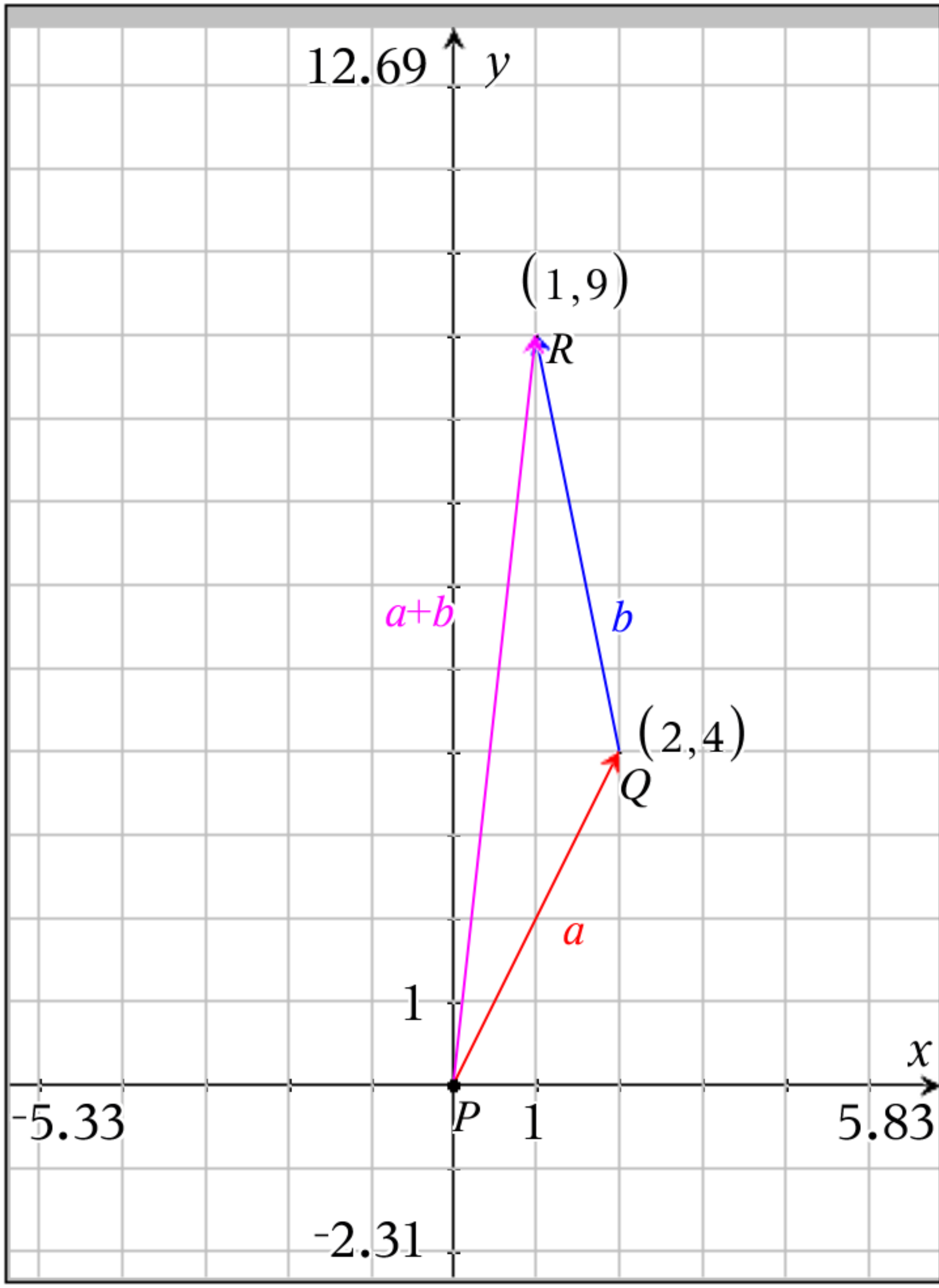
Problem 1

	K x_list6	L y_list6
=		
1	-1	-2
2	-4	-1
3	-8	-2
4	8	2
5		
6		
7		
8		
9		
10		
11		

L5



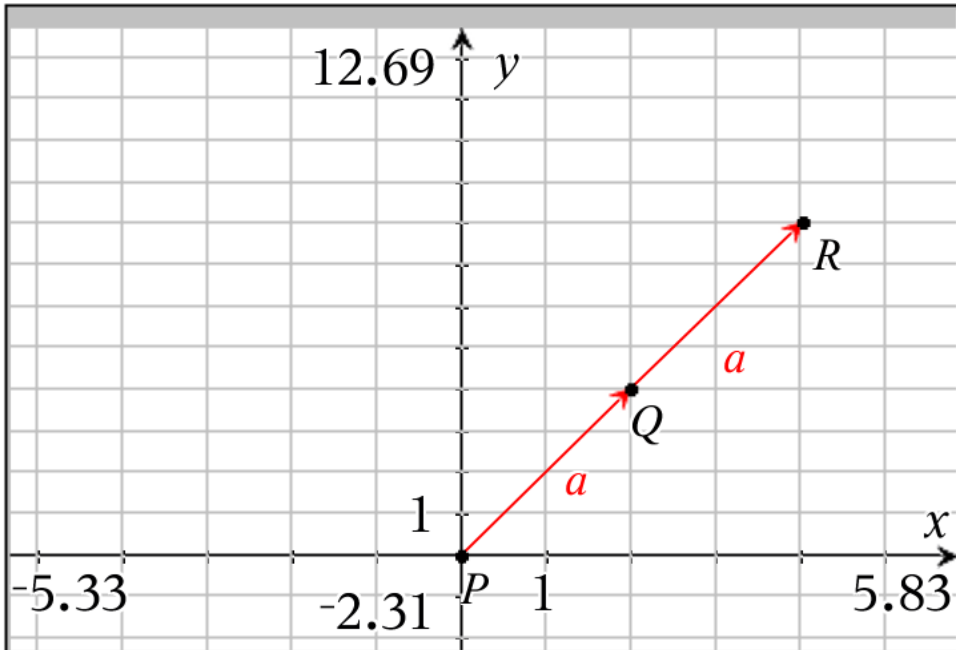


plot a+b

$a = \begin{bmatrix} \mathbf{a\_dx} \\ \mathbf{a\_dy} \end{bmatrix} \blacktriangleright \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $b = \begin{bmatrix} ^-1 \\ 5 \end{bmatrix}$

vector a+b $= \begin{bmatrix} 2 \\ 4 \end{bmatrix} + \begin{bmatrix} ^-1 \\ 5 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 9 \end{bmatrix}$

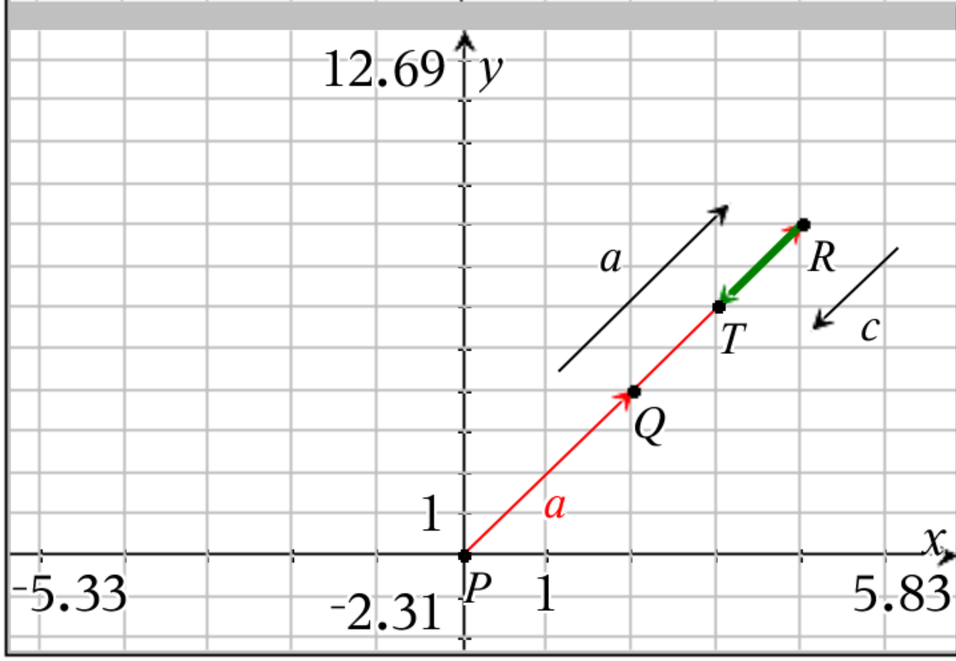


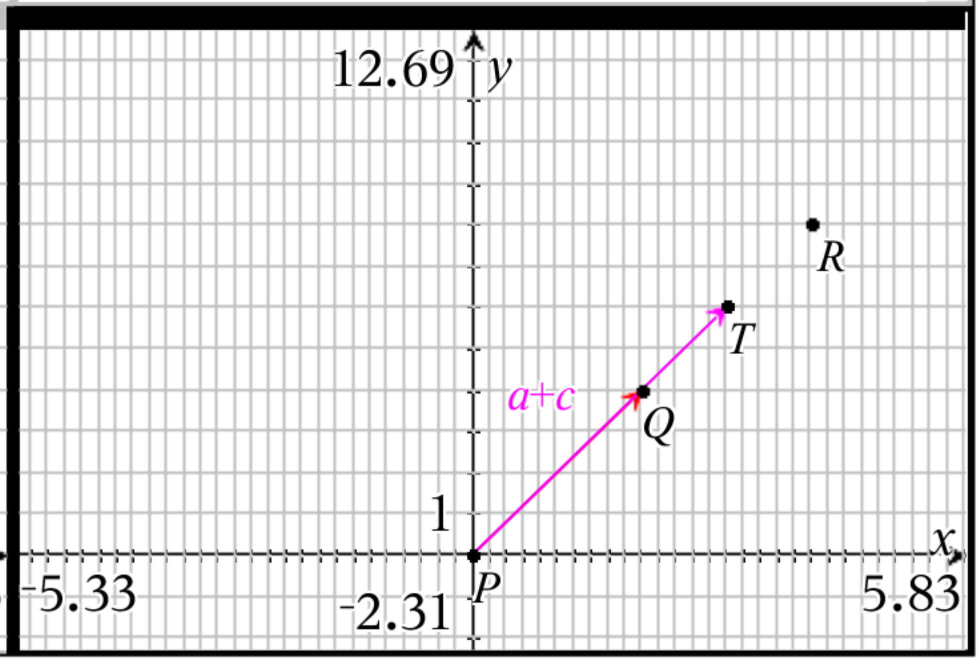
plot 2a+b

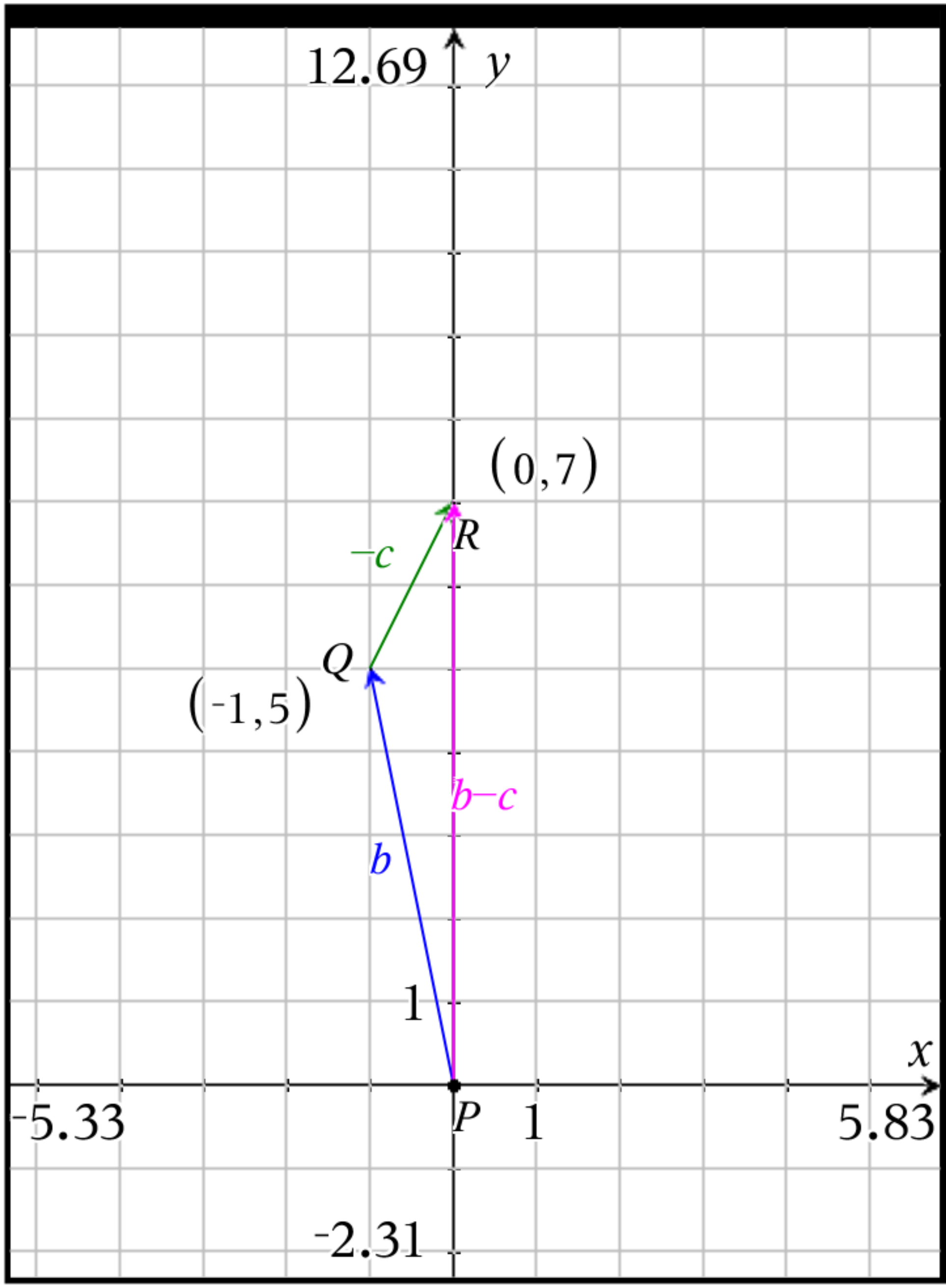
$a = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $c = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

vector 2a+c $= \begin{bmatrix} 4 \\ 8 \end{bmatrix} + \begin{bmatrix} -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

Note: Vector 2a+c is a scalar multiple of both a and c, a and c are parallel vectors





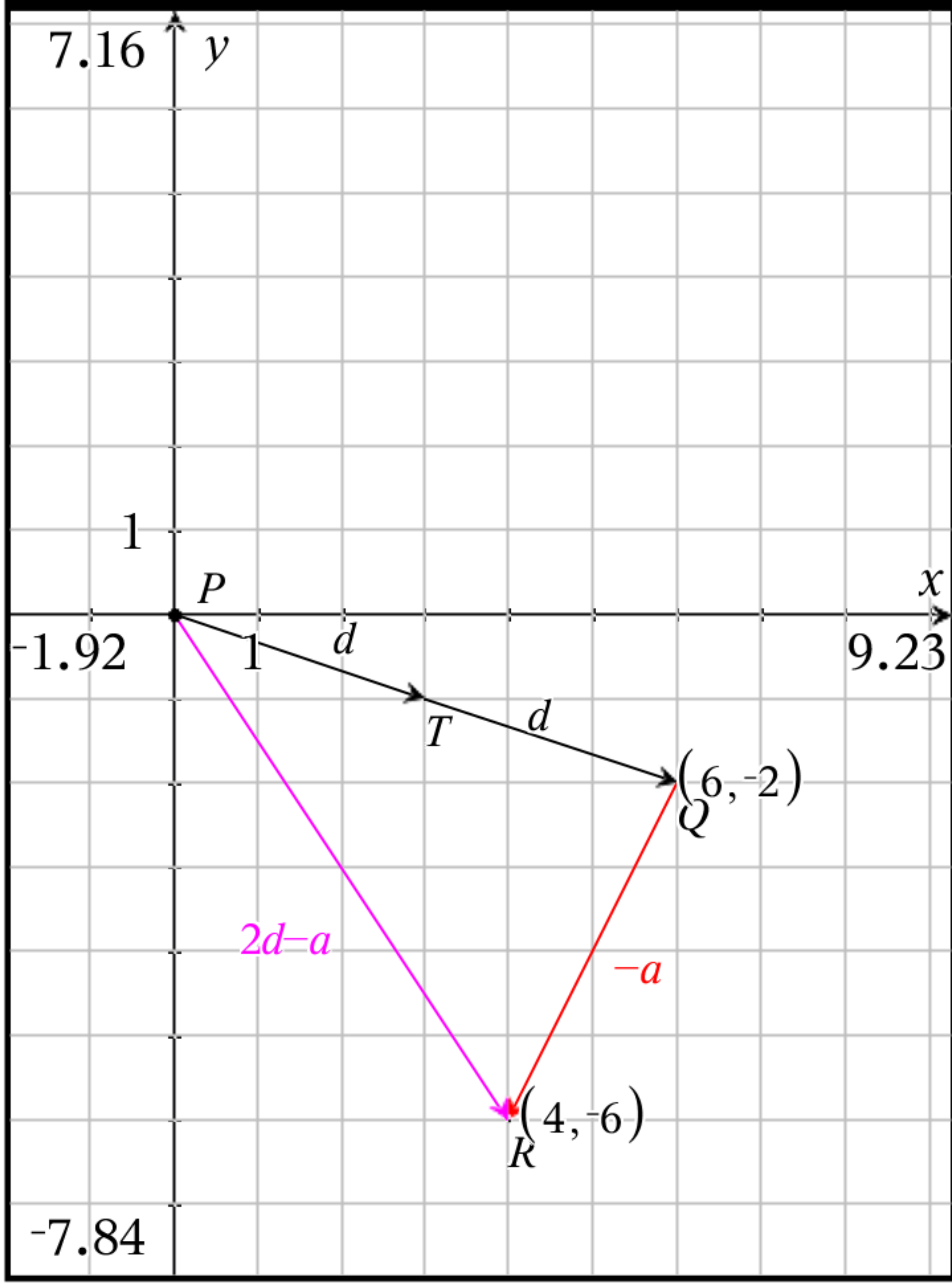


plot b−c

$b=\begin{bmatrix} -1 \\ 5 \end{bmatrix}$ $c=\begin{bmatrix} -1 \\ -2 \end{bmatrix}$

vector b−c $=\begin{bmatrix} -1 \\ 5 \end{bmatrix}-\begin{bmatrix} -1 \\ -2 \end{bmatrix}$

$=\begin{bmatrix} 0 \\ 7 \end{bmatrix}$

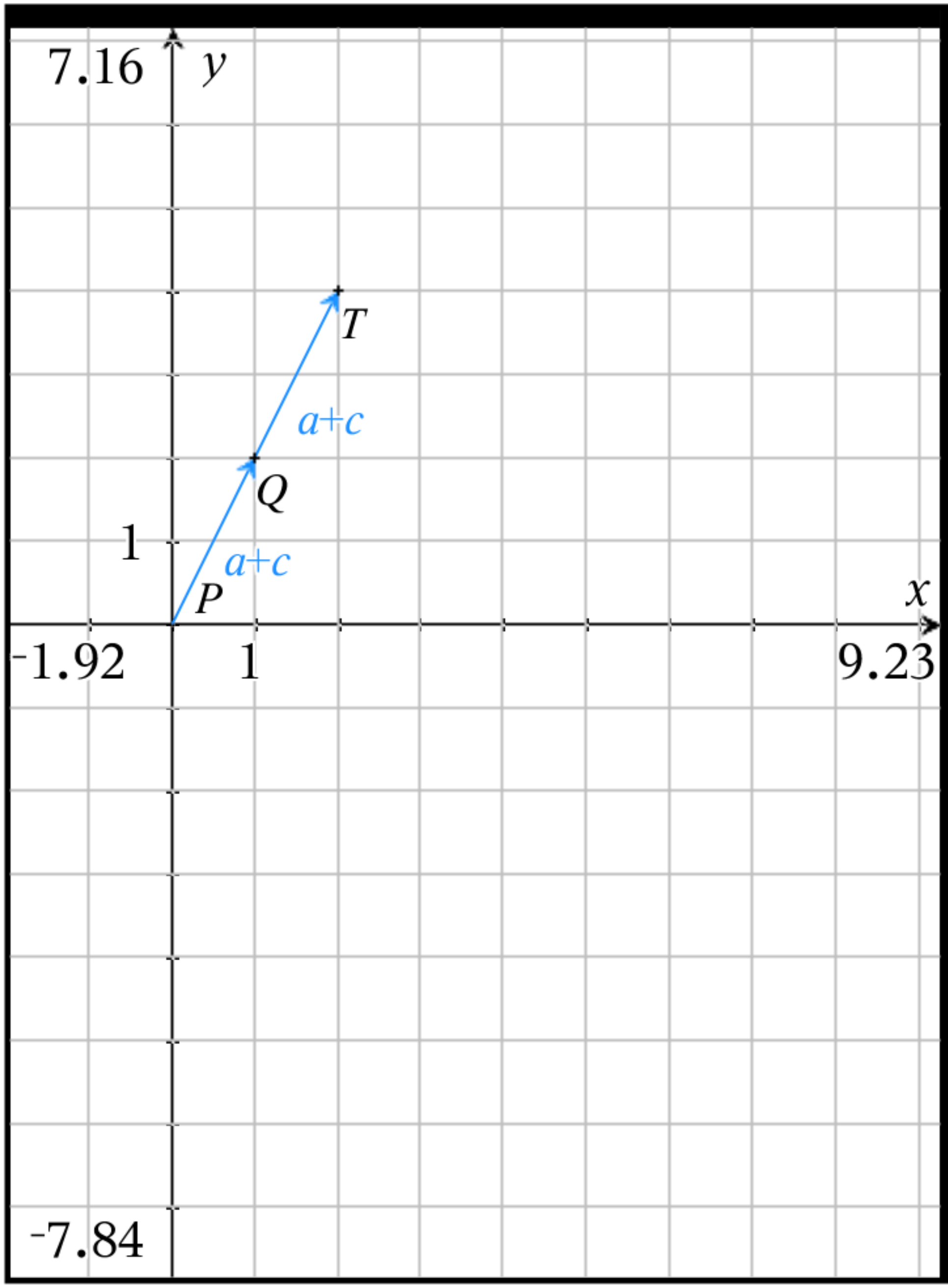


plot 2d−a

$a = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $d = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

vector 2d−a $= \begin{bmatrix} 6 \\ -2 \end{bmatrix} - \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$= \begin{bmatrix} 4 \\ -6 \end{bmatrix}$

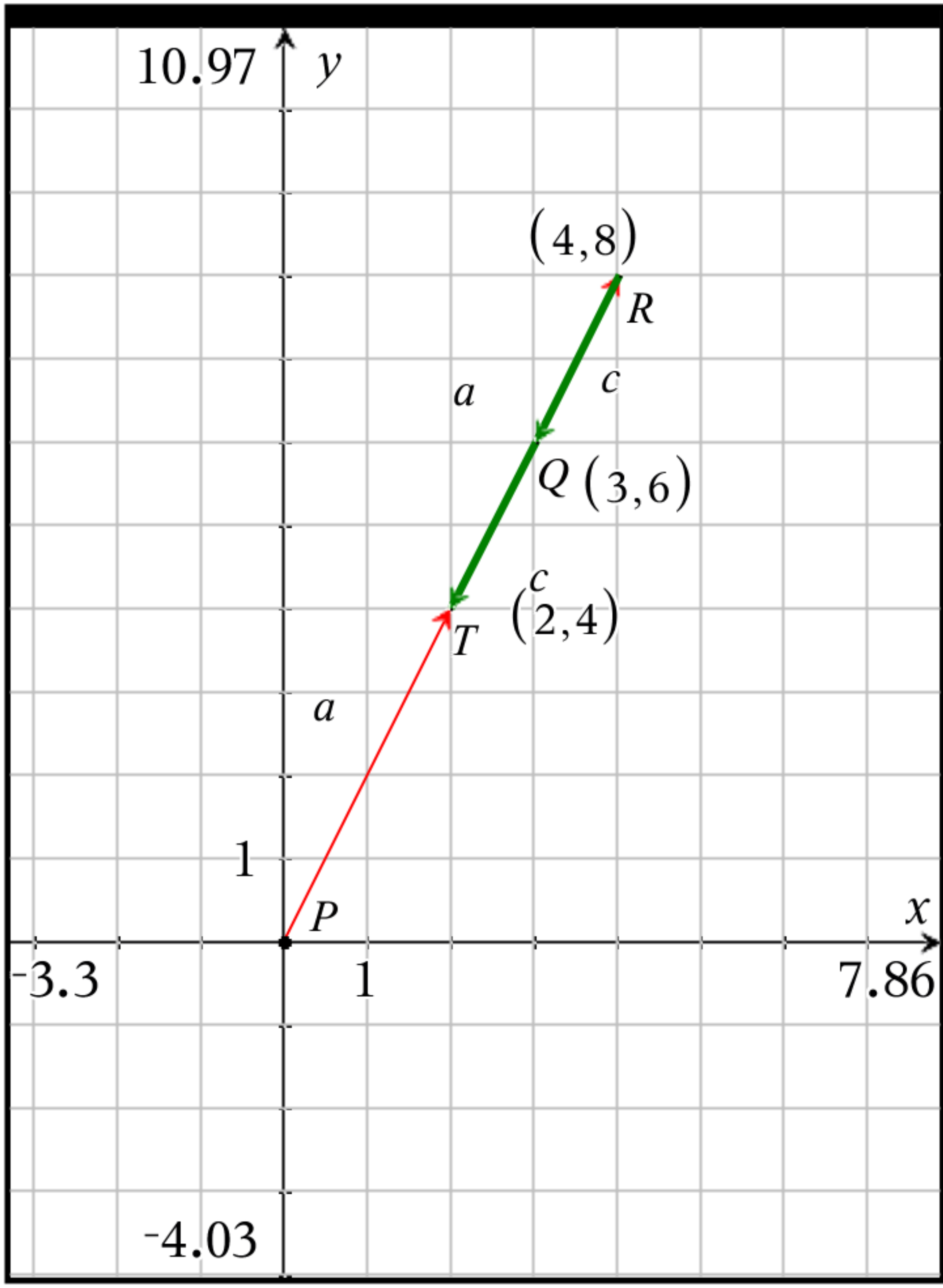


plot 2(a+c) DO NOT DISTRIBUTE

$a = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $c = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

vector a+c $\begin{bmatrix} 2 \\ 4 \end{bmatrix} + \begin{bmatrix} -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

vector 2(a+c) $= \begin{bmatrix} 2 \\ 4 \end{bmatrix}$



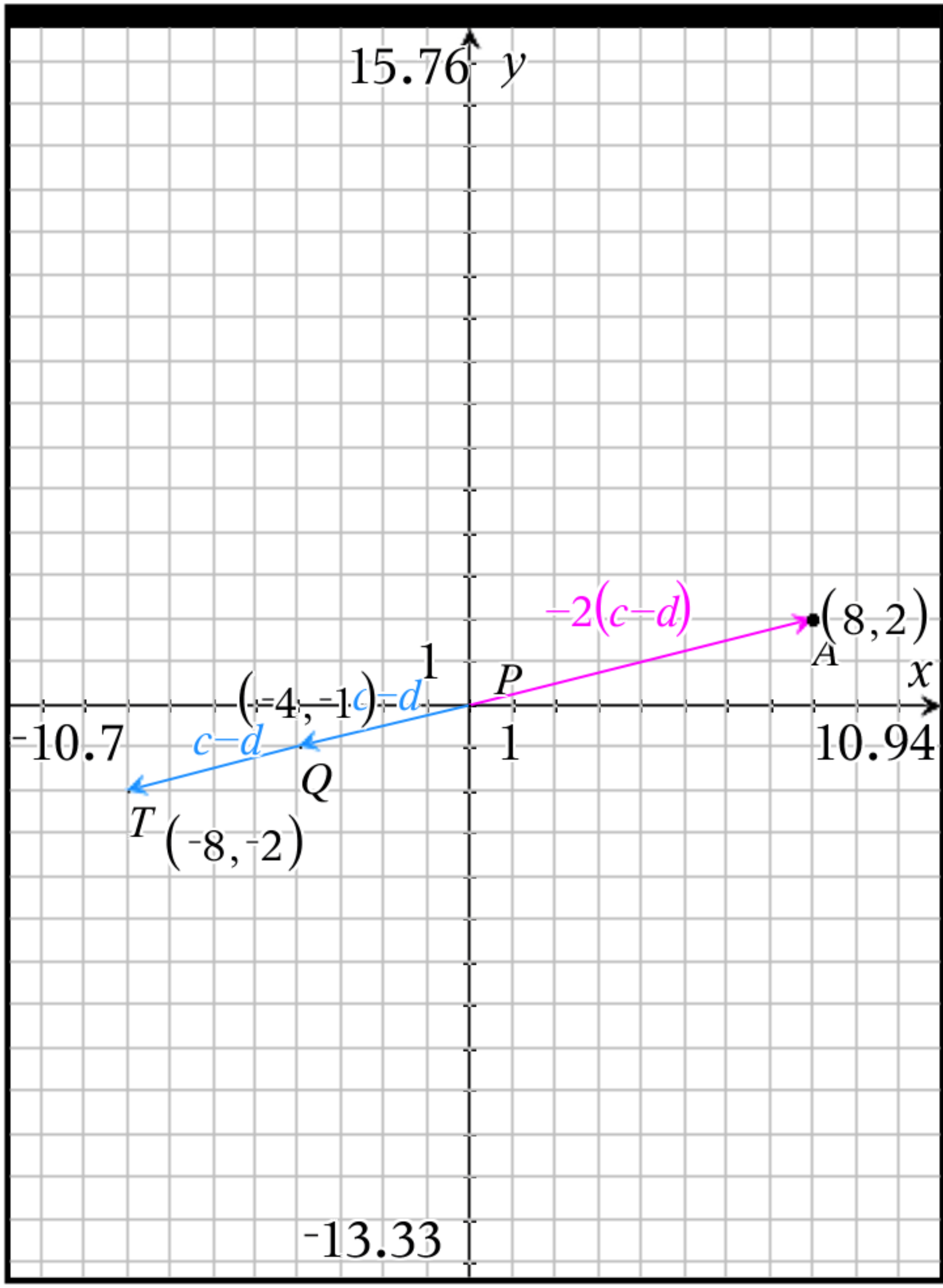
plot 2(a+c) DO DISTRIBUTE

$a = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $c = \begin{bmatrix} ^-1 \\ ^-2 \end{bmatrix}$

vector $2a = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

vector $2c = \begin{bmatrix} ^-2 \\ ^-4 \end{bmatrix}$

vector $2(a+c) = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

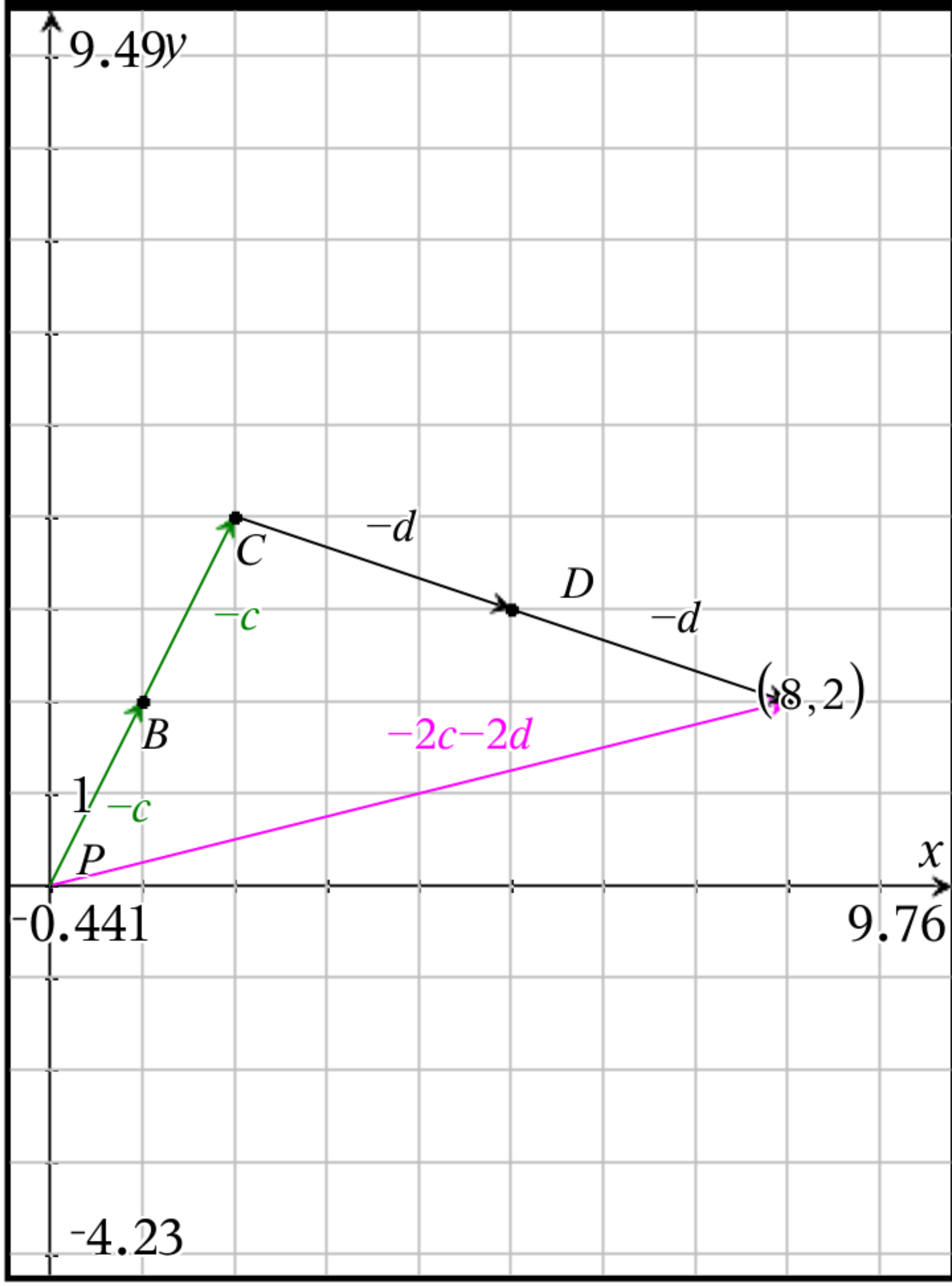


plot −2(c−d) DO NOT DISTRIBUTE

$c=\begin{bmatrix}-1\\-2\end{bmatrix}$ $d=\begin{bmatrix}3\\-1\end{bmatrix}$

vector c−d = $\begin{bmatrix}-1\\-2\end{bmatrix}-\begin{bmatrix}3\\-1\end{bmatrix}=\begin{bmatrix}-4\\-1\end{bmatrix}$

vector −2(c−d) $=\begin{bmatrix}8\\2\end{bmatrix}$



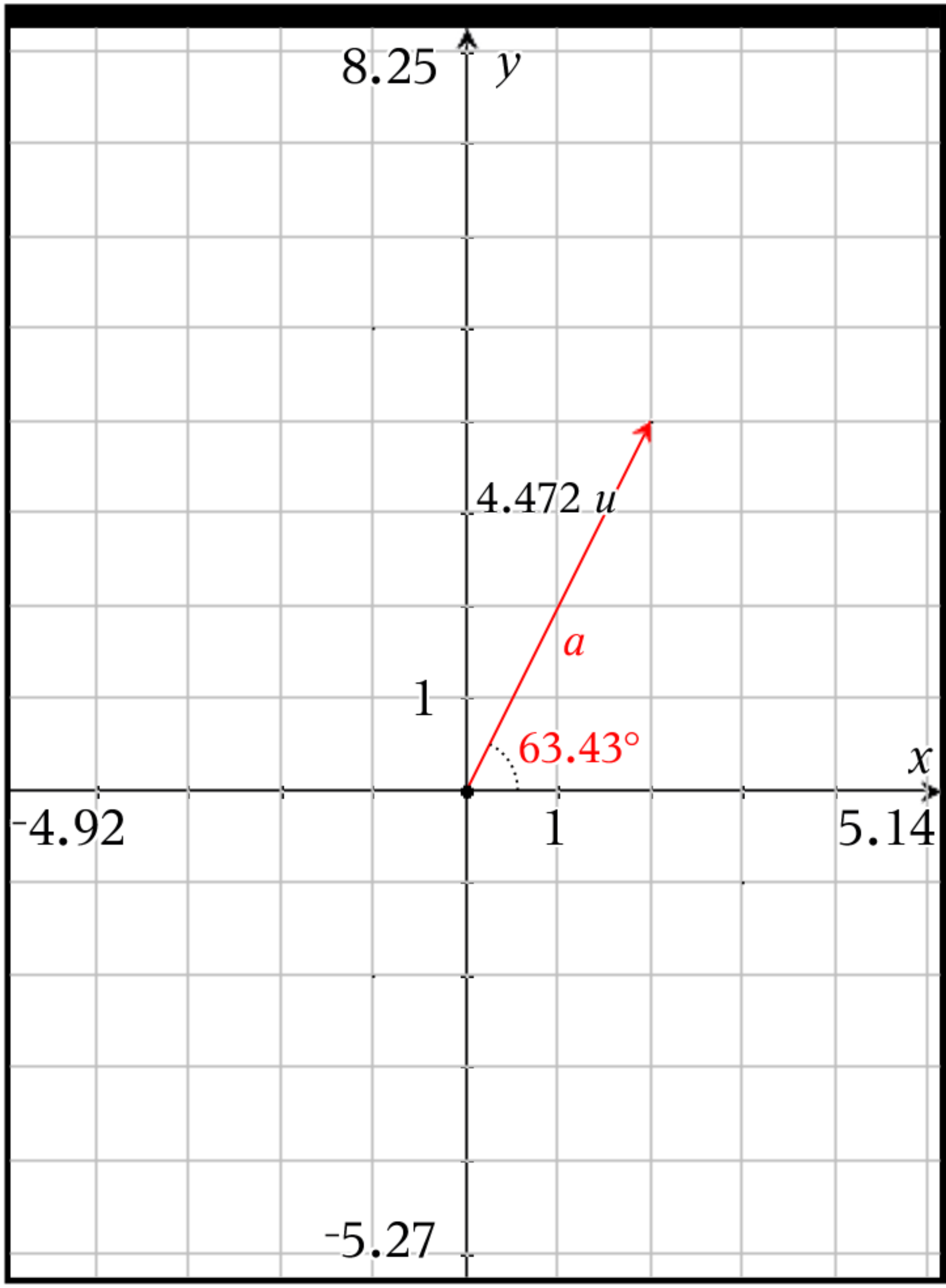
plot −2(c−d) DO DISTRIBUTE

$c=\begin{bmatrix} -1 \\ -2 \end{bmatrix}$ $d=\begin{bmatrix} 3 \\ -1 \end{bmatrix}$

vector $-2c=\begin{bmatrix} 2 \\ 4 \end{bmatrix}$

vector −2d $=\begin{bmatrix} -6 \\ 2 \end{bmatrix}$

vector $-2c-2d=\begin{bmatrix} 8 \\ 2 \end{bmatrix}$



Find directional angle related to vector a

Since this is a quadrant 1 angle (UP to RIGHT) there is NO REFERENCING

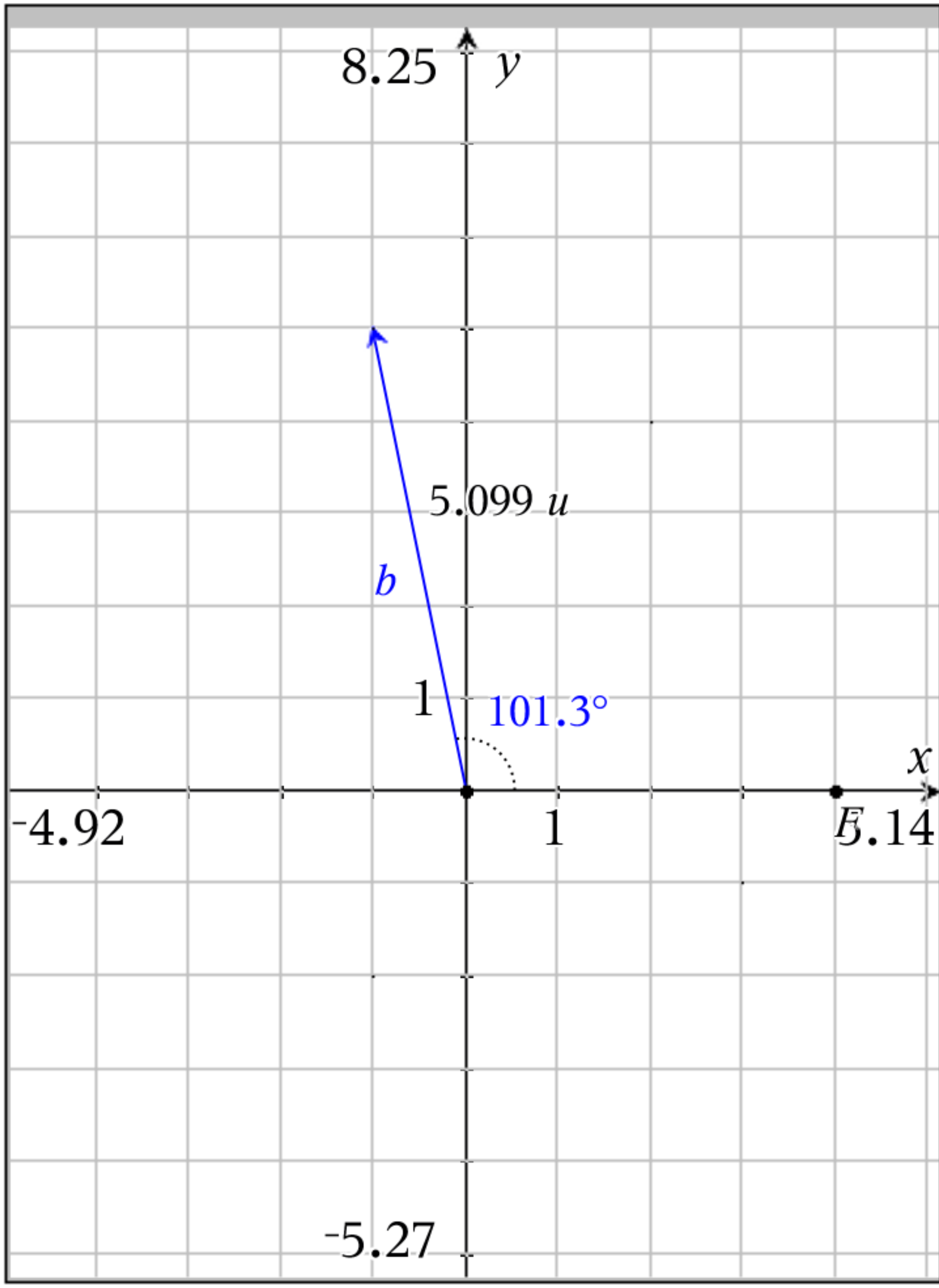
$a = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

angle $= \tan^{-1}(4/2) = 63.4349$ °

vector a has magnitude $= \sqrt{20}$

$= 2 \cdot \sqrt{5}$

≈ 4.47214



Find directional angle related to vector b

Since this is a quadrant 2 angle

(UP to LEFT) there is REFERENCING

$b = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

angle $= \tan^{-1}(5/-1) = -78.6901\ °$

BUT this is NOT in Quadrant 2

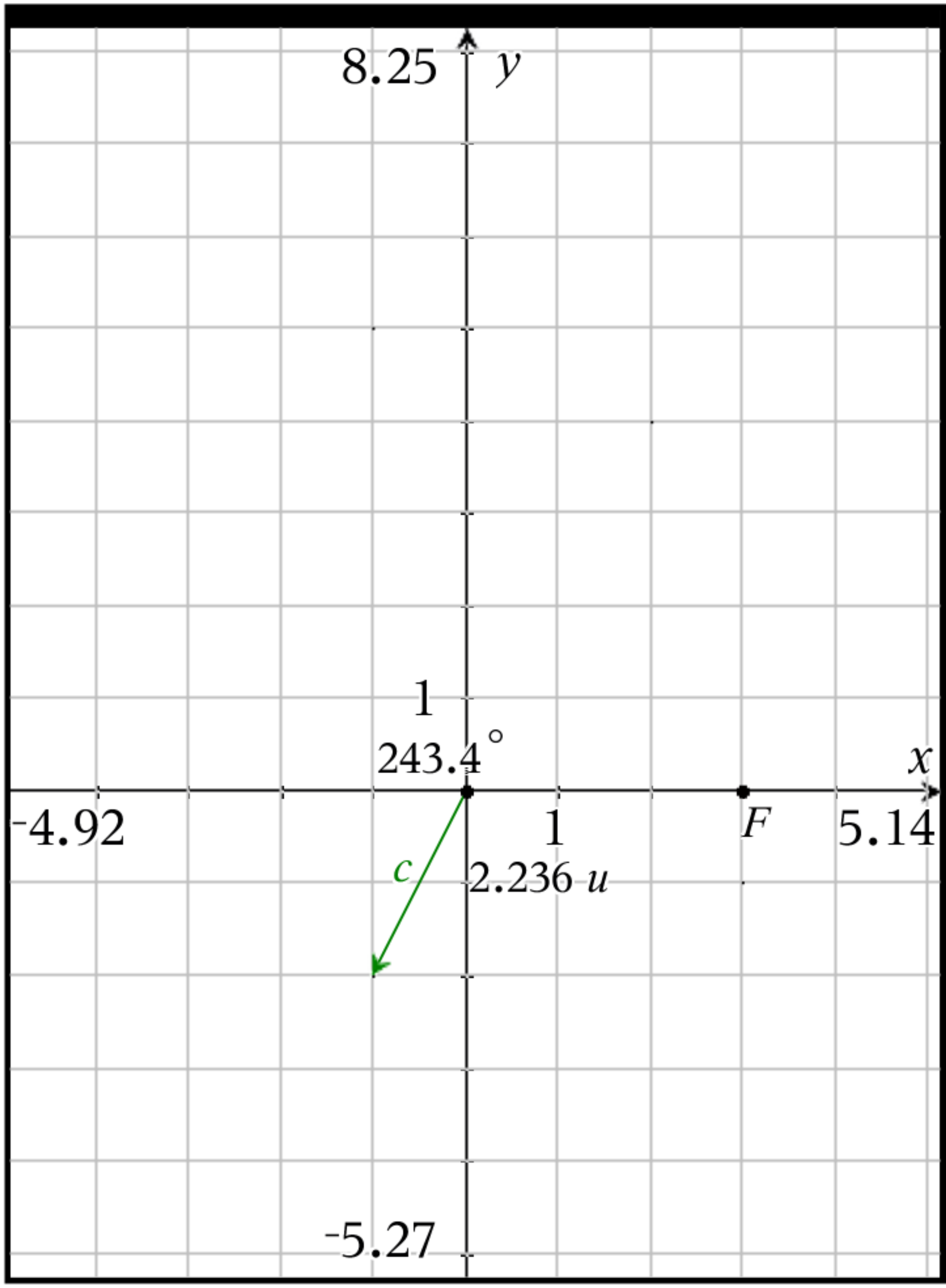
angle $= 180° + -78.6901\ °$

$= 101.31\ °$

vector b has magnitude $= \sqrt{26}$

$= \sqrt{26}$

≈ 5.09902



Find directional angle related to vector c

Since this is a quadrant 3 angle

(DOWN to LEFT) there is REFERENCING

$c = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

angle $= \tan^{-1}(-2/-1) = 63.4349°$

BUT this is NOT in Quadrant 3

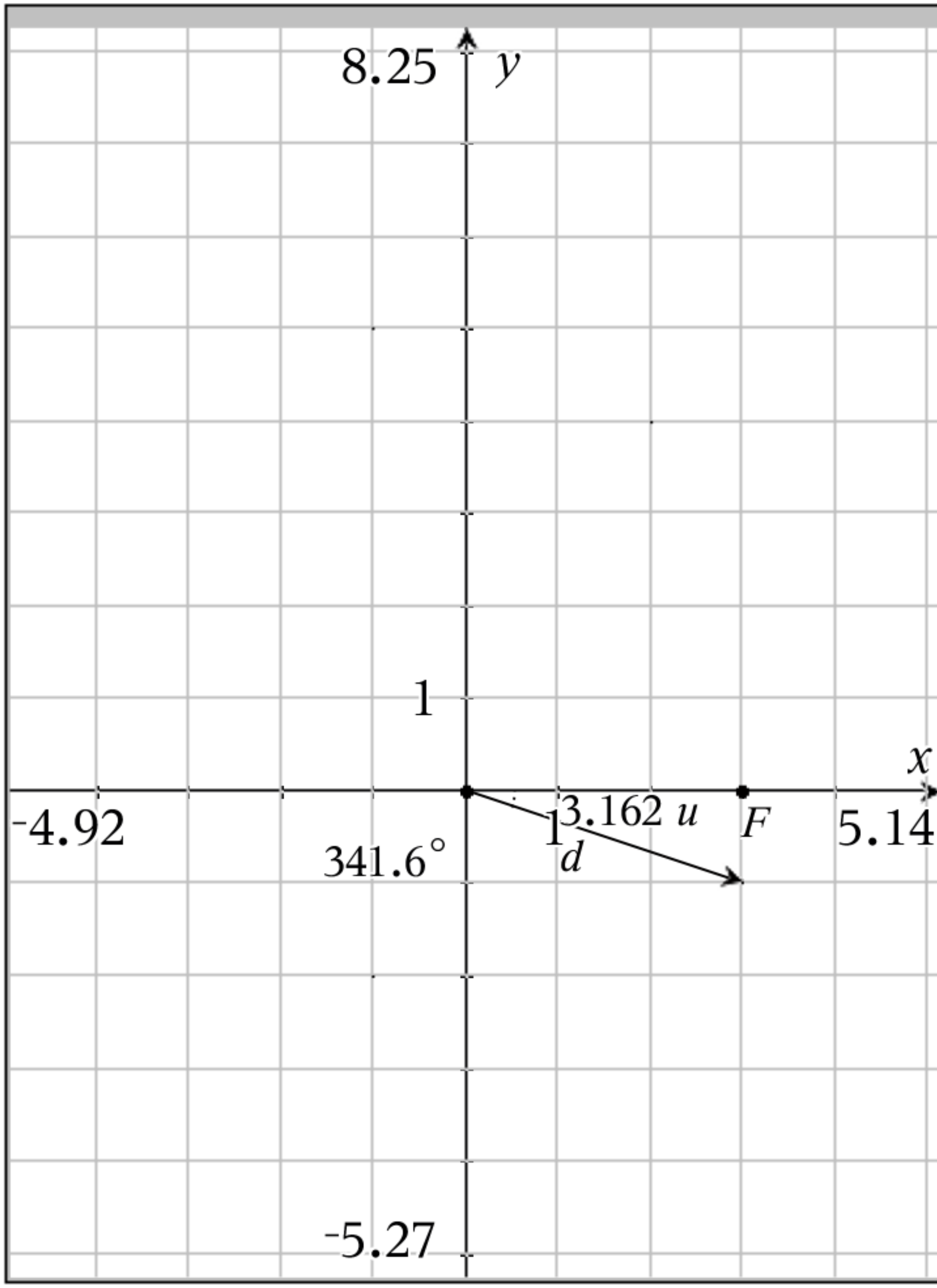
angle $= 180° + 63.4349°$

$= 243.435°$

vector c has magnitude $= \sqrt{5}$

$= \sqrt{5}$

≈ 2.23607



Find directional angle related to vector d

Since this is a quadrant 4 angle

(DOWN to RIGHT) there is REFERENCING

$d = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$\text{angle} = \tan^{-1}(-1/3) = -18.4349°$

this is in Quadrant 4

$\text{angle} = -18.4349°$

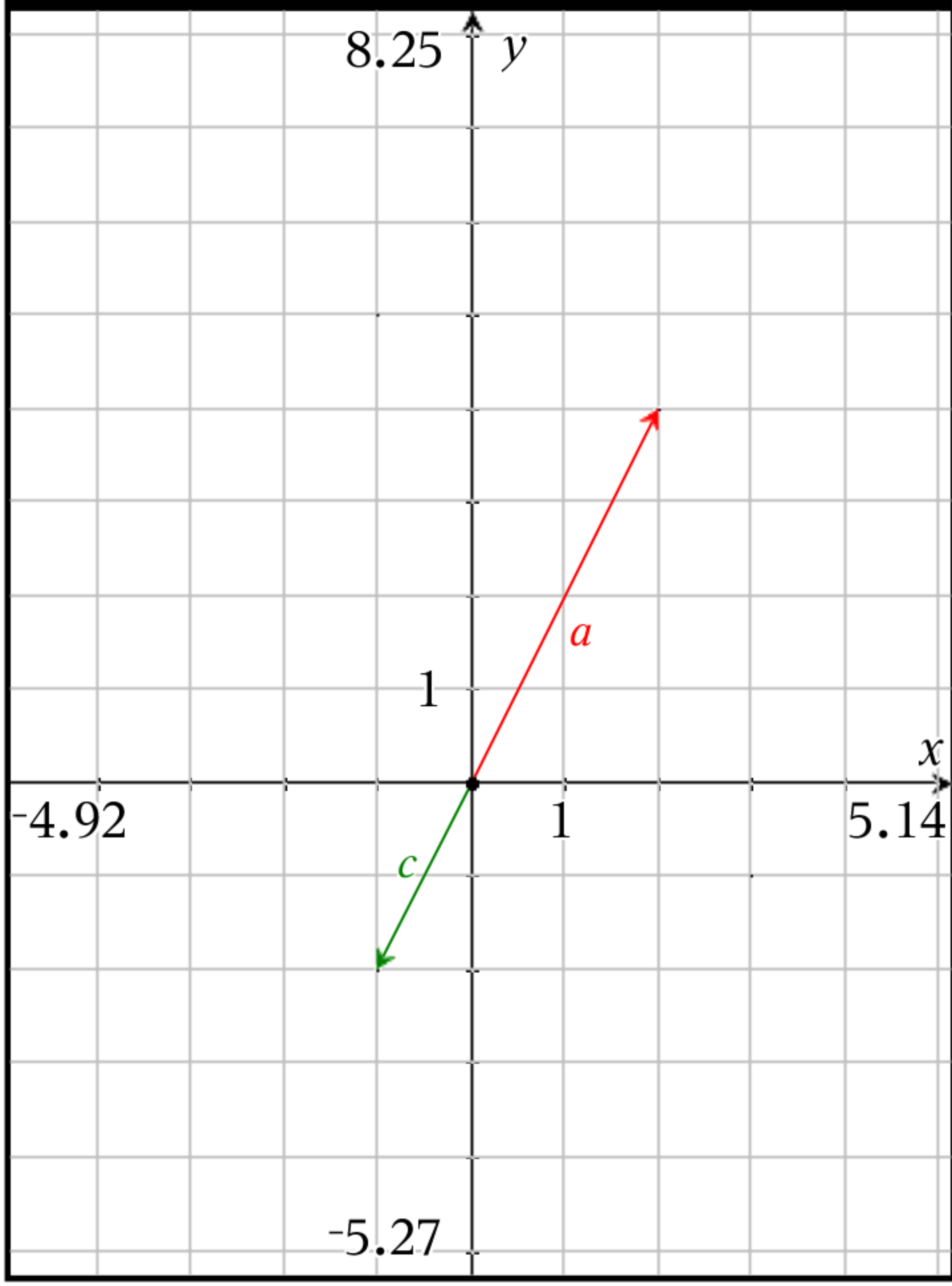
$\text{angle} = 360° + -18.4349°$

$= 341.565°$

vector b has magnitude $= \sqrt{10}$

$= \sqrt{10}$

≈ 3.16228



$a = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $c = \begin{bmatrix} ^-1 \\ ^-2 \end{bmatrix}$

dot product ac $= (2)(^-1) + (4)(^-2)$

$= ^-10$

magnitude of a = $\sqrt{20} = 2 \cdot \sqrt{5}$

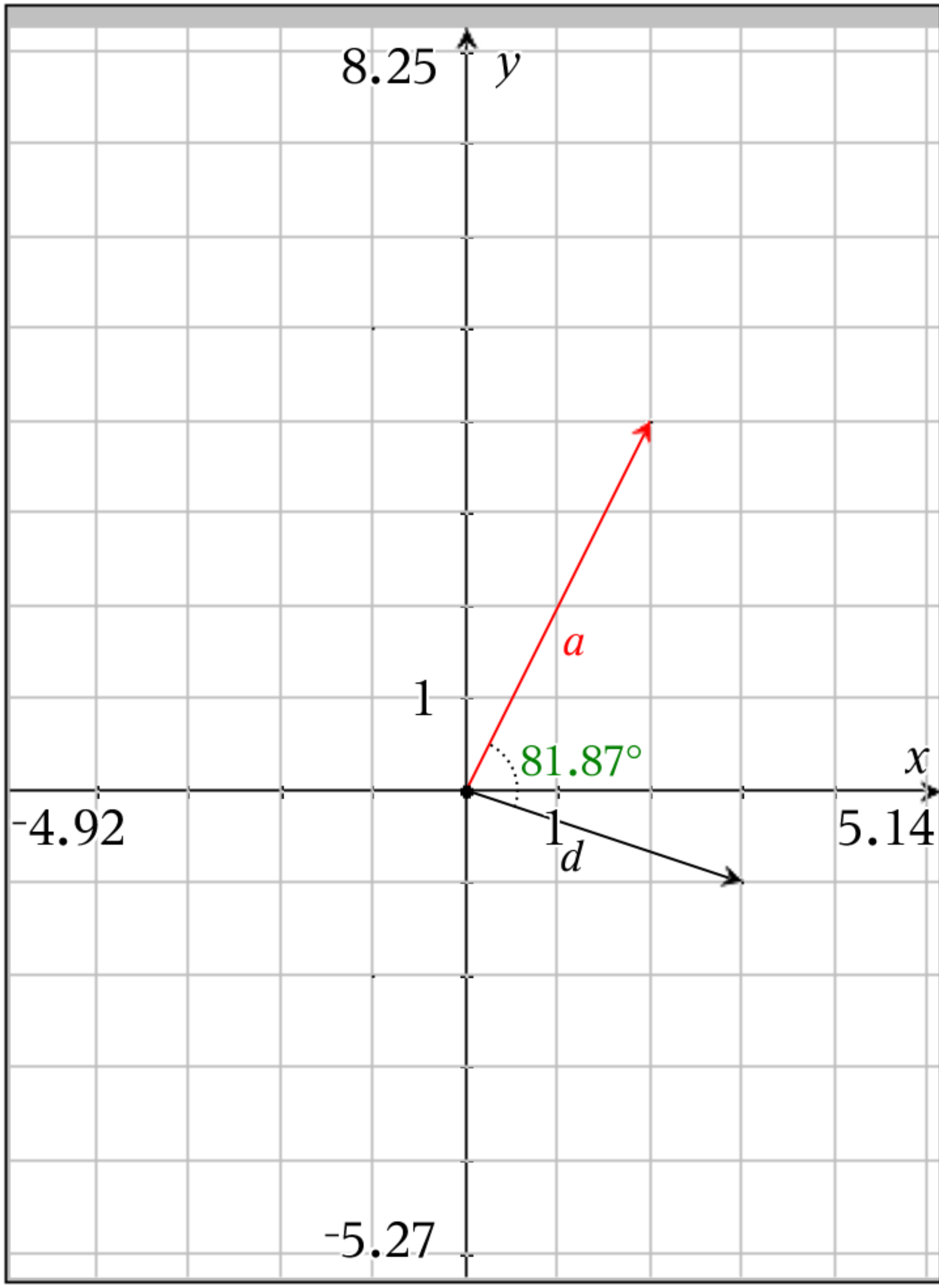
magnitude of c = $\sqrt{5}$

Since the dot product is NOT 0, these vectors meet at an angle other than 90°

$$\cos(\text{angle}) = \frac{dot \text{ product}}{[(magn\ of\ a)(magn\ of\ c)]}$$

$$= \frac{-10}{\sqrt{20} \cdot \sqrt{5}} = \frac{-10}{\sqrt{100}} = \frac{-10}{10} = -1$$

$$\text{angle} = \cos^{-1}\left(\frac{-10}{\sqrt{20} \cdot \sqrt{5}}\right) = 180°$$



$a = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $d = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

dot product ad $= (2)(3) + (4)(-1)$

$= 2$

magnitude of a = $\sqrt{20} = 2 \cdot \sqrt{5}$

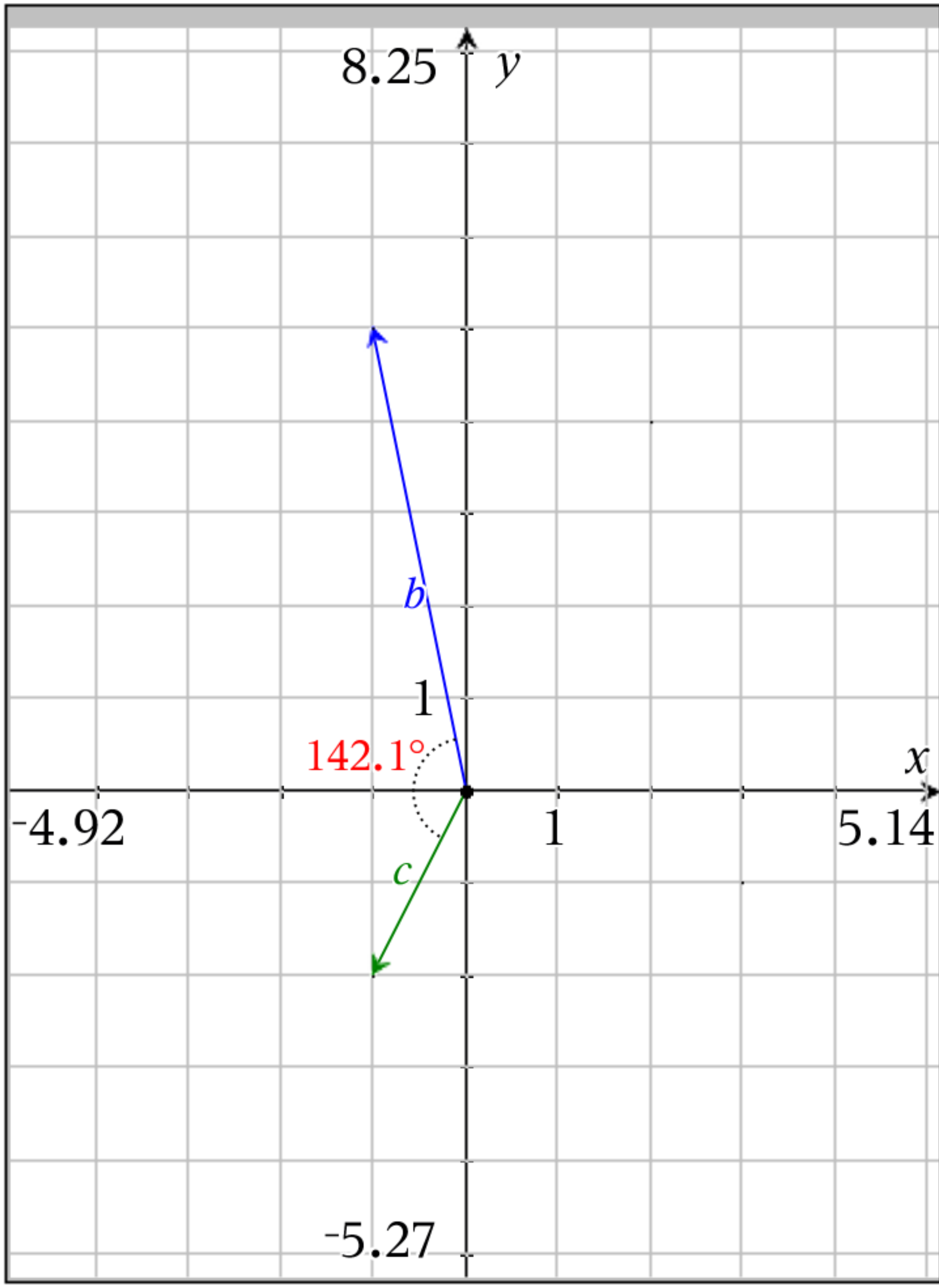
magnitude of d = $\sqrt{10}$

Since the dot product is NOT 0, these vectors meet at an angle other than 90°

$$\cos(\text{angle}) = \frac{dot \text{ product}}{\left[(magn\ of\ a)(magn\ of\ d)\right]}$$

$$= \frac{2}{\sqrt{20} \cdot \sqrt{10}} = \frac{2}{\sqrt{200}} = \frac{-\sqrt{2}}{10}$$

$$\text{angle} = \cos^{-1}\left(\frac{2.}{\sqrt{20} \cdot \sqrt{10}}\right) = 81.8699\ °$$



b = $\begin{bmatrix} ^-1 \\ 5 \end{bmatrix}$ c = $\begin{bmatrix} ^-1 \\ ^-2 \end{bmatrix}$

dot product bc = $(^-1)(^-1)+(5)(^-2)$

$= {}^-9$

magnitude of b = $\sqrt{26}$

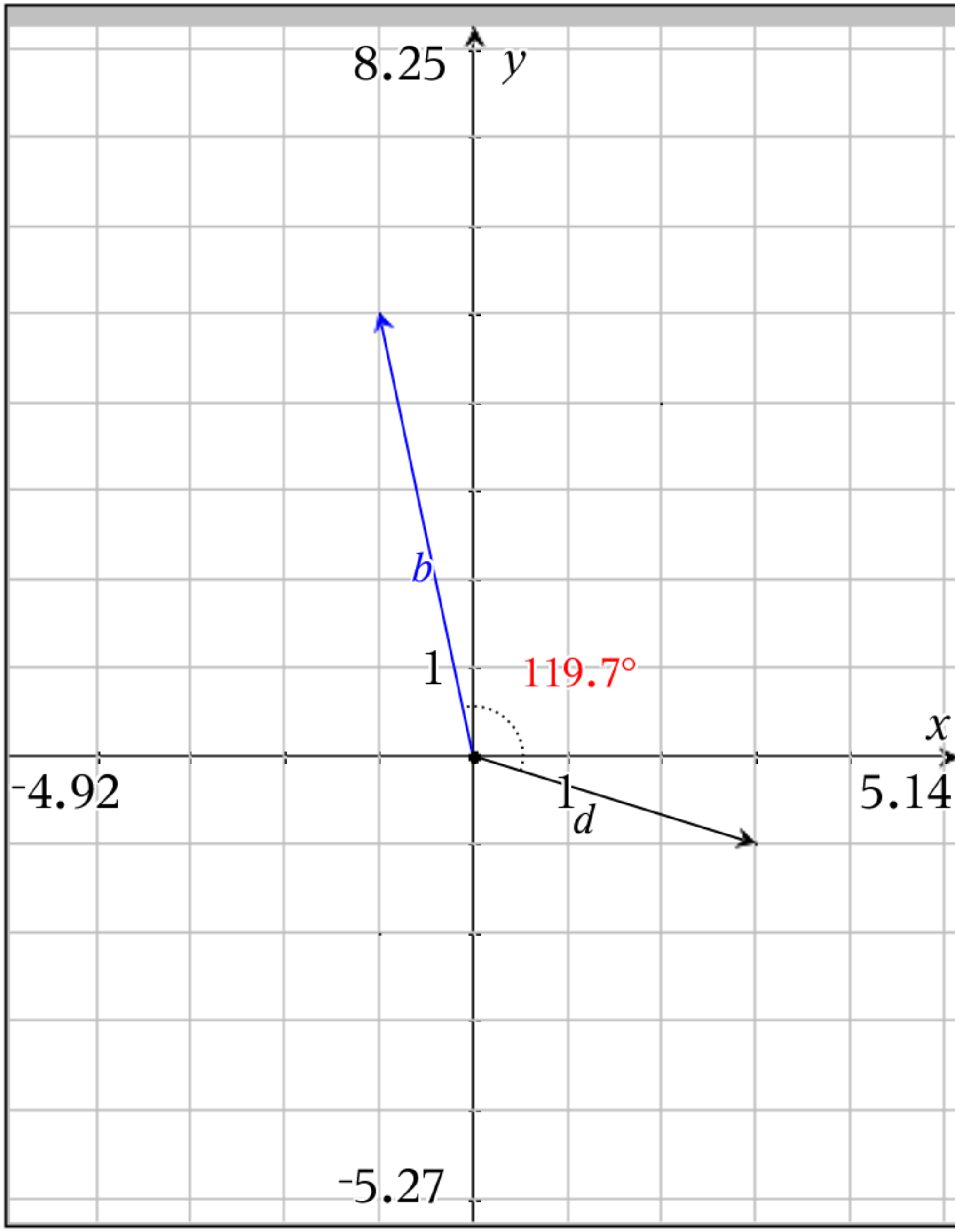
magnitude of c = $\sqrt{5}$

Since the dot product is NOT 0, these vectors meet at an angle other than 90°

$$\cos(\text{angle}) = \frac{\textit{dot}\text{ product}}{\left[(\textit{magn of } b)(\textit{magn of } c)\right]}$$

$$= \frac{-9}{\sqrt{26}\cdot\sqrt{5}} = \frac{-9}{\sqrt{130}} = \frac{^-9\cdot\sqrt{130}}{130}$$

$$\text{angle} = \cos^{-1}\left(\frac{-9.}{\sqrt{26}\cdot\sqrt{5}}\right) = 142.125\ °$$



$b = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$ $d = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

dot product bd $= (-1)(3) + (5)(-1)$

$= -8$

magnitude of b = $\sqrt{26}$

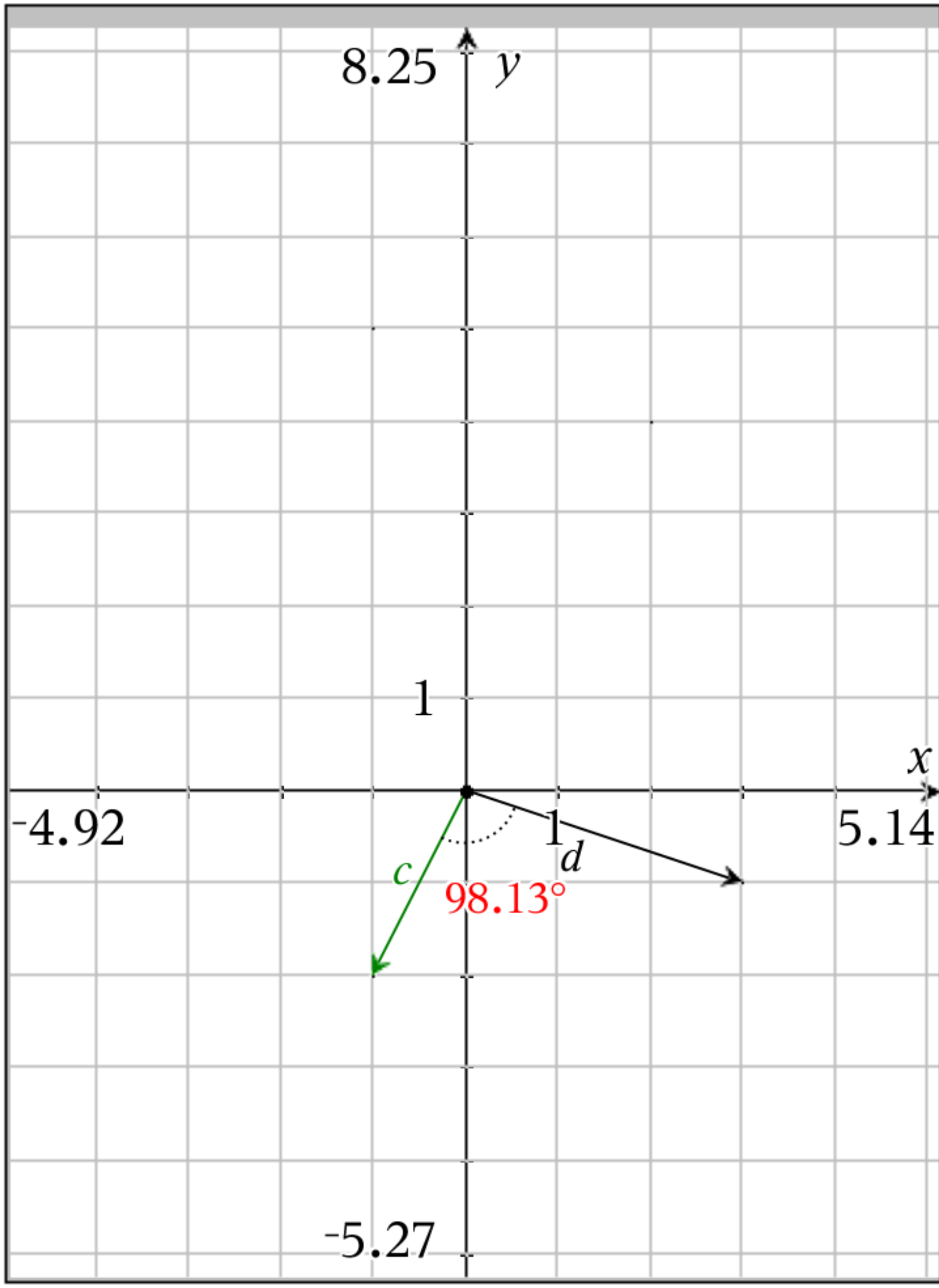
magnitude of d = $\sqrt{10}$

Since the dot product is NOT 0, these vectors meet at an angle other than 90°

$$\cos(\text{angle}) = \frac{\textit{dot}\ \text{product}}{\left[(\textit{magn of } b)(\textit{magn of } d)\right]}$$

$$= \frac{-8}{\sqrt{26}\cdot\sqrt{10}} = \frac{-8}{\sqrt{260}} = \frac{-4\cdot\sqrt{65}}{65}$$

$$\text{angle} = \cos^{-1}\left(\frac{-8.}{\sqrt{26}\cdot\sqrt{10}}\right) = 119.745\ °$$



$c = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ $d = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

dot product bd $= (-1)(3) + (-2)(-1)$

$= -1$

magnitude of c = $\sqrt{5}$

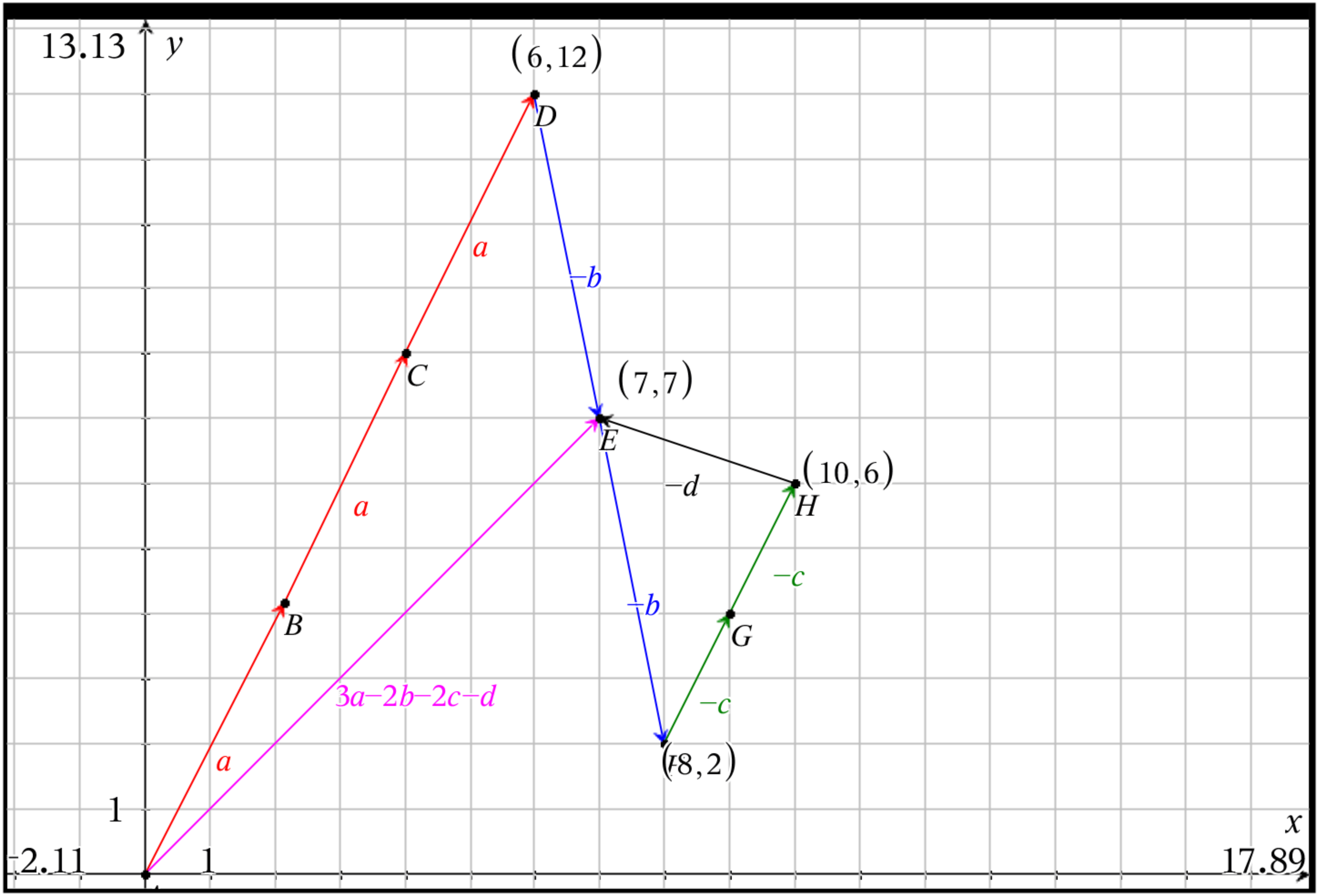
magnitude of d = $\sqrt{10}$

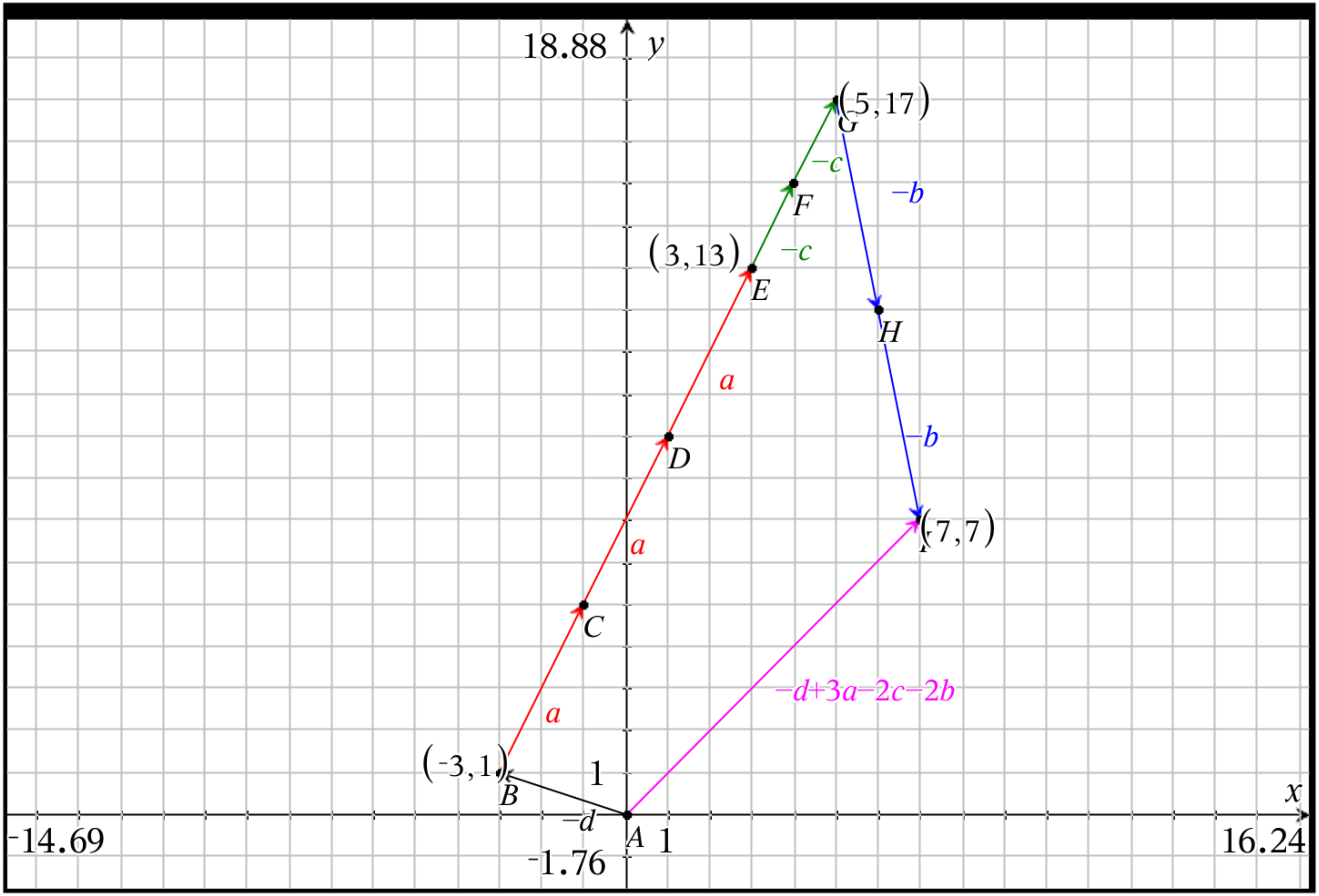
Since the dot product is NOT 0, these vectors meet at an angle other than 90°

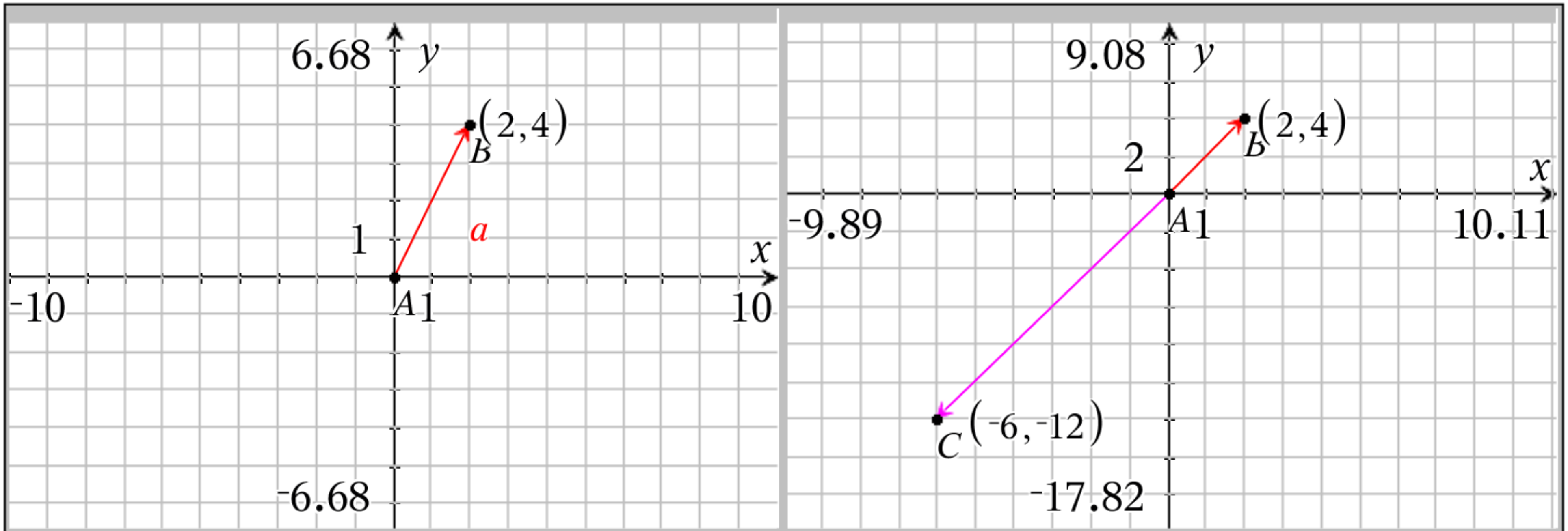
$$\cos(\text{angle}) = \frac{dot\ \text{product}}{\left[(magn\ of\ c)(magn\ of\ d)\right]}$$

$$= \frac{-1}{\sqrt{5}\cdot\sqrt{10}} = \frac{-1}{\sqrt{50}} = \frac{-\sqrt{2}}{10}$$

$$\text{angle} = \cos^{-1}\left(\frac{-1.}{\sqrt{5}\cdot\sqrt{10}}\right) = 98.1301\ °$$


13.13
y
(6,12)
D
a
−b
C
(7,7)
E
(10,6)
−d
H
a
−c
−b
B
G
3a−2b−2c−d
−c
a
(8,2)
1
x
-2.11
1
17.89


18.88
y
(5, 17)
G
−c
F
−b
(3, 13)
−c
E
H
a
D
−b
(7, 7)
a
C
−d+3a−2c−2b
a
(-3, 1)
1
B
x
−d
A
1
-14.69
-1.76
16.24



Recall all vectors that are parallel are scalar multiples of each other

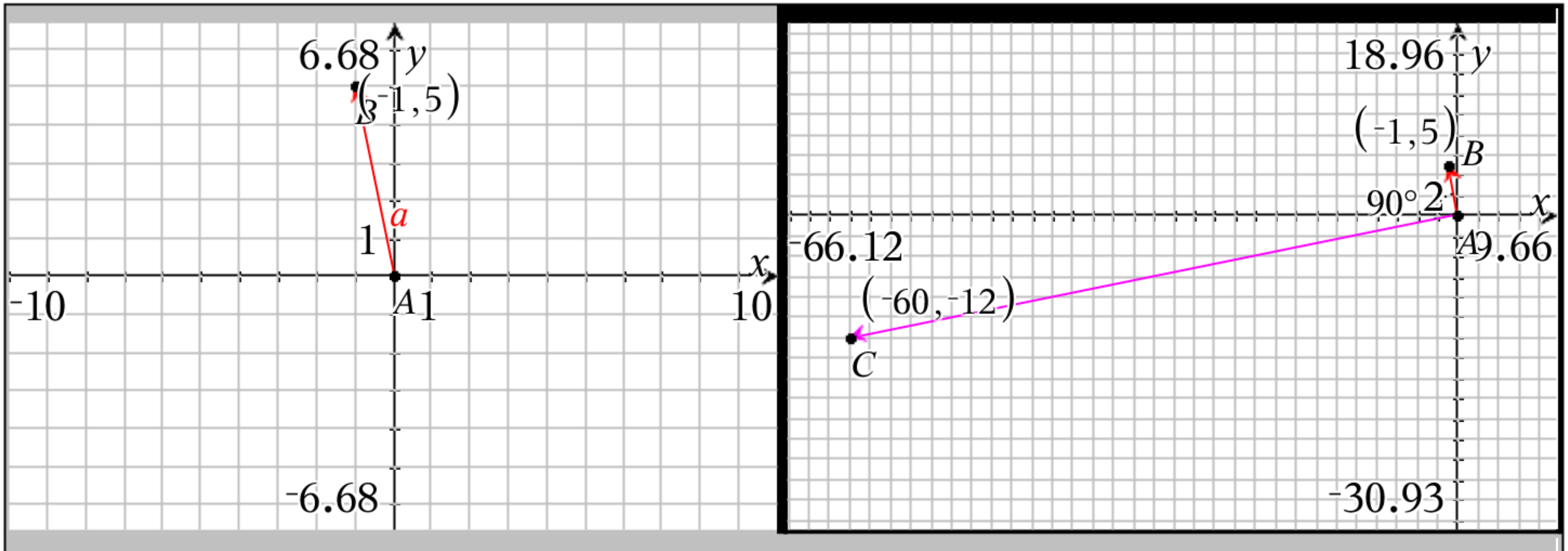
$a = \langle 2,4 \rangle$

if unknown vector $= \langle x,-12 \rangle$ is parallel,

then there is a scalar n such that $n\langle 2,4 \rangle = \langle x,-12 \rangle$

This implies $2n=x$ and $4n = -12$

we can solve for n using $4n = -12$ $n = \frac{-12}{4}$ ▸ -3 if $n = -3$ then $x = 2(-3) = -6$



Recall all vectors that are perpendicular if their dot product is 0

b = <−1,5>

if unknown vector =<w,−12> is parallel,

then the dot product is −1w+5(−12) = −1w−60

Set this equal to 0 and solve for w

−1w−60=0 tells us w = −60

Problem 2

Click here to add an application